1^{st} *Singapore Physics League*

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All diagrams were designed in-house. Some problems' diagrams involved the use of free external graphics acknowledged below:

- *Holding a Pen:* Image of fingers made by Smashicons from www.flaticon.com.
- *Self-Supporting Table:* Image of bucket made by Freepik from www.flaticon.com.
- *Tug of War:* Images of people pulling rope made by Freepik from www.flaticon.com.
- *Tilted Mirrors, Lights over Nuremberg, Mirage, Shortsighted Swimmer:* Image of eye made by Freepik from www.flaticon.com.

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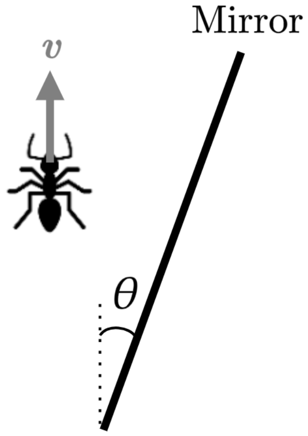
Problem 1: Deeply Reflective Ant (3 points)

An ant travels upwards at constant velocity $v = 4.0\ \mathrm{cm\ s^{-1}}$. Beside it, there is a mirror inclined at angle $\theta = 15°$ from the vertical. How fast does the ant perceive its mirror image to be moving?

Leave your answer to 2 significant figures in units of $\mathrm{cm\ s^{-1}}$.



Problem 2: Measuring Contact Angles

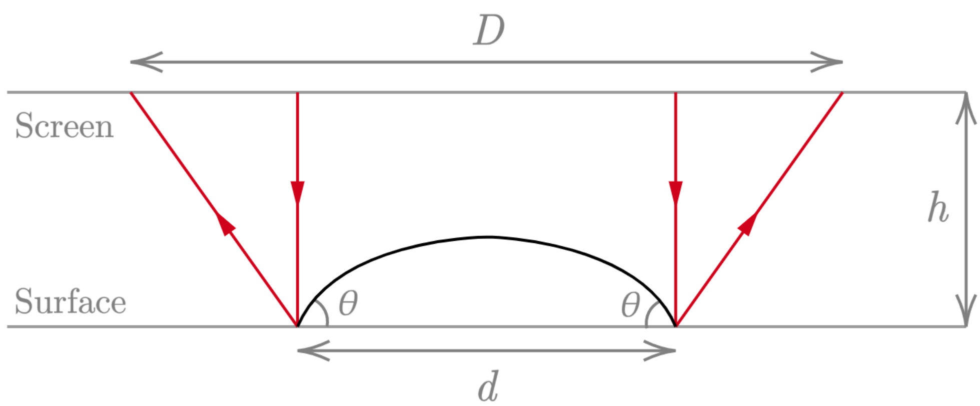
A drop of liquid of diameter $d = 5.0$ mm rests on a horizontal, flat solid surface. In order to find the contact angle θ between the drop and the surface, a collimated laser beam of identical diameter is shone vertically onto the liquid drop, and a translucent screen is set up horizontally at a height $h = 12$ cm above the drop. Assume that $\theta < 90°$. The diagram is not drawn to scale.

(a) Given that an image of diameter $D = 5.10$ cm is formed on the screen, find the contact angle θ between the liquid and solid surface.

Leave your answer to 2 significant figures in units of degrees. (3 points)

(b) The image has blurry edges, and the experimenters could only confidently determine the diameter of the image up to an uncertainty of $\delta D = 0.10$ cm. What is the corresponding uncertainty $\delta\theta$ in the measured contact angle?

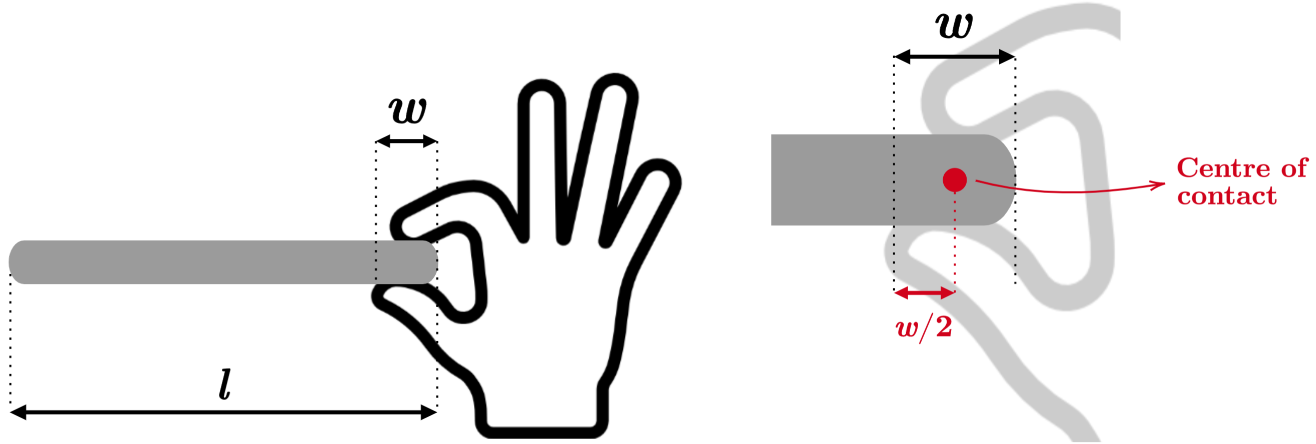
Leave your answer to 2 significant figures in units of degrees. (3 points)



Problem 3: Holding a Pen (3 points)

If you try to hold a stationary pen horizontal by clamping its right end with two fingers, your fingers have a tendency to rotate anticlockwise. Consider the pen to be a thin uniform stick of mass $m = 25$ g and length $l = 14$ cm, and suppose that your fingers have contact width $w = 2$ cm with the pen. Calculate the anticlockwise torque exerted by the pen on your fingers about the centre of contact of your fingers.

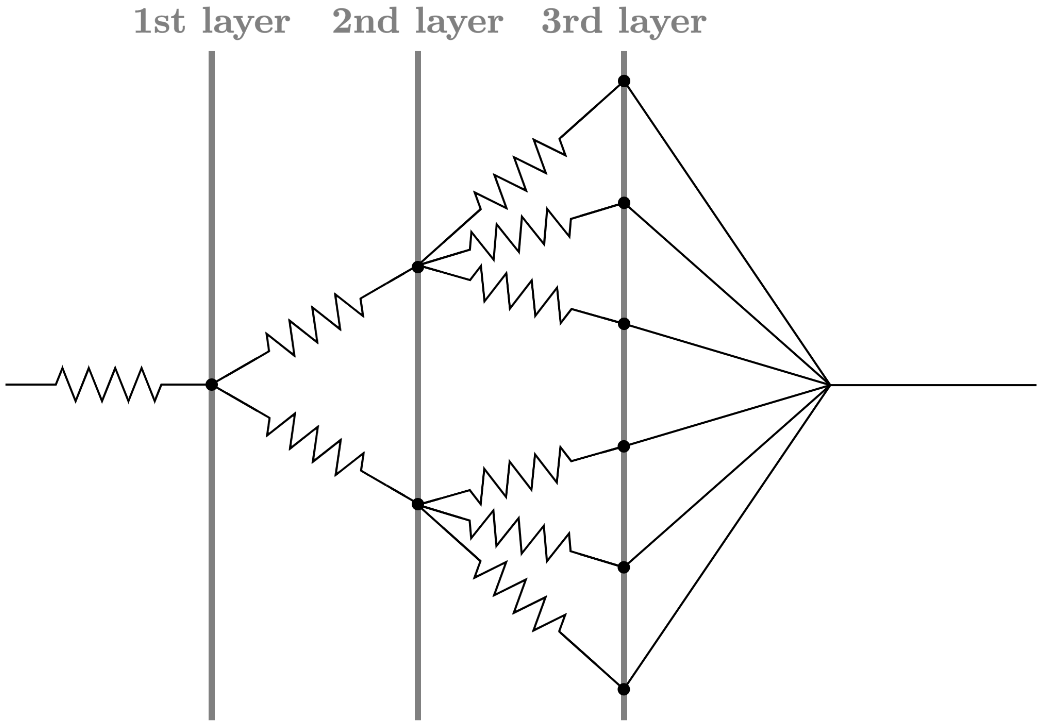
Leave your answer to 2 significant figures in units of N m.



Problem 4: Circuitree (4 points)

Consider a tree of resistors with a number of layers, with nodes on each layer that resistor branches are connected to. Each node at the n^{th} layer branches out to form $(n+1)$ nodes at the $(n+1)^{\text{th}}$ layer. Each branch has resistance $R = 10\ \Omega$. A tree with 3 layers has been illustrated below. What is the effective resistance, R_{eff}, of an infinite-layered variant of the tree (with an infinite number of layers)?

Leave your answer to 4 significant figures in units of Ω.



Problem 5: Cannonballs (4 points)

Two cannons, one situated on a cliff at coordinates $P = (80\text{ m},\ 60\text{ m})$ and the other at the origin, are aimed directly at each other. They fire simultaneously, projecting their cannonballs at identical speeds of $v = 50\text{ m s}^{-1}$. Let the coordinates of the position where the cannonballs collide be $(X\text{ m},\ Y\text{ m})$. Find $X + Y$. You can assume that the cannonballs are point objects, and that the trajectories of the cannonballs are not obstructed by the cliff.

Leave your answer to 3 significant figures.

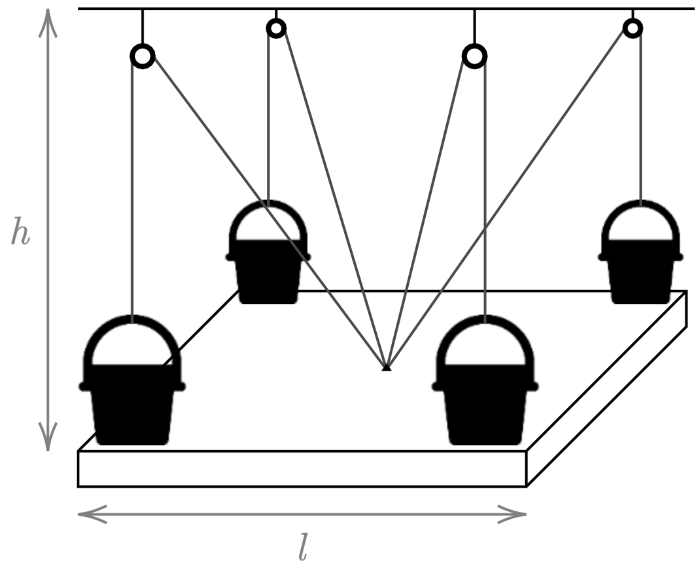
Problem 6: Self-Supporting Table (4 points)

"This table is held up by its own weight." - *r/mildlyinteresting*

A horizontal square table of side length $l = 50.0$ cm is located at vertical distance $h = 100$ cm below the ceiling. The table has mass M when it is empty. Four buckets, each of mass $m = 0.500$ kg, are placed on the corners of the table. Tied to each bucket is a taut light string that loops over a small support at the ceiling, whose other end is attached to the centre of the table. This way, the table stands by itself without requiring any legs.

What is the maximum value of M such that the table can be in equilibrium like that? Assume that the buckets are negligibly small, and that each support is directly above a bucket. Also take the distance between the supports and the ceiling to be negligible.

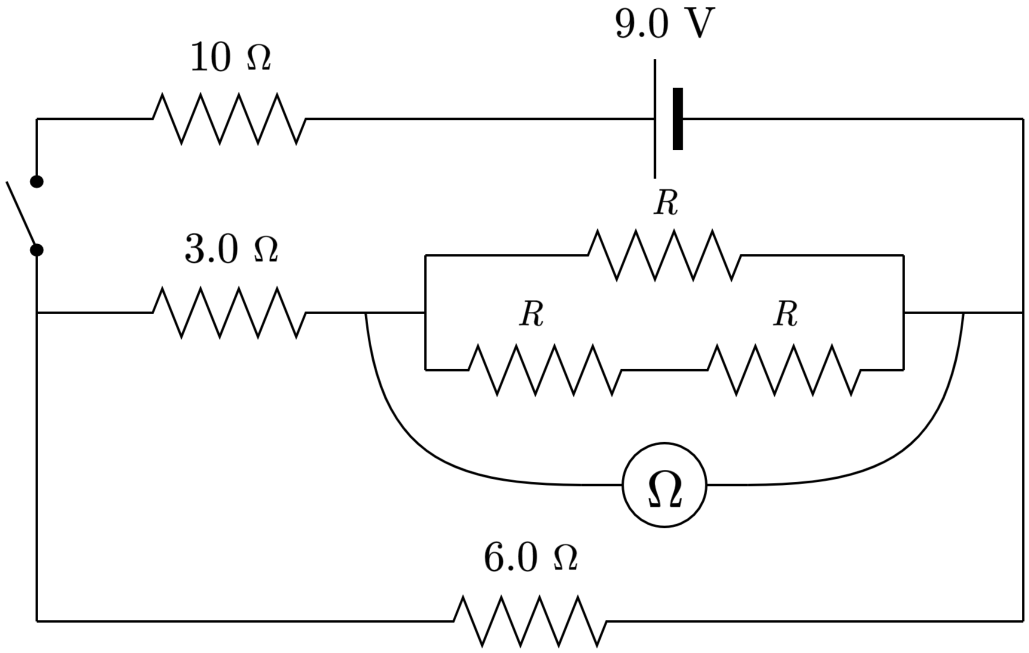
Leave your answer to 3 significant figures in units of kg.



