2^{nd} *Singapore Physics League*

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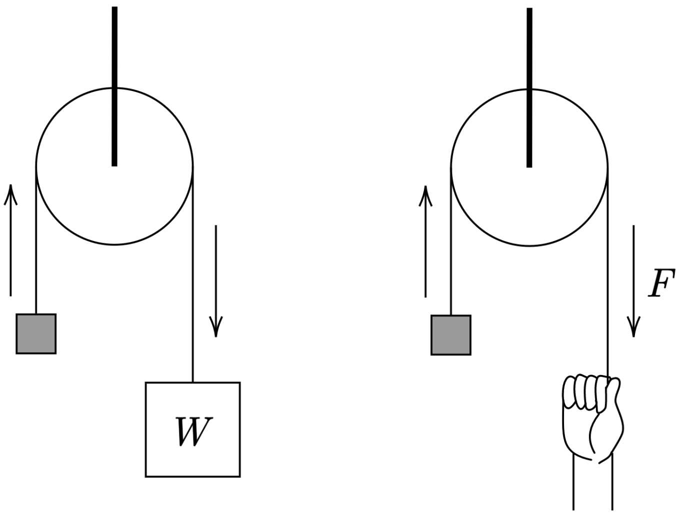
Problem 1: Pulley Pull (2 points)

A block is connected to a light inextensible string that loops over a massless pulley and is pulled by weight W. As a result, the block travels with acceleration a.

Now, instead of pulling on the string with weight W, we directly exert downward force F on the string. For the block to travel with the same acceleration a, how must F and W compare?



(1) $F = W$

(2) $F > W$

(3) $F < W$

Problem A: No Speeding! (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

Along a stretch of road of length $L = 1.2$ km, a maximum speed limit of $v_s = 100$ km h^{-1} is imposed. Cars must strictly adhere to this at every point along their journey.

To enforce this, two speed sensors are installed, one at the start of the road and one at the end, each measuring the instantaneous speeds of cars passing by. But does this really help? Even if a car registers speeds lower than v_s at both sensors, we can only conclude that it was travelling within the speed limit at the start and end of the road. We still wouldn't know whether the car complied with the speed limit across its entire journey.

In view of this, clever engineers decided to reconfigure the sensors to now measure the time interval T between a car passing the first sensor and the same car passing the second sensor. If $T < T_s$, we can be absolutely sure that the car must have been speeding. Find the maximum value of T_s.

Leave your answer to 3 significant figures in units of s.

Problem B: Airdrop (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

Two identical crates are each loaded with different masses and sealed. When the loaded crates are dropped separately from positions high above the ground, the heavier crate reaches terminal velocity $U = 8.0 \text{ m s}^{-1}$, while the lighter crate reaches terminal velocity $u = 2.0 \text{ m s}^{-1}$.

The two crates are now joined together by a light rope and, with the heavier crate dangling by the rope below the lighter one, are dropped once more from a position high above the ground. What is the new terminal velocity of the joined crates?

Assume the air drag to be linear, and the length of the rope to be much greater than the size of the crates.

Leave your answer to 2 significant figures in units of m s^{-1}.

Problem C: Messing with Springs

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

Two ideal light springs have the same unstretched length $L = 10.0$ cm, but different spring constants $k_1 = 40$ N cm^{-1} and $k_2 = 80$ N cm^{-1}.

(a) Each spring is stretched by the same distance of $x = 3.0$ cm. Find the ratio E_1/E_2 of the energy stored in each spring.

Leave your answer to 3 significant figures. (2 points)

(b) Each spring is now stretched with the same force of $F = 100$ N. Find the ratio E_1/E_2 of the energy stored in each spring.

Leave your answer to 3 significant figures. (2 points)

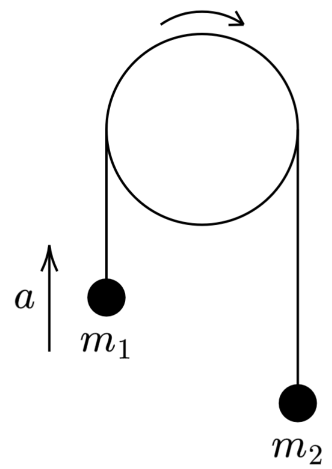
(c) Each spring is now stretched to store the same energy of $E = 50$ J. Find the ratio x_1/x_2 of the extension of each spring.

Leave your answer to 3 significant figures. (2 points)

Problem D: A Simple Pulley

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

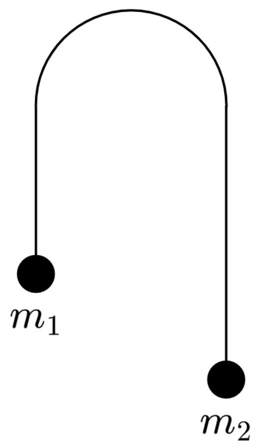
Two point masses, $m_1 = 1.0$ kg and $m_2 = 2.0$ kg, are attached to the ends of a light inextensible string that is threaded over a light frictionless pulley.



(a) Find the upward acceleration a of mass m_1.