Problem 7: Ohmmeter (3 points)

An ohmmeter is a device that measures the resistance between two points in a circuit. In the circuit below, the ohmmeter produces a reading of $r = 6.0\ \Omega$. Find R.

Leave your answer to 2 significant figures in units of Ω.



Problem 8: Distorted Stick (3 points)

A stationary observer on Earth sees a stick travelling at velocity v in the horizontal direction, with the stick angled at $\theta = 60°$ from the horizontal. However, a second observer who rides along with the stick views it to be oriented at $\phi = 30°$ from the same horizontal. Determine the value of v/c, where c is the speed of light.

Leave your answer to 3 significant figures.

Problem 9: A Confident Batter (5 points)

There was an oversight for this question. Calculations would yield a negative value of h_1, implying the ball would "phase through the ground", which is physically impossible in the context of the question. Therefore, this question is voided.

Joey is a world-class baseball player, admired and adored by many. However, he has suffered a recent string of poor performances. Thus, he arranges a batting session to prove that he is, indeed, the best baseball player of all time. Joey stands at a horizontal distance of $d = 20.0$ m from the pitcher. The pitcher throws a ball of mass $m = 142$ g at a speed $u_0 = 30$ m s^{-1} horizontally towards Joey, from a height $h_0 = 1.7$ m above the ground. Joey, being a strong, well-trained batter, hits the ball back at an angle $\theta = 32°$ above the horizontal, with a force of $F = 5000$ N for contact duration $t = 0.002$ s.

At what horizontal distance D away from Joey does the ball land after contact with the bat? Ignore effects of air resistance, and any rotational motion of the ball.

Leave your answer to 3 significant figures in units of m.

Problem 10: Straw Trick (4 points)

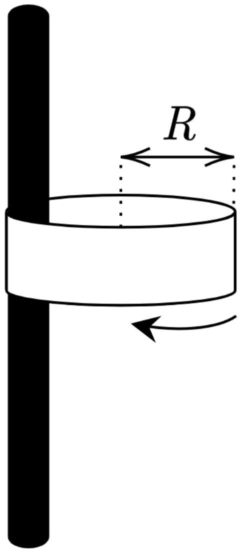
A vertical cylindrical straw of length $l = 15.0$ cm is partially immersed into a basin of water, causing the inside of the straw to be filled with water to depth $h = 10.0$ cm. Brian caps the top opening of the straw using his finger, and *slowly* raises the still-capped straw. By the time the whole straw has been lifted out of the basin of water, some of the original water column remains held within the straw. Calculate the percentage of water that is retained inside the straw. Assume that surface tension is negligible, and that the process is slow enough to be treated as quasistatic.

Leave your answer to 3 significant figures as a percentage. (For example, if you think the final answer should be 51.0%*, input your answer as* 51.0*)*

Problem 11: Ring around the Rod (3 points)

A vertical rod is placed through a thin ring of radius $R = 5.0$ cm, with coefficient of static friction $\mu_s = 0.30$ between the rod and the ring. When the ring is spun at a sufficiently high frequency f_{min} around the rod, it will remain at the same height for some time before it falls. Determine the value of f_{min}. Assume that the axis of the ring always remains vertical, and that the radius of the rod is negligible.

Leave your answer to 2 significant figures in units of s^{-1}.



Problem 12: Hopeless Romance (3 points)

Joshua is stranded alone on an island. In hopes of being found, he uses his radio to transmit signals. His radio operates on voltage $V = 12$ V and draws current $I = 1.0$ A.

As distance makes the heart grow fonder, Isabelle is determined to find him. She attempts to set up a satellite dish, whose geometry is perfectly hemispherical with radius R. The satellite receiver can detect the source of signals that have a minimum strength $P_{\text{min}} = 24$ μW.

Given that Joshua is located at a distance $x = 1600$ km away from Isabelle, what is the minimum radius R of the satellite dish that Isabelle must construct in order to locate him? Assume that Joshua's radio is 100% efficient in emitting signals, and behaves like a point wave source that transmits signals uniformly in all directions. You may neglect the Earth's curvature, and any obstructions in the path of the signals.

Leave your answer to 2 significant figures in units of m.

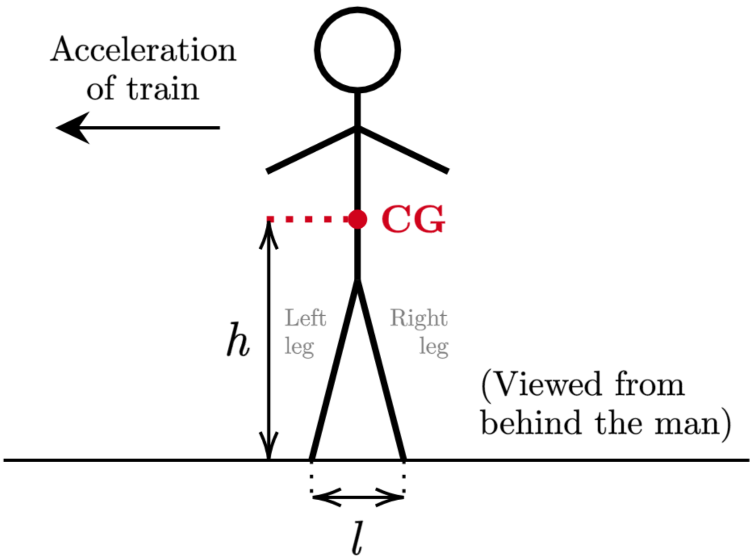
Problem 13: Accelerating MRT (4 points)

A man is in an MRT train that was originally at rest. He stands straight and faces the right side of the train, with his legs at horizontal distance $l = 0.40$ m apart from each other. The centre of gravity of the man is at vertical distance $h = 0.90$ m above the floor. The train starts travelling with uniform acceleration $a = 1.5$ m s^{-2} forward on a level track. Assuming that the man continues standing straight, what percentage of the man's total weight is transferred between his legs?

Leave your answer as positive if you think that weight is transferred from his left leg to his right leg.

Leave your answer as negative if you think instead that weight is transferred from his right leg to his left leg.

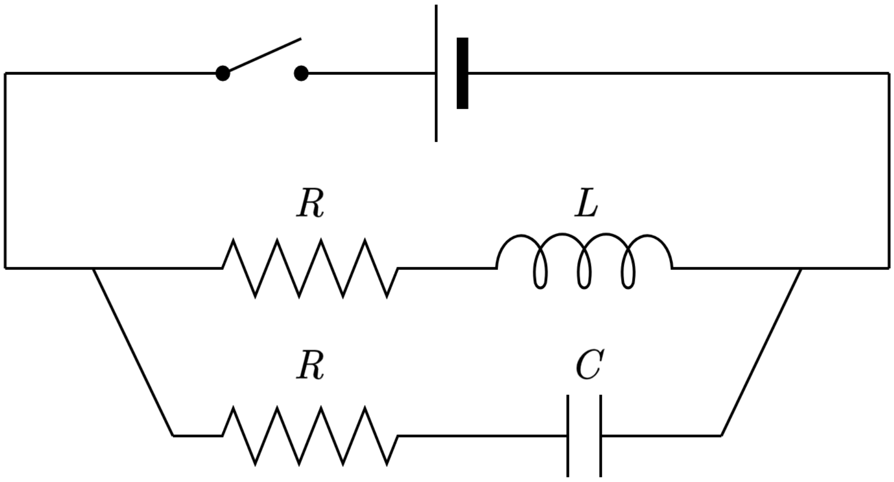
Leave your answer to 3 significant figures as a percentage. (For example, if you think the final answer should be 51.0%*, input your answer as* 51.0*)*



Problem 14: Current Stabiliser (4 points)

In a highly inductive circuit with inductance $L = 15.0$ H and resistance $R = 8.00\ \Omega$ connected to a battery, it takes some time after closing the switch for the current supplied by the battery to reach its maximum value. This may be avoided by connecting a much less inductive branch (which can be assumed to have zero inductance) with the same resistance $R = 8.00\ \Omega$ and an initially uncharged capacitor with capacitance C in parallel with the original circuit, as drawn below. This way, upon closing the switch, the current drawn from the battery may instantly reach its final value and stay there indefinitely. Determine the value of C required to make this possible.

Leave your answer to 3 significant figures in units of F.

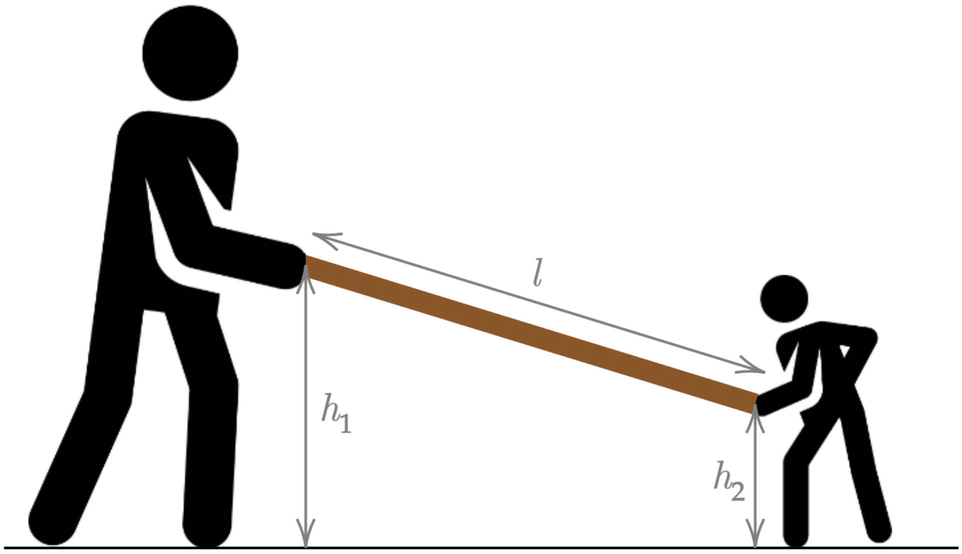


Problem 15: Tug of War (4 points)

Alice and Bob engage in a game of Tug of War. Alice has mass $m_1 = 20$ kg and pulls the rope at height $h_1 = 1$ m above the ground. Bob pulls the other end of the rope at a height of $h_2 = 0.2$ m above the ground. The rope has length $l = 2$ m and negligible mass. The coefficient of static friction between the feet of either person and the ground is $\mu = 0.5$.

Find the minimum mass of Bob, m_2, such that he can beat Alice.

Leave your answer to 3 significant figures in units of kg.



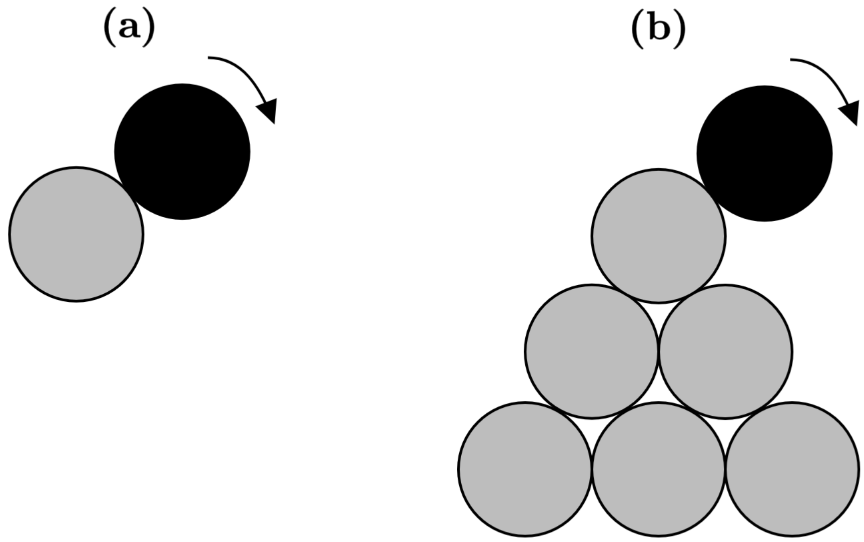
Problem 16: Rolling Pennies

(a) One penny rolls without slipping around another identical penny which is held stationary. How many revolutions does the penny complete relative to you, when it returns to its starting position?

Leave your answer to 2 significant figures. (2 points)

(b) Six pennies are arranged in the triangle formation shown below and held stationary. A seventh penny rolls around the six pennies without slipping. How many revolutions will it complete, relative to you, when it returns to its starting position?