Leave your answer to 2 significant figures in units of m s^{-2}. (2 points)

In the remaining parts, consider the combined system of the two point masses and the string that connects them, as drawn below.



(b) Find the downward acceleration a_{CM} of the centre of mass of this system.

Leave your answer to 2 significant figures in units of m s^{-2}. (2 points)

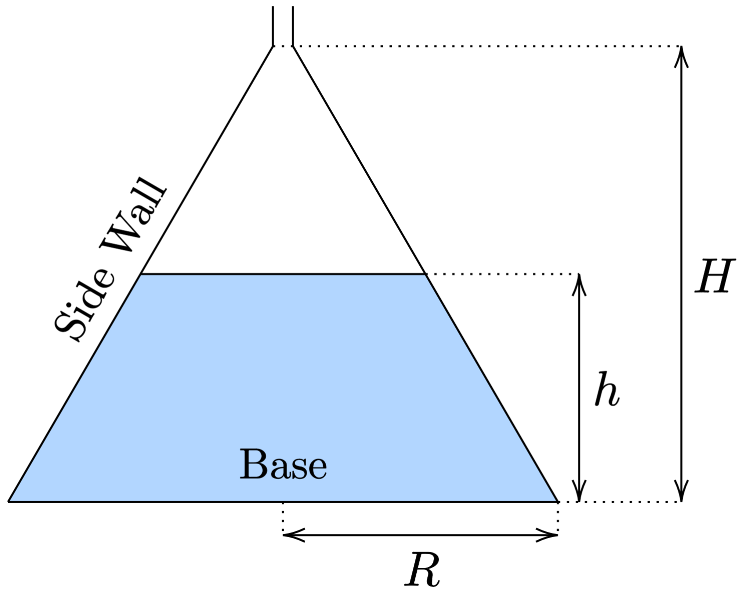
(c) By considering the external forces acting on this system, along with your answer in (b), find the net contact force N exerted on the string by the pulley.

Leave your answer to 2 significant figures in units of N. (2 points)

Problem E: Conical Flask

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

A conical flask has the shape of a perfect cone with radius $R = 10.0$ cm and height $H = 15.0$ cm, except that there is a small opening (of negligible size) at the top. Water is poured into the flask to a depth of $h = 5.0$ cm.



(a) Find the weight W of the water.

Leave your answer to 2 significant figures in units of N. (2 points)

The water experiences contact forces, both from the circular base of the conical flask, as well as from the side walls of the conical flask.

(b) Find the net contact force F_b exerted by the circular base of the conical flask on the water.

Leave your answer to 2 significant figures in units of N. (2 points)

(c) Find the net contact force F_s exerted by the side walls of the conical flask on the water.

Leave your answer to 2 significant figures in units of N. (2 points)

Problem 2: Freefall Photography (3 points)

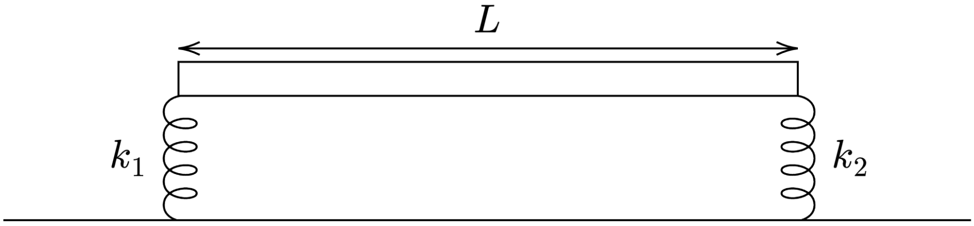
A ball that is initially at height h above the ground is released from rest. Between its time of release and its time of landing, a camera is calibrated to capture one photograph of the ball at a random time. Find the probability that in the captured photograph, the height of the ball above the ground is smaller than $h/2$.

Leave your answer to 3 significant figures.

Your answer should range between 0 and 1.

Problem 3: Springs in Parallel

Two ideal light springs have the same unstretched length, but different spring constants $k_1 = 30$ N m^{-1} and $k_2 = 50$ N m^{-1}. The springs are vertical, with their bottom ends fixed onto a horizontal table, and their top ends respectively attached to the left and right ends of a horizontal board of length $L = 1.00$ m and uniform density, as shown below. Neglect gravity.



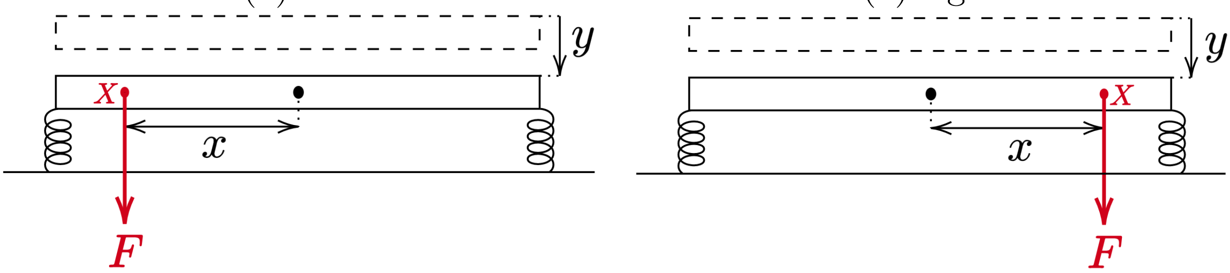
A vertical force F is now applied on the board, causing the board to have displacement y at equilibrium. Textbooks commonly claim that:

$$F = k_{\text{eff}} y, \quad \text{where } k_{\text{eff}} = k_1 + k_2$$

However, this will only be true under certain conditions. In particular, force F must be applied at a specific point X on the board for this relation to be valid.