Leave your answer to 2 significant figures. (3 points)



Problem 17: Releasing a Spring

An ideal light spring with spring constant $k = 25$ N m^{-1} is placed on the ground with its axis vertical. The spring's top end is attached to a massless plate, while its bottom end is fixed onto the ground. A mass $m = 80$ g is placed on the plate and pressed down, compressing the spring by distance x compared to its unstretched state. The mass is then released from rest.

(a) If the relaxed (unstretched) length of the spring were $l = 10$ cm, find the *maximum* value of x such that the subsequent motion of the mass can be exactly described as simple harmonic motion.

Leave your answer to 2 significant figures in units of cm. (3 points)

(b) If the relaxed (unstretched) length of the spring were $l = 5$ cm, find the *minimum* value of x such that the subsequent motion of the mass can be exactly described as simple harmonic motion.

Leave your answer to 2 significant figures in units of cm. (3 points)

Problem 18: Gravitational Blueshift (4 points)

Photons have an effective gravitational mass that is always equal to their momentum divided by their speed. Suppose a photon with some initial frequency f is emitted from a satellite orbiting $x = 20000$ km above the surface of the Earth. When it reaches the surface of the Earth, its new frequency is f'. By treating the photon as a massive particle with mass equal to its effective gravitational mass, find the natural logarithm of fractional change in frequency of the photon, $\ln \frac{f'-f}{f}$. Take the radius of the Earth to be $R = 6370$ km, and the mass of the Earth to be $M = 5.97 \times 10^{24}$ kg. You may assume the photon travels at constant speed c.

Leave your answer to 3 significant figures.

Problem 19: Eddy Current Braking (3 points)

A long, thin, hollow, conducting cylinder of radius a and length $l \gg a$ is rotated about its axis at angular velocity ω in a uniform magnetic field B normal to its axis. This induces the formation of eddy currents on the cylinder and results in a retarding torque. If the cylinder has sheet resistance s and permeability μ, then the torque τ can be written in the form:

$$\tau = \frac{4\pi\omega s a^n B^2 l}{4s^2 + \omega^2\mu^2 a^2}$$

Find the constant n.

Leave your answer to 3 significant figures.

Problem 20: Heating a Blackbody (4 points)

A heat pump of ideal efficiency consumes power $W = 90.7$ kW from an external supply and draws from a reservoir held at constant temperature $T_0 = 200$ K. It is used to heat a blackbody that simultaneously loses heat through radiation. Given that the total surface area of the blackbody over which radiation occurs is $A = 125$ m^2, find the equilibrium temperature T of the blackbody.

Leave your answer to 3 significant figures in units of K.

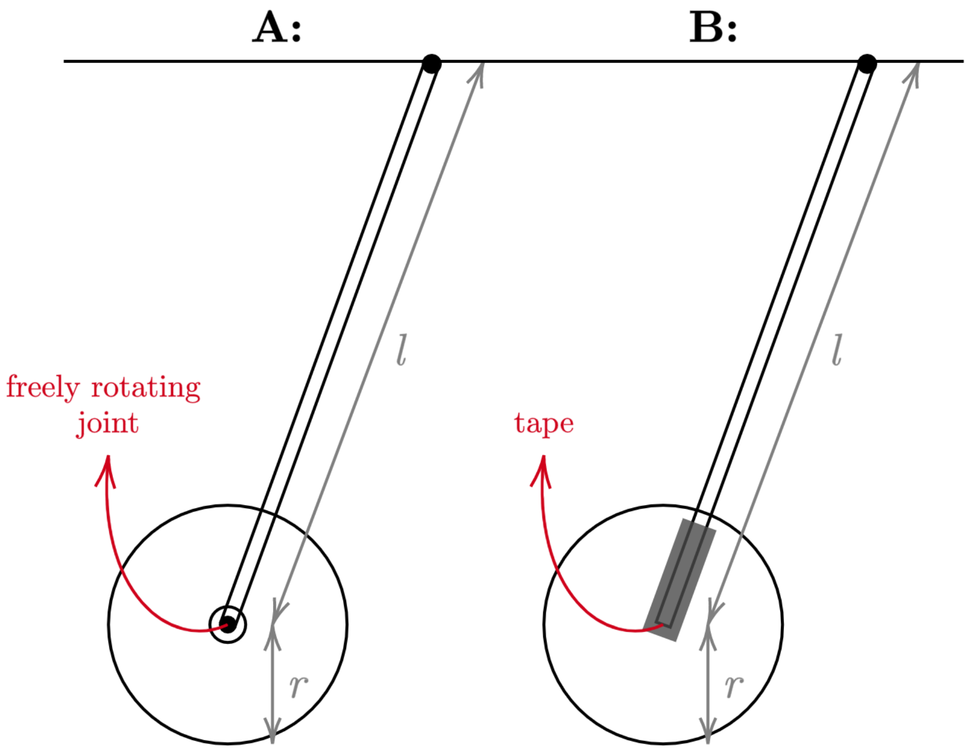
Problem 21: Disc Pendulums (4 points)

Two pendulums, A and B are pivoted from the ceiling. Each pendulum is made from a flat disc of mass $m = 1.0$ kg and radius $R = 0.2$ m which is joined at its centre to a massless rod of length $l = 0.5$ m. The oscillations occur in the plane of the disc.

In pendulum A, the rod only contacts the disc at a single point at the disc's centre, via a joint that can rotate without friction. On the other hand, in pendulum B, the rod is taped securely onto the disc with many points of contact.

Let the periods of pendulum A and B be T_A and T_B respectively. For oscillations of small angles, find the ratio T_A/T_B. Assume that there are no dissipative forces in the system.

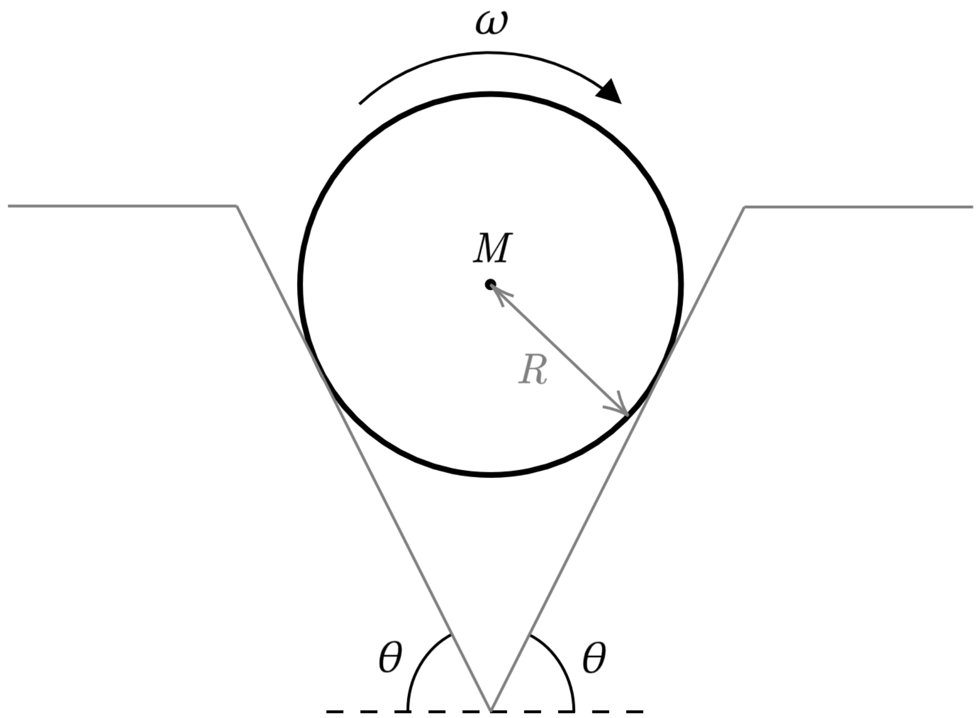
Leave your answer to 3 significant figures.



Problem 22: Cylinder in Groove (4 points)

A cylinder is placed in a V-shaped groove, with its axis parallel to the groove. Each side of the groove makes an angle of $\theta = 30^\circ$ with the horizontal. The cylinder has mass $M = 500$ g and radius $R = 25$ cm, and is driven to rotate at a constant angular velocity $\omega = 30$ rad s^{-1}. The coefficient of friction between the cylinder and each surface is $\mu = 0.35$. Calculate the external torque τ required to rotate the cylinder.

Leave your answer to 2 significant figures in units of N m.



Problem 23: Strange Release

A light inextensible string of length $l = 5$ cm has its top end fixed at point P and its bottom end tied to a mass. Treat air resistance to be negligible.

(a) The mass is held at horizontal distance $x_0 = 3$ cm from P, and vertical distance $y_0 = 4$ cm below P. It is then released from rest. What is the velocity of the mass at the later instant when the string is vertical?

Leave your answer to 2 significant figures in units of cm s^{-1}. (2 points)

(b) Solve again for $x_0 = 2$ cm and $y_0 = 4$ cm.

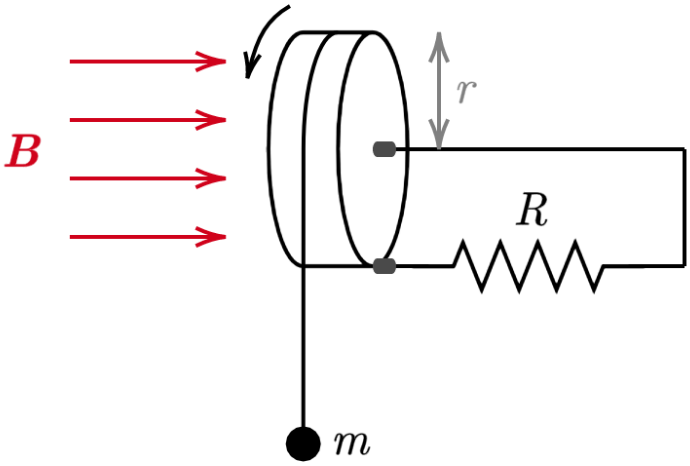
Leave your answer to 2 significant figures in units of cm s^{-1}. (4 points)

Problem 24: Faraday's Wheel (4 points)

A conducting wheel of radius $r = 0.50$ m is mounted on a horizontal axle, and connected in series with a resistor of resistance $R = 1.2\ \Omega$ as shown in Figure 1. The resistor is connected via stationary frictionless brush contacts at its ends. One end touches the conducting central axis, while the other end touches the edge of the wheel. A uniform magnetic field of $B = 1.8$ T is directed along the rotational axis of the wheel.

A thin insulating string is wound around the circumference of the wheel, from which a weight of mass $m = 5.0$ kg is hung. Upon release, the mass begins to fall and eventually reaches a constant velocity. Find P, the power dissipated by the resistor in this steady state. You can assume that the string does not slip in this process.

Leave your answer to 2 significant figures in units of W.

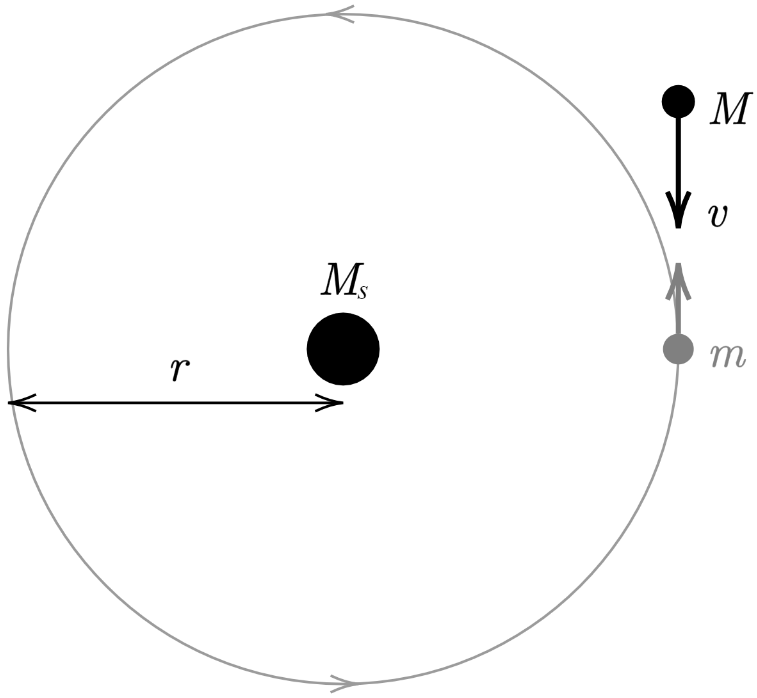


Problem 25: Colliding Asteroids (4 points)

An asteroid of mass $m = 3.35 \times 10^5$ kg is initially in a circular orbit of radius $r = 1.60 \times 10^{12}$ m about the sun. Another asteroid of mass M travelling in the opposite direction strikes it with velocity $v = 1.50 \times 10^5$ m s^{-1}. Both asteroids subsequently stick together to form a new body. Given that the sun's mass is $M_s = 1.99 \times 10^{30}$ kg, what is the minimum mass M required for the resulting body to enter an unbound orbit?

Leave your answer as 0 if you think the resulting body will always be in a bound orbit for all M.

Leave your answer to 2 significant figures in units of kg.



Problem 26: Thermodynamic Board

A mass $m = 0.100$ kg sits on a very long board of mass $M = 0.500$ kg, which rests on horizontal frictionless ground. The mass and the board start out at the same temperature $T_i = 20.0°\text{C}$, and share the same specific heat capacity $c = 400\ \text{J kg}^{-1}\ \text{K}^{-1}$. Kinetic friction exists between the mass and the board, with coefficient $\mu = 0.800$. Now, the mass is imparted a small horizontal velocity u. Neglect any energy transfer to the surroundings (including the ground), and assume that the mass does not rotate.

(a) The final change in temperature of the mass can be written as αu^2, where α is a numerical constant. Calculate the value of α.

Leave your answer to 2 significant figures in SI units. (3 points)

If you were unable to solve (a), you may use $\alpha = 1$ for (b). However, the maximum attainable score for (b) will be reduced by 1 point.

(b) The final change in entropy of the universe can be approximated as βu^2, where β is a numerical constant. Calculate the value of β.

Leave your answer to 2 significant figures in SI units. (3 points)

Problem 27: Enlarged Merlion (5 points)

The Merlion has height $h = 8.6$ m. A water pump, which operates at power P with ideal efficiency, delivers water to the Merlion at a constant rate, via a pipe that transports water at uniform speed from a stationary reservoir at ground level to the top of the Merlion. There, water is ejected horizontally, forming the iconic fountain. The horizontal range of the Merlion's fountain is $l = 13$ m.

Authorities plan to double the fountain's horizontal range. To perform this, the operating power of the pump is to be increased to P'. Find the ratio P'/P.

Leave your answer to 3 significant figures.

Problem 28: Flat Earth Experimentalists (3 points)

In the 2018 documentary *Behind the Curve*, Flat Earthers devised an experiment to test the Earth's curvature. Two measuring posts are placed at geographical distance $l = 2.50$ km apart. The posts are adjusted to be perfectly vertical using a spirit level. A laser is calibrated such that the emitted beam intersects each post at height $h = 3.00$ m above the ground. An identical vertical post is then placed at the exact midpoint of the two existing posts. Keeping the laser's calibration unchanged, the intersection between the laser beam and this newly added post is measured to be at height H above the ground. What is the percentage deviation of measured H from the prediction of Flat Earth theory? You may treat the Earth to be spherical with radius $R = 6371$ km.

Leave your answer to 2 significant figures as a percentage. (For example, if you think the final answer should be 51%*, input your answer as* 51*)*

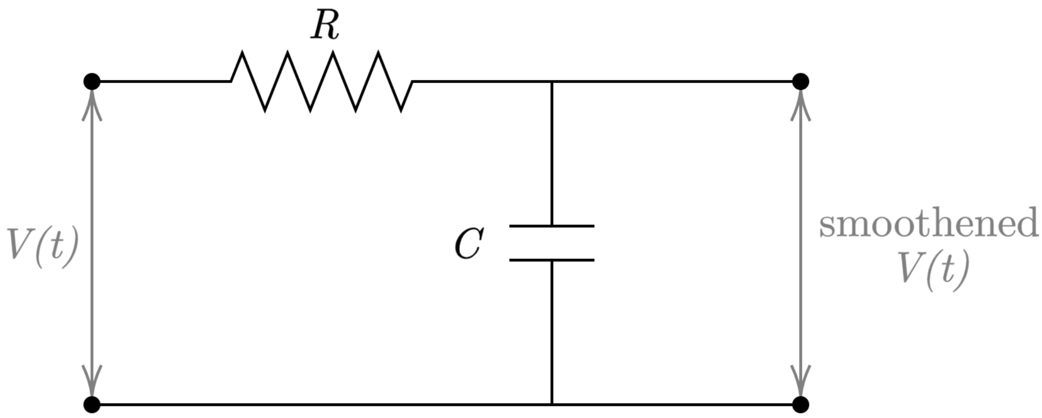
Your answer should be positive.

Problem 29: Noisy Voltage (5 points)

When the operation of a regular direct voltage source is affected by significant periodic fluctuations, its output voltage is best described as a sinusoidal variation centred around a positive value, taking the form $V(t) = A\cos(Bt) + C$ for positive constants A, B, C. As a result of these fluctuations, the ratio $V_{\max}/V_{\min} = 1.5$, where $V_{\max}$ and $V_{\min}$ are the maximum and minimum voltages of the source respectively. The fluctuations come at frequency $f = 200$ Hz, which may be taken to be the frequency of $V(t)$.

In efforts to reduce this noise, the voltage of the source is now fed through a low-pass filter (shown below). This comprises a resistor $R = 15.0\ \Omega$ and a capacitor $C = 120\ \mu$F. The new output voltage, taken as the voltage across the capacitor, is a smoothened version of $V(t)$ with a lowered $V_{\max}/V_{\min}$ ratio. Find this ratio.

Leave your answer to 3 significant figures.



Problem 30: Nailed It (4 points)

A uniform rod lies flat on a horizontal table. One of its ends is nailed to the table. This nailed end serves as a pivot about which the rod is free to rotate. On its other end, a horizontal force $|\vec{F}| = 60$ N is applied perpendicular to the length of the rod. What is the force exerted by the nail on the rod at this instant? Assume that there is no friction between the rod and the nail or the table.

Leave your answer as positive if you think the force by the nail on the rod acts in the direction of $\vec{F}$.

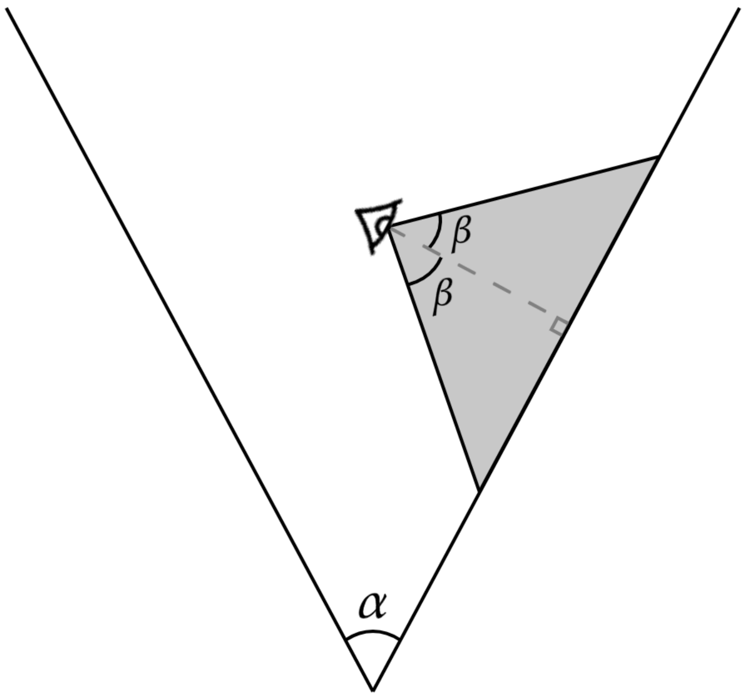
Leave your answer as negative if you think the force by the nail on the rod opposes the direction of $\vec{F}$.

Leave your answer to 2 significant figures in units of N.

Problem 31: Tilted Mirrors (4 points)

Two large plane mirrors are oriented at an angle $\alpha = 12°$ with respect to each other. A man stands halfway between the two mirrors and orients himself such that his line of sight is perpendicular to one of the mirror surfaces. The field of view of the man is $\beta = 39°$ towards each side. How many images of himself does he see?

Leave your answer as an integer.



Problem 32: Dazzling Supernovae (3 points)

In observational astronomy, the magnitude of an object is a dimensionless measure of the intensity of light from it. Magnitude is measured on a logarithmic scale. For every 100-fold increase in light intensity, the magnitude will decrease by 5 units. Therefore, if light from object A is 100 times more intense than light from another object B whose magnitude is 3.2, the magnitude of A will be $3.2 - 5 = -1.8$.

The apparent magnitude refers to the magnitude of an object as viewed from Earth while the absolute magnitude refers to the magnitude of an object as viewed from a fixed distance of 32.6 light years.

A type-1A supernova has an absolute magnitude of $M = -19.3$. If such a supernova occurred 1000 light years from Earth, what would its apparent magnitude, m, be? Assume that nothing obstructs the line of sight between the supernova and Earth.

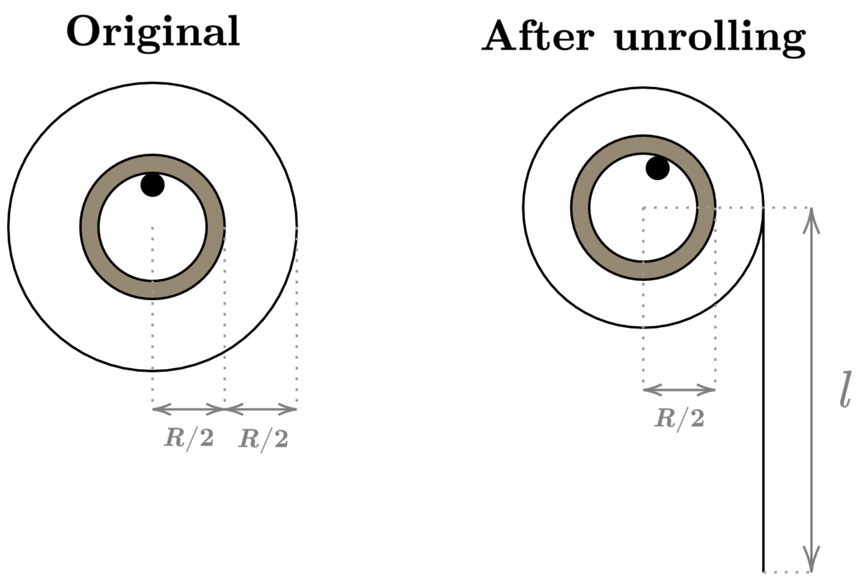
Leave your answer to 3 significant figures.

Problem 33: A Toilet Nightmare (5 points)

A toilet roll with initial outer radius R is made by winding length L of paper with uniform mass per unit length λ, around a cardboard tube of radius $r = R/2$ and negligible thickness and mass. The cardboard roll in the centre experiences friction with the toilet roll holder, a thin rod upon which the inner surface of the cardboard tube rests. The static friction is characterised by coefficient $\mu_s = 0.300$.

The roll is unrolled by grabbing its end and pulling it downwards slowly. When a minimum length, l, of toilet paper has been unrolled, it is released and the remaining toilet paper unrolls spontaneously. Find l/L. Assume that there is sufficient friction between the paper and the roll such that the paper never slips relative to the roll. You may also assume that the roll is elevated such that the paper never touches the floor.

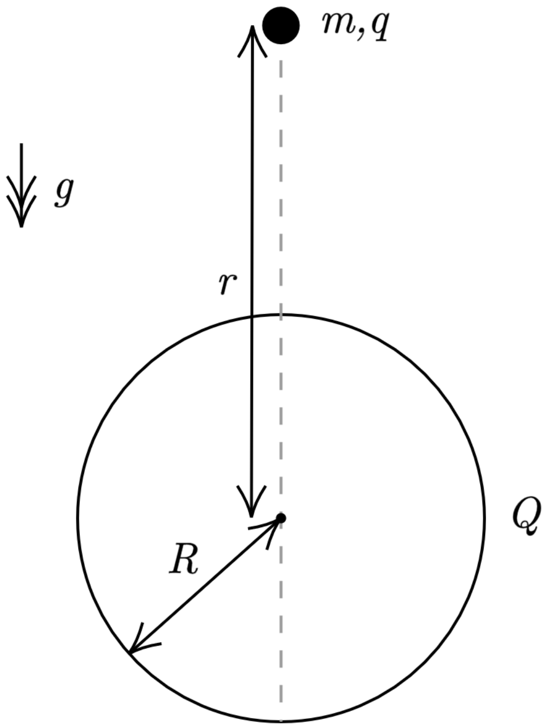
Leave your answer to 3 significant figures.



Problem 34: A Charged Conducting Sphere (5 points)

A conducting sphere of radius $R = 0.500$ m has an unknown charge Q initially distributed uniformly over its surface. To find this unknown charge, a particle with known mass $m = 1.00$ g and charge $q = 0.520$ μC is placed above the sphere and constrained to move along a vertical line passing through the centre of the sphere. It was found that the particle settled into a stable equilibrium at a distance $r = 2R$ above the sphere's centre. Find the charge Q on the conducting sphere.

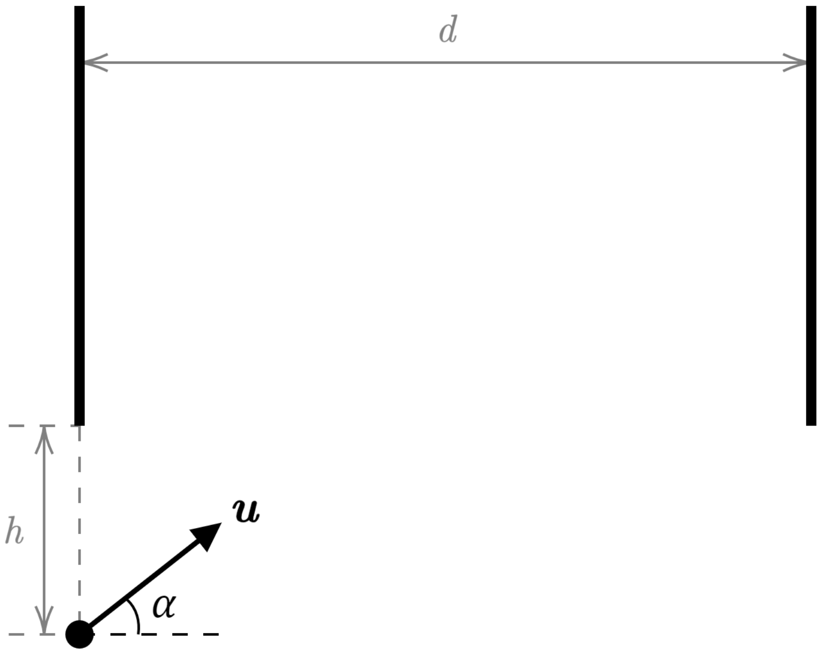
Leave your answer to 3 significant figures in units of μC.



Problem 35: Boing Boing (5 points)

Two vertical walls of infinite height have their bases elevated $h = 3.40$ m above the ground, and are separated by a distance $d = 10.0$ m. A mass on the ground directly under the left wall is fired towards the right wall at angle α above the horizontal with an initial speed $u = 30$ m s^{-1}. For the angles $\alpha_1 < \alpha < \alpha_2$, the mass undergoes a maximal number of collisions with the walls before falling to the ground. Assume all collisions are perfectly elastic and ignore resistive forces. Find the ratio α_1/α_2.

Leave your answer to 2 significant figures.



Problem 36: Stacked Blocks

A block of mass $m = 3.0$ kg is stacked above another block of mass $M = 5.0$ kg, which lies on horizontal ground. The ground is frictionless, but there exists friction between the two blocks, with coefficient $\mu_s = 1.0$ for static friction and coefficient $\mu_k = 0.7$ for kinetic friction. With both blocks initially at rest, the lower block is pushed with a constant horizontal force F. Assume that the lower block is sufficiently long and wide, such that the upper block never falls off, and the blocks do not rotate.

(a) Determine the value of F required so that the lower block has an acceleration $a = 5.0$ m s^{-2}.

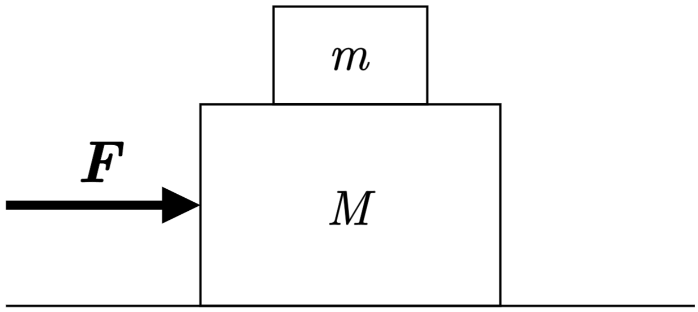
Leave your answer to 3 significant figures in units of N. (2 points)

(b) Determine the value of F required so that the lower block has an acceleration $a = 15.0$ m s^{-2}.

Leave your answer to 3 significant figures in units of N. (2 points)

(c) There is a range of accelerations $a_0 < a < a_1$ that is impossible for the lower block to attain regardless of the value of F. Find $a_1 - a_0$.

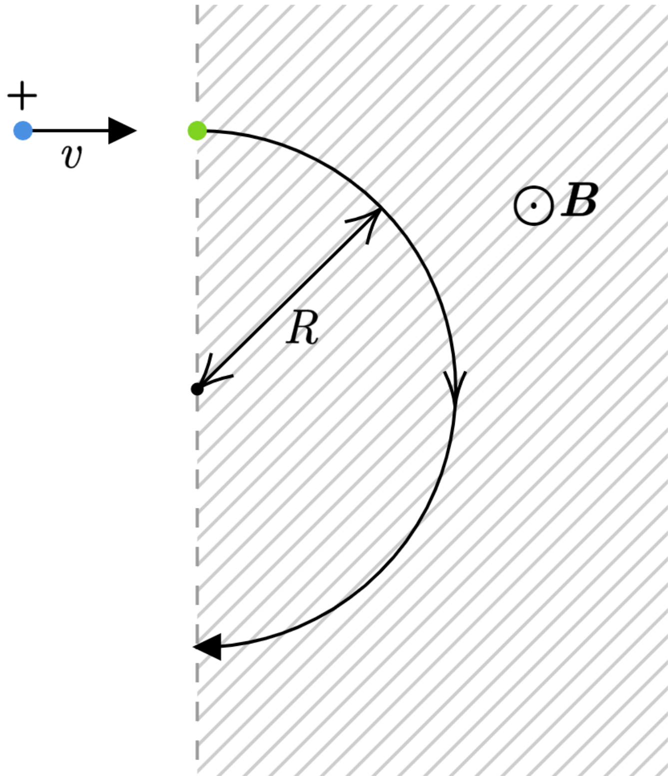
Leave your answer to 3 significant figures in units of m s^{-2}. (4 points)



Problem 37: Collisions in a Magnetic Field (5 points)

A proton moving at relativistic speed v in the lab frame collides with and sticks to a stationary neutron at the edge of a magnetic field of strength $B = 0.100$ T. The magnetic field is directed perpendicular to the plane of the proton's motion. If the resulting particle moves in a circular arc of radius $R = 1.00$ m through the magnetic field, find the value of v/c, where c is the speed of light. Take the neutron to have the same rest mass as the proton.

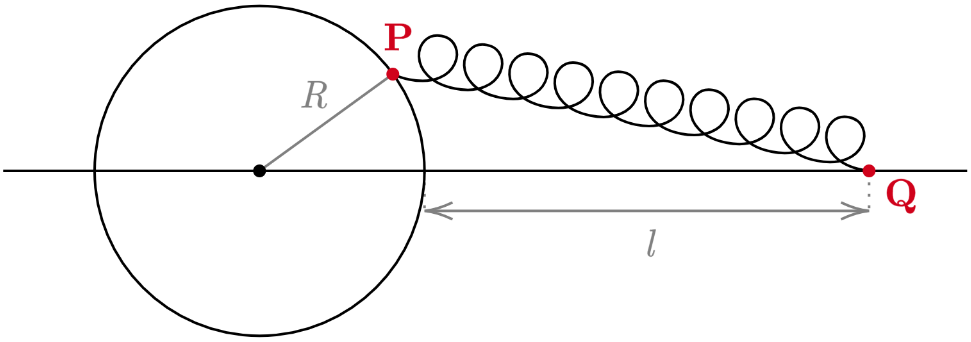
Leave your answer to 2 significant figures.



Problem 38: Springs and Discs (4 points)

A uniform solid disc with mass $m = 1.0$ kg and radius $R = 0.30$ m is free to rotate around a fixed frictionless axle through its centre. An ideal light spring with zero rest length and spring constant $k = 5.0 \times 10^3$ N m^{-1} has one end attached to the disc's edge, at the point P, while the other end is attached to a fixed point a distance $l = 1.00$ m from the disc's edge, at the point Q. What is the frequency f of small oscillations of the disc about its equilibrium position?

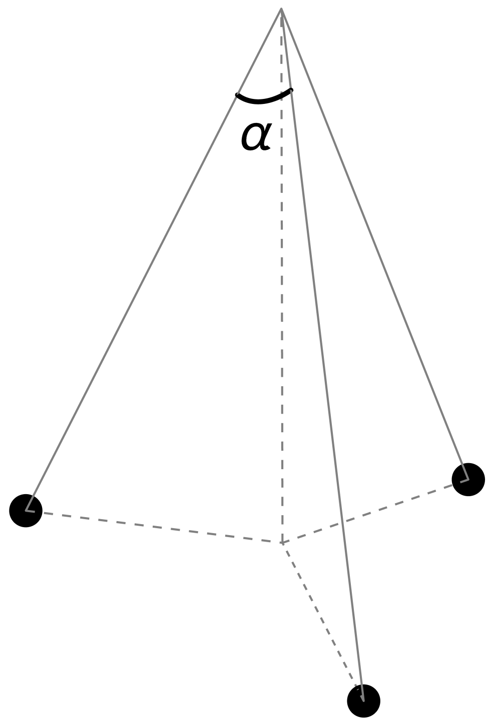
Leave your answer to 3 significant figures in units of s^{-1}.



Problem 39: 3 Charged Balls (5 points)

Three identical point charges, each with a charge of $Q = 10^{-6}$ C and mass $m = 1.0$ kg are suspended from the same point using massless, insulating threads of fixed length $l = 1.0$ m. Find the angle α between two strings when the system is at its stable equilibrium configuration.

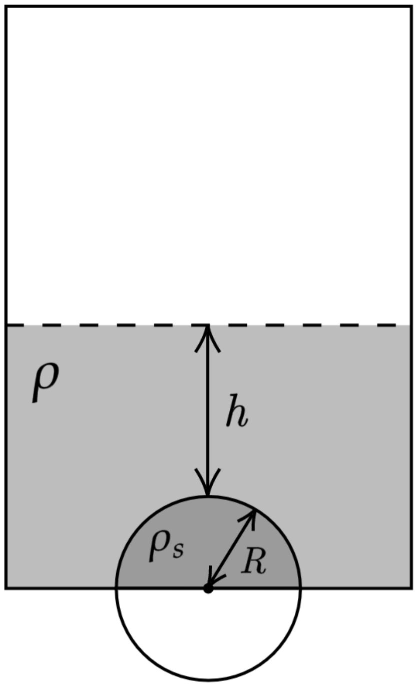
Leave your answer to 2 significant figures in units of degrees.



Problem 40: Force on Sphere (5 points)

A circular hole of radius $R = 15.0$ cm is cut out at the bottom of a large tank. A copper sphere with density $\rho_s = 8960$ kg m^{-3} and radius infinitesimally larger than R is lodged into the hole. The tank is then filled with water until the top of the sphere is a distance $h = 20$ cm below the water surface. Hence, the upper half of the sphere is now surrounded by water, and the lower half surrounded by air. Calculate the total upward force F exerted on the sphere by the hole at equilibrium. You may take atmospheric pressure to be constant all around the set-up.

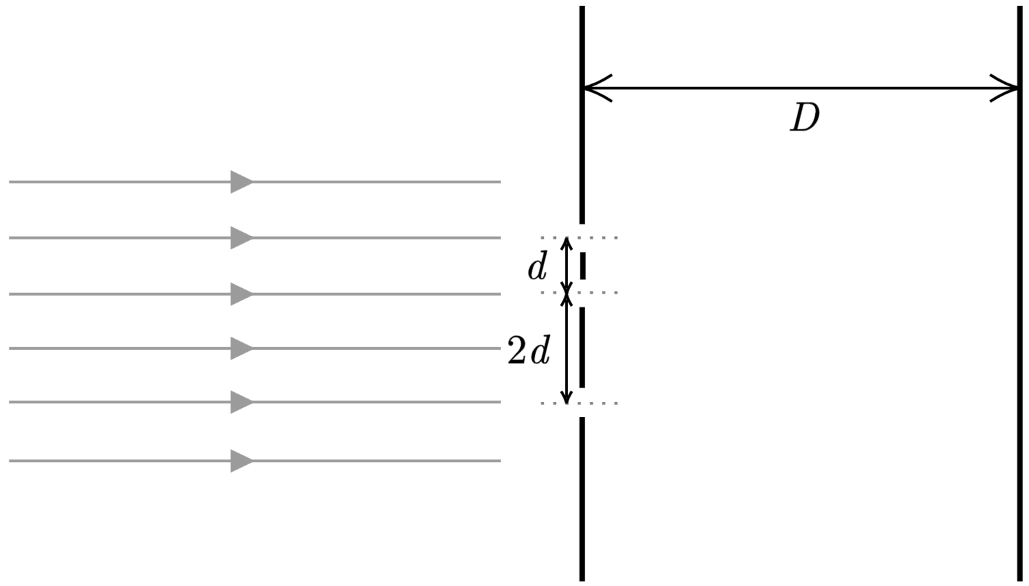
Leave your answer to 3 significant figures in units of N.



Problem 41: Triple Slit Interference (5 points)

A triple slit interference experiment is set up. The slit separation is unequal, with one slit being a distance $d = 18\ \mu$m away from the central slit, while the other slit is a distance $2d$ away from the central slit. The screen is placed a distance $D = 1.5$ m from the slits, and monochromatic light of wavelength $\lambda = 600$ nm is shone normally onto the slits. Compute the distance x between the first intensity minima on either side of the central maxima. Take the central maxima as the maxima directly in front of the central slit. You may take the slit width to be negligible.

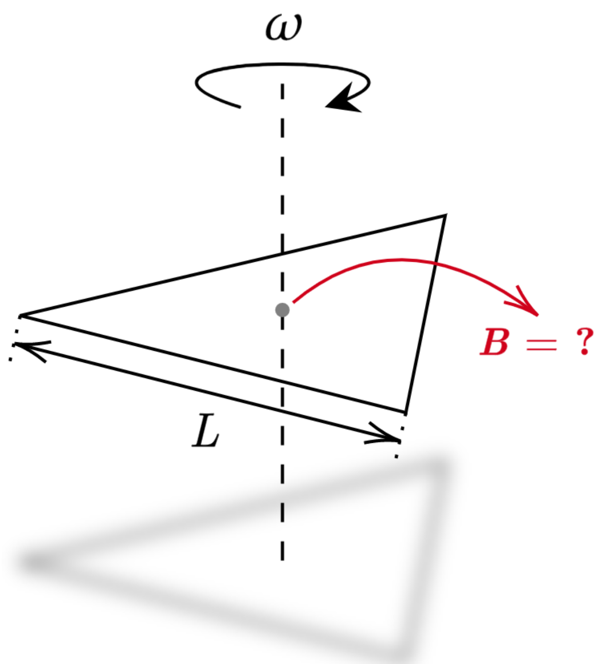
Leave your answer to 2 significant figures in units of mm.



Problem 42: Rotating Triangle (5 points)

A perfectly insulating thin rigid wire, in the shape of an equilateral triangle, has side length $L = 12$ cm and a linear charge density of $\lambda = 10$ C m^{-1}. It is rotated about an axis perpendicular to its plane and passing through its centre at an angular velocity of $\omega = 320$ rad s^{-1}. Find the magnetic field B at the centre of the triangle. You may assume that ω is sufficiently large, such that the currents formed from the motion of the triangle are steady.

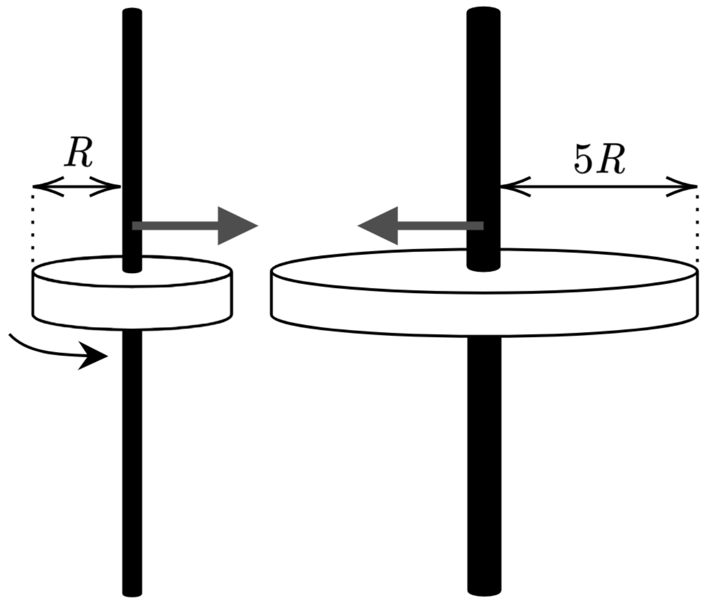
Leave your answer to 2 significant figures in units of T.



Problem 43: Contacting Discs (5 points)

Two uniform discs made from the same material and of equal thickness have radii R and $5R$ respectively. Each disc has a fixed axle through its centre that lets it rotate frictionlessly about its central axis. The disc with radius R is imparted some initial rotational velocity, while the other disc is initially stationary. The two discs are brought in contact and allowed to settle to a steady state. What percentage of the system's mechanical energy is dissipated in the process?

Leave your answer to 2 significant figures as a percentage. (For example, if you think the final answer should be 51.0%*, input your answer as* 51.0*)*



Problem 44: Joule Heating (5 points)

A long cylindrical copper wire has radius $R = 0.5$ mm, electrical resistivity $\rho = 1.68 \times 10^{-8}\ \Omega\ \mathrm{m}^{-1}$, and thermal conductivity $\kappa = 385\ \mathrm{W\ m^{-1}\ K^{-1}}$. The wire carries uniform current density $J = 2.55 \times 10^{6}\ \mathrm{A\ m^{-2}}$ directed along the axis of the wire. The surface of the wire is maintained at temperature $T_0 = 300$ K. Given that T is the temperature at a distance $R/2$ from the central axis of the wire at steady state, find $T - T_0$.

Leave your answer to 3 significant figures in units of mK.

Problem 45: Damped Oscillator (5 points)

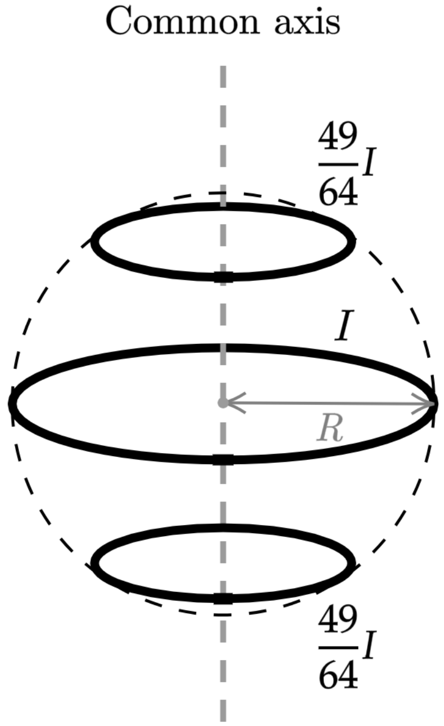
A mass $m = 0.50$ kg is connected to an ideal light spring with spring constant $k = 60$ N m^{-1} on a rough horizontal surface. The mass is pulled, giving the spring an initial extension $A = 1.2$ m from its rest length, and released from rest. Throughout its subsequent motion, there is a resistive frictional force with constant magnitude $f = 0.40$ N opposing the oscillatory motion of the mass. Determine the time taken, t, for the mass to come to a permanent rest. Neglect static friction.

Leave your answer to 3 significant figures in units of s.

Problem 46: Super Uniform Field (5 points)

James sets up a magnetic field by using 3 coils. The planes of the coils are parallel to one another, and the centres of the coils are collinear (in other words, the coils share a common axis). The central coil has a radius of $R = 20$ cm, and carries a current of $I = 1.0$ A, while the other 2 coils are smaller and identical in size. They each carry a current of $49I/64$, and are equidistant from the central coil. Furthermore, a sphere of radius R can be drawn such that all 3 coils lie on its surface. The first, second, third, fourth and fifth derivatives of the magnetic field strength with respect to position along the common axis, taken at the centre of the central coil, are all zero. Find the magnetic field strength at the centre of the central coil.

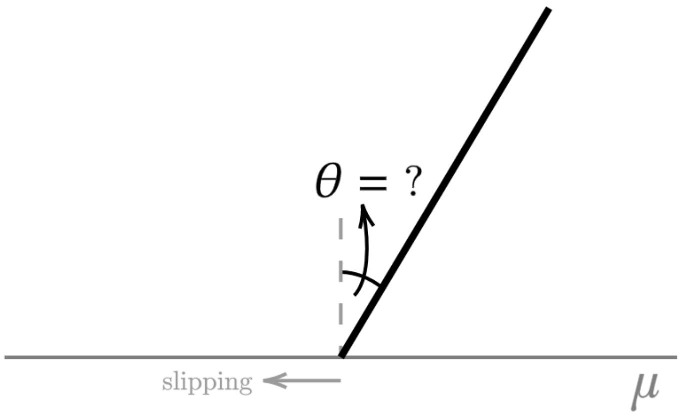
Leave your answer to 2 significant figures in units of μT.



Problem 47: Slipping Stick (6 points)

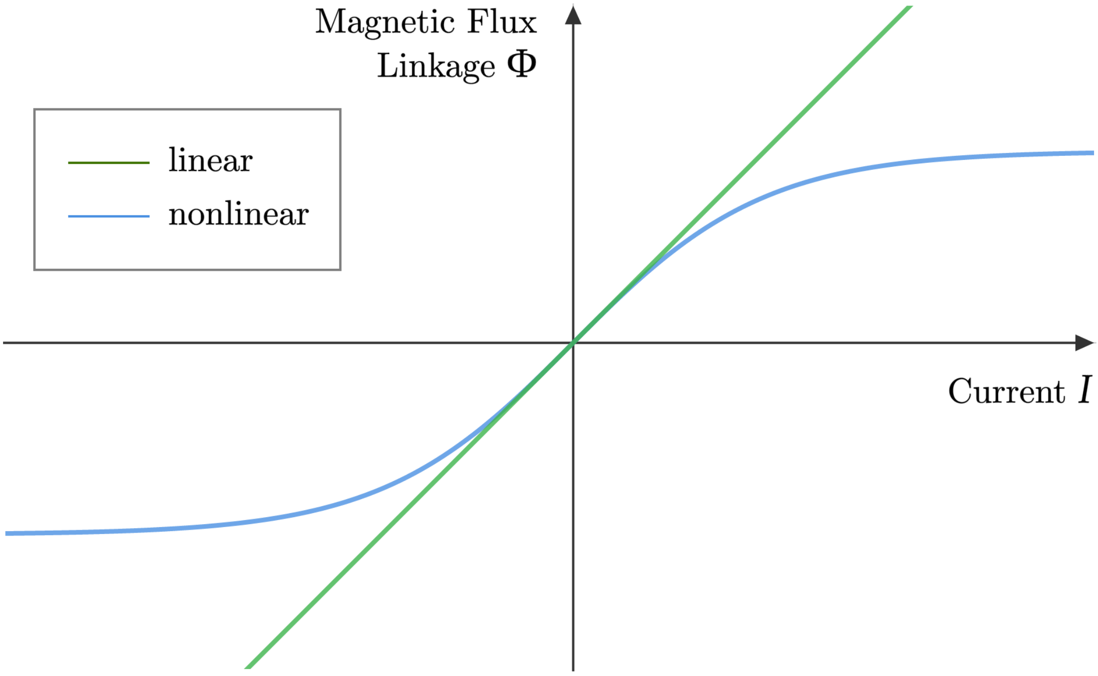
A uniform stick is initially held vertical on a rough horizontal surface. The coefficient of static friction between the stick and the surface is $\mu = 0.250$. The stick is released from rest and falls. What is the angle θ between the stick and the vertical when the bottom end of the stick first starts slipping backwards across the surface?

Leave your answer to 3 significant figures in units of degrees.



Problem 48: Nonlinear Inductors (5 points)

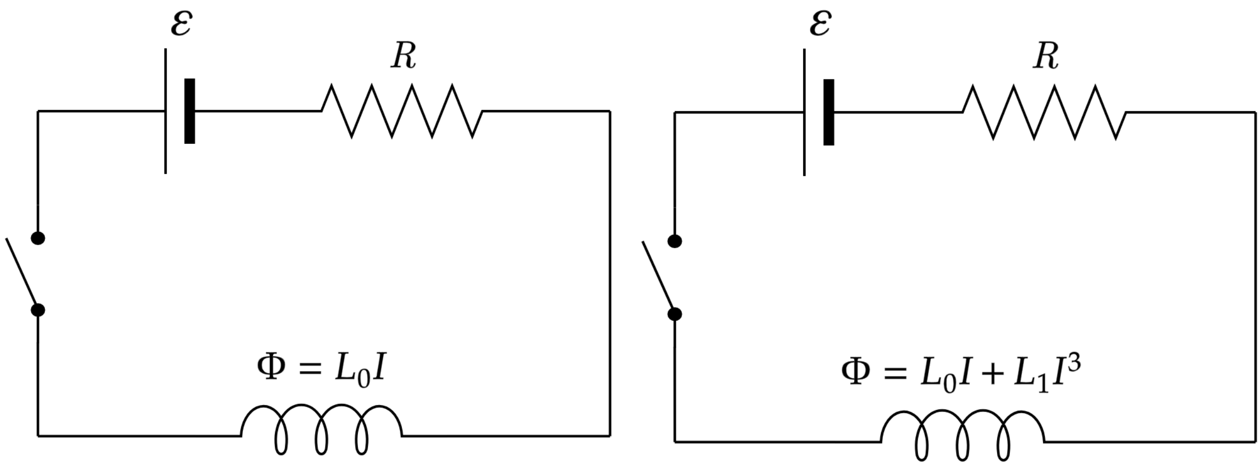
An inductor models the relationship between the magnetic flux linkage Φ and the current I through a coil. Generally, the magnetic flux linkage is modelled to be directly proportional to current in the coil, i.e. $\Phi(I) = LI$, where the inductance L is a constant independent of current. We shall call these *linear inductors*. In practice, however, many of these inductors are built with magnetic cores that exhibit saturation and other effects that cause the flux to "taper off" from proportionality when the current grows large, and these inductors are therefore described as *nonlinear*.



One way to capture these effects is to expand $\Phi(I)$ as an odd power series in I:

$$\Phi(I) = L_0 I + L_1 I^3 + L_2 I^5 + \ldots \tag{1}$$

where $L_0, L_1, L_2, \ldots$ are constants. For small I, all terms but the first become negligible and we recover the direct proportionality relationship $\Phi(I) \approx L_0 I$. However, for slightly larger currents, we will keep the first two terms in Eq. (1) and consider a nonlinear inductor with the flux-current relationship $\Phi(I) = L_0 I + L_1 I^3$. We will then compare its behavior to a linear inductor with the ordinary flux-current relationship $\Phi(I) = L_0 I$.



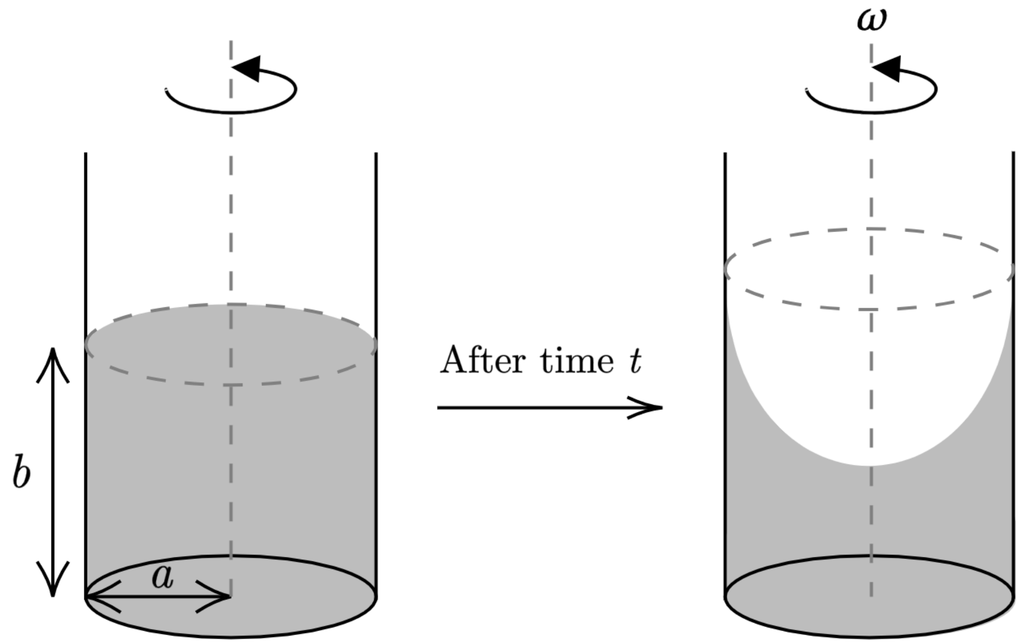
An RL circuit consists of a resistor, inductor, voltage source and an open switch in series, where the resistor has resistance $R = 1.00\ \Omega$ and the voltage source drives with emf $\mathcal{E} = 1.00$ V. Suppose that two such circuits are set up using the two different inductors mentioned previously. The switch is closed and the time taken for the current to reach half its maximum possible value, t_0 and t_1, is measured for the linear and nonlinear inductor circuits respectively. What is the relative change in this time, $\frac{t_1 - t_0}{t_0}$, when switching from the linear inductor to the nonlinear one? Take $L_0 = 1.00$ H, $L_1 = -1.00 \times 10^{-2}$ H A^{-2}. Note that L_1 is negative.

Leave your answer to 3 significant figures.

Problem 49: Spinning up a Bucket (6 points)

A tall cylindrical bucket of radius $a = 1.10$ m contains incompressible fluid of density $\rho = 100$ kg m^{-3} up to a height $b = 0.80$ m. A constant torque $\tau = 0.38$ N m then rotates the bucket and the fluid within it from rest. How much time t does it take for the whole system to reach an angular velocity of $\omega = 2.70$ rad s^{-1}? Suppose that the mass of the bucket is negligible compared to that of the fluid, that the time scale over which the angular velocity of the system changes is far longer than the time scale over which internal movement and deformations of the fluid surface reach equilibrium, and neglect any effects of surface tension.

Leave your answer to 3 significant figures in units of s.



Problem 50: A Strong Glow (6 points)

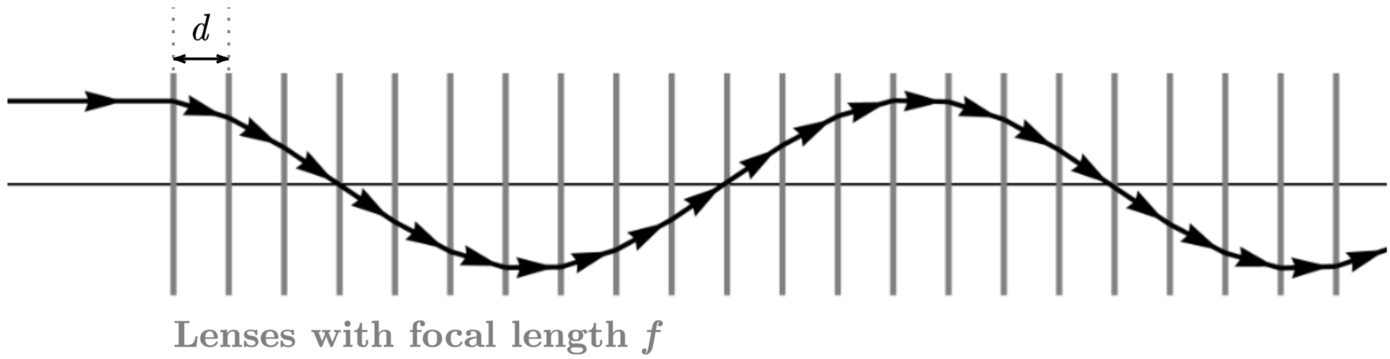
An exceedingly high-powered lamp shines on a perfectly reflective, horizontal circular mirror of diameter $d = 0.10$ m and mass $m = 0.20$ kg from $H = 0.25$ m below the mirror's centre. Taking the lamp to be a point source which radiates uniformly in all directions with ideal efficiency, what is the minimum power of the lamp, P, required to support the weight of the mirror?

Leave your answer to 3 significant figures in units of GW.

Problem 51: Compound Lens System (7 points)

Identical thin convex lenses of focal length $f = 100$ mm are positioned along the same line, with equal separation $d = 19.8$ mm between adjacent lenses. The lenses are aligned such that they share the same horizontal principal axis. When a horizontal ray of light is shined through the first lens above its centre, the resulting path of the ray is periodic. Determine the wavelength of this path.

Leave your answer to 2 significant figures in units of mm.



Problem 52: The Sun is a Deadly Laser (7 points)

A rigid, hollow, thin-walled, airtight sphere filled with an ideal gas lies on the equator of Earth. The sphere has a surface area of $A = 1.00\ \mathrm{m}^2$ and perfect thermal contact with the gas. Suppose the entire sphere and gas system behaves as an ideal blackbody with heat capacity $C = 50\ \mathrm{kJ\ K}^{-1}$. During the day, the intensity of light incident at the position of the sphere can be assumed to be constant at $I = 1.361\ \mathrm{kW\ m}^{-2}$, while during the night, no light is incident on the sphere. As such, the pressure of the gas in the sphere varies over time. Let the minimum and maximum pressures of the gas be P_1 and P_2 respectively. Find the ratio P_1/P_2.

You may assume that the sphere is thermally isolated from the Earth by enclosing it in a transparent vacuum chamber, so that radiation is the only form of heat transfer.

Leave your answer to 3 significant figures.

Half Hour Rush M1: Passing IPPT (3 points)

Rick and Morty are pre-enlistees. As part of their IPPT 2.4 km running assessment, they run around a circular track. They begin at the same starting point and start running simultaneously in the same direction. To complete each round, Rick takes $T_1 = 1$ min 30 sec, while Morty takes $T_2 = 2$ min 20 sec. How long does it take for Rick to lap Morty after they start running?

Leave your answer to 3 significant figures in units of s.

Half Hour Rush M2: Ferry to Tekong (4 points)

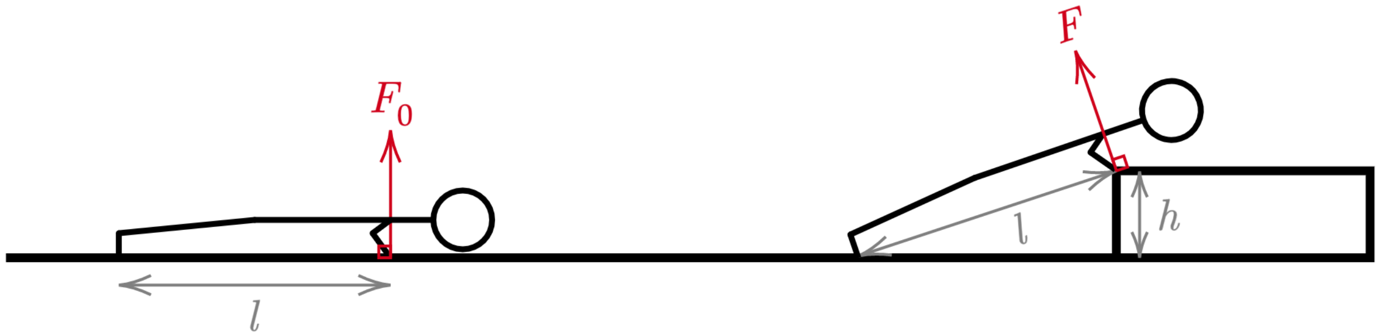
A newly enlisted recruit travels by ferry from mainland Singapore to Pulau Tekong. At the start of the journey, the ferry accelerates forward uniformly. There is a marking on the floor of the ferry at the original position of the recruit. He jumps vertically upward relative to the ferry, such that his body's centre of mass rises by a maximum height $h = 0.50$ m above its original position. When he lands, he finds that he is now at distance $x = 0.15$ m behind the marking. What is the acceleration, a, of the ferry?

Leave your answer to 2 significant figures in units of $\mathrm{m\ s^{-2}}$.

Half Hour Rush M3: Inclined Push-ups (4 points)

An inclined push-up is a push-up where one places their hands on a surface that is elevated relative to that on which their feet are resting, while in a normal push-up the hands and feet are on level ground. We will model a person as a rigid rod with two points of contact with the floor: one at the feet, and another at the hands, as shown. Suppose that the feet-to-hand distance is $l = 1.50$ m. We will assume that the forces on the hands are directed perpendicular to the person's body. If the combined force on the hands is $F_0/W = 64\%$ of body weight in a normal push-up, what will F/W be when the hands are elevated by height $h = 50$ cm above the level of the feet?

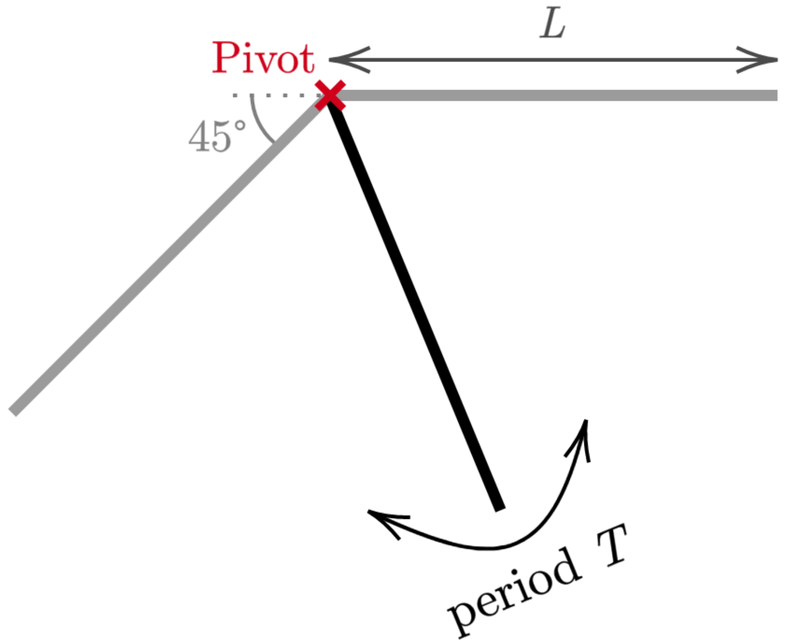
Leave your answer to 2 significant figures as a percentage of body weight. (For example, if you think the final answer should be $F/W = 0.51 = 51\%$*, input your answer as* 51*)*



Half Hour Rush M4: Marching in the SAF (4 points)

In the Singapore Armed Forces, marching is done using the 90-45 rule. Soldiers swing their arms from a horizontal position in front of their body (90° to the vertical), to a diagonal position behind their body (45° to the vertical) and back repeatedly. To model the effect of the swinging motion on blood pressure, we treat the blood vessel in the arm as a sealed isolated narrow cylindrical tube completely filled with blood. As such, we may neglect atmospheric pressure and interactions with the rest of the body. The tube has length $L = 65$ cm and swings about its fixed end sinusoidally, taking $T = 1$ s to complete one oscillation. Given that the density of blood is $\rho = 1060$ kg m^{-3}, find the maximum blood pressure in the arm during the whole swinging process. You may neglect gravity.

Leave your answer to 3 significant figures in units of kPa.



Half Hour Rush E1: Plate Levitation 1 (3 points)

Jim wishes to prank his co-workers into believing that he has telekinetic powers. To do this, he wants to make an object levitate mid-air via electrostatic means.

He chooses to use a square metal plate of mass $m = 0.150$ kg. He charges the plate until it acquires positive charge $q = +1.0\ \mu$C. Then, he orients it such that it is horizontal, and secretly applies an upward electric field E, causing it to levitate upon release.

Determine the minimum E that Jim needs to apply for his plan to work.

Leave your answer to 2 significant figures in units of MN C^{-1}.

Half Hour Rush E2: Plate Levitation 2 (4 points)

Jim wishes to prank his co-workers into believing that he has telekinetic powers. To do this, he wants to make an object levitate mid-air via electrostatic means. Thanks to your calculation earlier, Jim knows that his prank can certainly work.

He chooses to use a square metal plate of side length $l = 0.20$ m. He charges the plate until it acquires positive charge $q = +1.0\ \mu$C. Then, he orients it such that it is horizontal, and secretly applies an upward electric field $E = 6.0$ MN C^{-1}, causing it to levitate upon release.

As the plate levitates at equilibrium, determine the overall charge that resides across the entire area of the lower surface of the plate. You may neglect edge effects.

Leave your answer to 2 significant figures in units of μC.

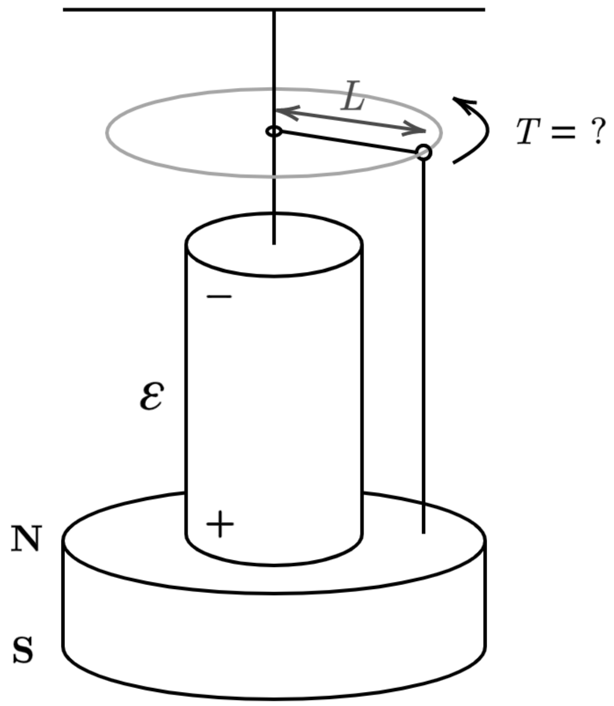
Half Hour Rush E3: An Inconspicuous Motor (4 points)

Jim's co-workers eventually got bored of the levitating plate. Jim feels that a little more motion is needed to reignite his co-workers' interests. So, he designs an inconspicuous motor.

The motor is constructed from a battery, wire, and magnet, as shown in the figure. The L-shaped wire is constrained to a vertical plane by a conducting vertical shaft and an insulating ring. The magnet produces a uniform vertical magnetic field of strength $B = 0.1$ T. The battery has voltage $U = 4.8$ V, and the wire has resistance $R = 0.52\ \Omega$. The horizontal section of the wire has length $L = 0.10$ m. Assume that the magnet, battery and shaft have no resistance.

A resistive torque $\tau_{\text{resistive}} = -k\omega$ acts on the wire, where ω is the angular frequency of the wire and damping coefficient $k = 0.060$ kg m s^{-1}. As a result, the wire will eventually rotate with a constant period T. Find T.

Leave your answer to 3 significant figures in units of s.



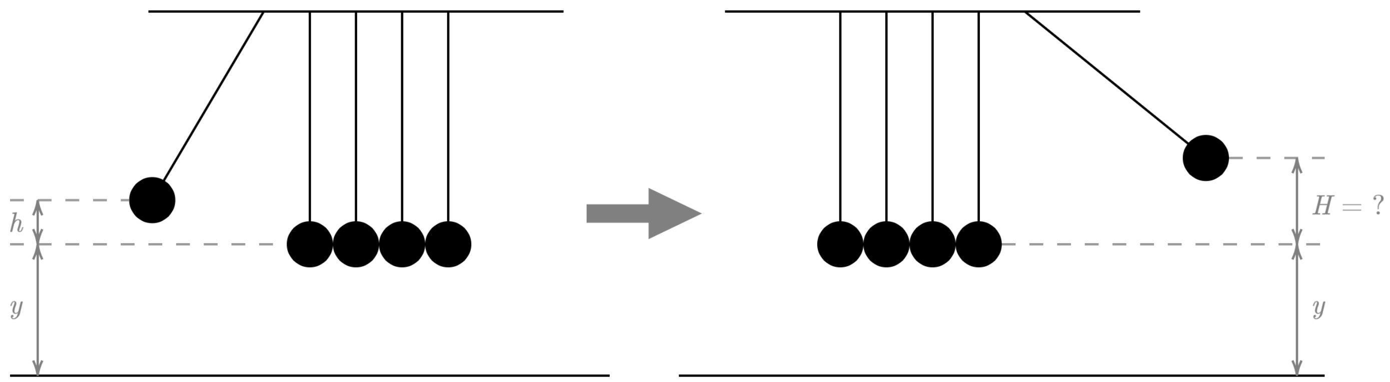
Half Hour Rush E4: Phony Newton's Cradle (4 points)

Jim's co-worker Dwight remains unconvinced of his telekinetic powers. Jim now opts to deceive Dwight using a phony Newton's Cradle. He designs a Newton's Cradle with balls of insulating material, which have identical size and mass $m = 100$ g.

However, he secretly charges the leftmost ball with $Q = +3.0\ \mu$C, while all other balls remain electrically neutral. His demonstration is done on a large, flat and horizontal metal table, which is grounded. There is a vertical distance $y = 20.0$ cm between the balls' resting positions and the tabletop.

When he displaces the leftmost ball by vertical height $h = 5.0$ cm (with the string remaining taut) and releases it from rest, the rightmost ball rises to a larger maximum height $H > h$. Find H. You may assume that all collisions are perfectly elastic and that the balls are point masses. Neglect any effects of polarisation or magnetism.

Leave your answer to 2 significant figures in units of cm.



Half Hour Rush X1: Magnifying Glass (3 points)

When an object of length $l = 1.0$ cm is placed in front of a magnifying glass, its image has length $L = 4.0$ cm. The magnifying glass is now moved such that its distance from the object is halved. What is the new length of the image formed? Model the magnifying glass as an ideal thin converging lens.

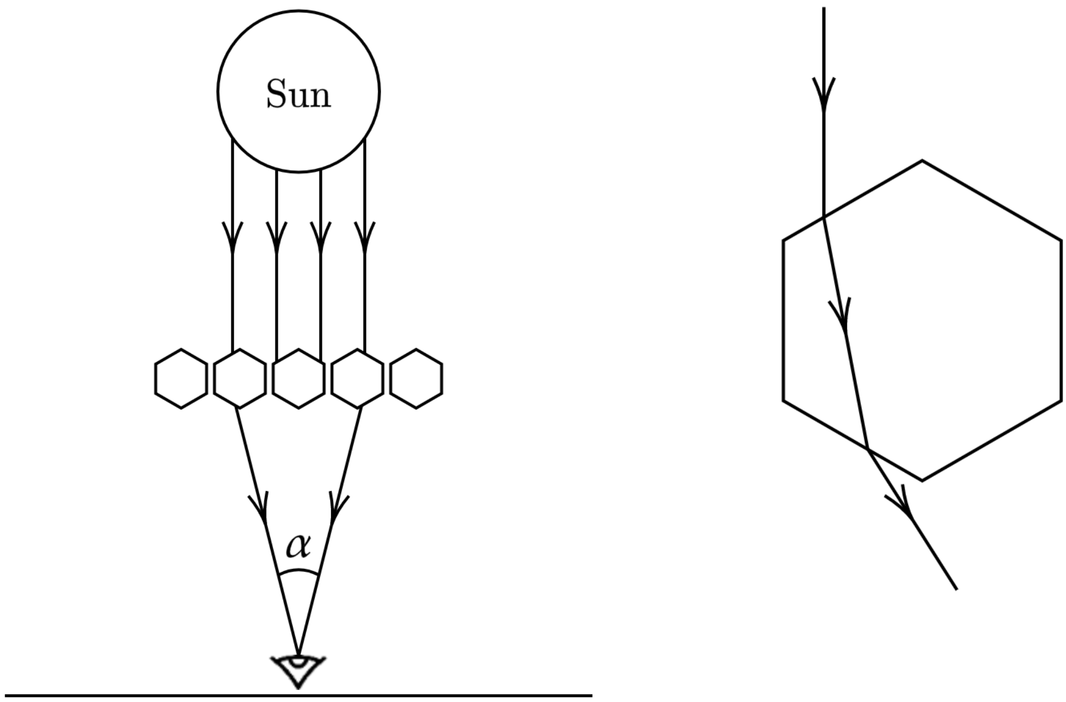
Leave your answer to 2 significant figures in units of cm.

Half Hour Rush X2: Lights over Nuremberg (4 points)

In April 1561, strange geometric objects were observed over the city of Nuremberg in Germany, seemingly engaged in aerial battle. Many explanations for this strange occurrence have been put forth, ranging from warnings from divine powers, to a space war between extra-terrestrials. However, most scientists today believe that the sightings are best explained by an optical phenomenon known as a *sun-dog*, consisting of a circular ring of light around the Sun. One theory suggests that these rings are formed when light from the Sun is refracted by regular hexagonal ice crystals suspended within the atmosphere.

For this problem, assume that the Sun is directly overhead and that all the ice crystals adopt the configuration shown below. Find the angle α subtended by the ring. Take the refractive index of ice to be $\eta_{\text{ice}} = 1.31$, and the refractive index of air to be $\eta_{\text{air}} = 1.00$.

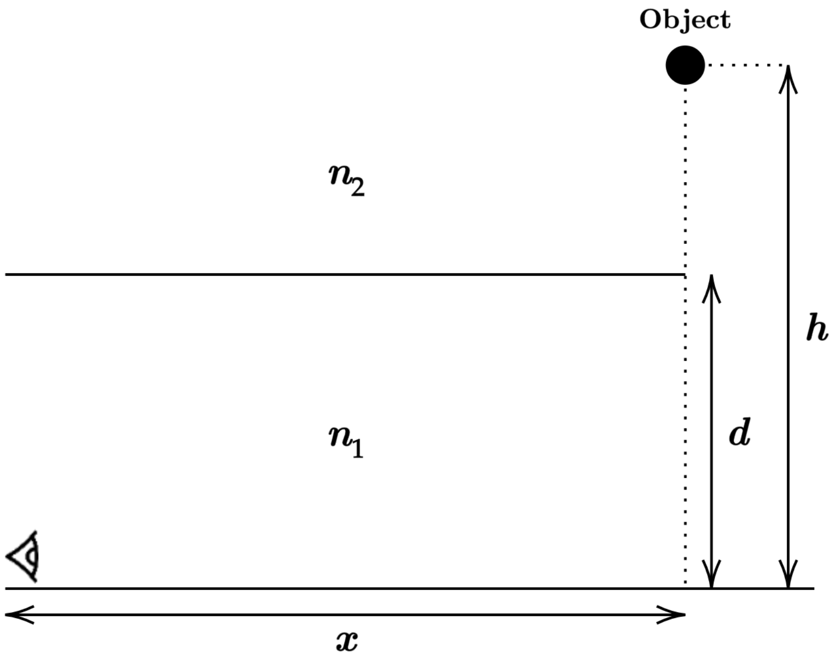
Leave your answer to 5 significant figures in units of degrees.



Half Hour Rush X3: Mirage (4 points)

In polar regions, air tends to be colder near the ground, causing the refractive index of air to vary with height. This results in an optical phenomenon known as a mirage. As a simplified model of this effect, let us suppose the air within distance $d = 3.00$ m from the horizontal ground has uniform refractive index $n_1 = 1.30$, while all the air above has refractive index $n_2 = 1.05$. If an object is placed at height $h = 5.00$ m above the ground and horizontal distance $x = 10.0$ m away from an observer, how high above the ground does the observer perceive the object to be?

Leave your answer to 2 significant figures in units of m.



Half Hour Rush X4: Shortsighted Swimmer (4 points)

A biconcave thin lens is used to correct myopic vision and can be described with a radius of curvature R on both faces. In glasses, light from the surroundings travels through the media air $\to$ glass $\to$ air before reaching the eye, while in swimming goggles, light travels through the media water $\to$ plastic $\to$ air. A swimmer has glasses and goggles, both of which suit his myopia perfectly. Find the ratio R_1/R_0, where R_1 is the radius of curvature of the swimming goggles and R_0 is that of the glasses.

Take the refractive indices of glass and plastic to be $\eta = 1.5$ and the refractive index of water to be $\eta_{\text{water}} = 1.33$. Incident light may be assumed to consist of paraxial, parallel rays. The eye's position relative to the lens is the same in both cases, and lies along the lenses' principal axes.

Leave your answer to 2 significant figures.