(a) Point X must be to the ________ of the centre of the board. (1 point)

(1) left (2) right



(b) What must the horizontal distance x between the centre of the board and point X be?

Leave your answer to 2 significant figures in units of m. (3 points)

Problem 4: Dice Roll (3 points)

A uniform cube rests on a surface that is initially horizontal. We slowly increase the angle of the surface from the horizontal, keeping the cube's bottom face parallel to the surface. At some point, the cube starts moving relative to the surface. When that happens, we see that the cube rolls down rather than slides down the surface.

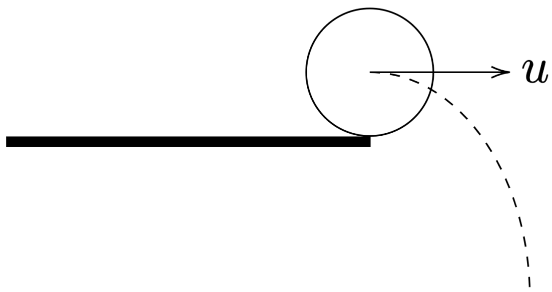
Find the minimum coefficient of static friction μ between the cube and the surface.

Leave your answer to 3 significant figures.

Problem 5: Parabolic Fall (3 points)

A spherical ball of radius $r = 0.04$ m rests exactly on the rightmost end of a horizontal platform. The ball is given a quick push that causes it to instantaneously acquire speed u towards the right. The subsequent trajectory of the ball's falling motion traces a perfect parabola if and only if $u > u_{\text{min}}$. Find u_{min}. Neglect friction and air resistance.

Leave your answer to 2 significant figures in units of m s^{-1}.



Problem 6: Twisted Stick (4 points)

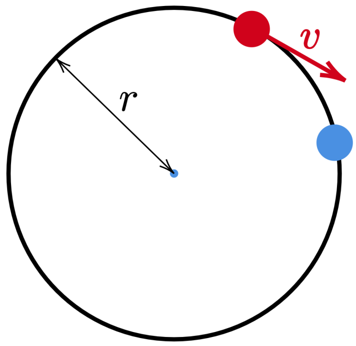
A stick is placed in a transparent rectangular container, and fixed at angle $\theta = 25°$ from the vertical. The container is partially filled with water of refractive index $n = 1.33$, such that some (but not all) parts of the stick are underwater. If you view the stick from near the top, what is the acute angle ϕ that you perceive the underwater portion of the stick to make from the vertical?

Leave your answer to 2 significant figures in units of degrees.

Problem 7: Boing Boing (3 points)

Two small identical beads are threaded on a fixed frictionless horizontal circular hoop of radius $r = 0.2$ m. One bead is given an initial kick such that it slides with speed $v = 0.8 \text{ m s}^{-1}$ around the hoop. It then collides with the other stationary bead.

Determine the number of collisions that take place between the two beads within time $T = 50$ s after the first collision. (Do not count the first collision.) Assume that all collisions are instantaneous and perfectly elastic.



Leave your answer as an integer.

Problem 8: Light but Powerful

Chris the child prodigy is attempting to build a solar cell. His attempt consists of two vacuum-sealed, parallel conducting plates with a small separation. When sunlight is incident upon one of these plates, charges are liberated and travel to the other plate where they accumulate, thus creating a potential difference between the plates.

(a) Chris uses platinum, with a work function of $\Phi_{\mathrm{Pt}} = 6.35$ eV, to make the plates. What is the maximum wavelength, λ_{max}, that would cause a potential difference to form between the plates?

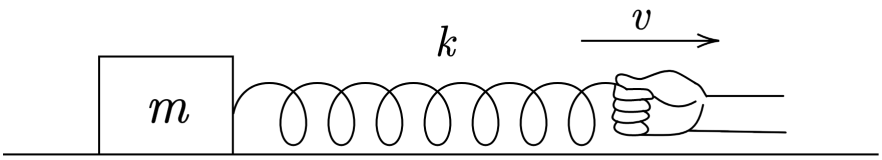
Leave your answer to 2 significant figures in units of nm. (2 points)

(b) Chris then uses a different material, with a work function $\Phi = 2.00$ eV, and light of wavelength $\lambda = 400$ nm. After a long time, what is the final voltage generated by the solar cell?

Leave your answer to 2 significant figures in units of V. (2 points)

Problem 9: Catch-Up Spring (3 points)

A mass $m = 0.10$ kg rests attached to the left end of an ideal light spring of relaxed length $l = 0.15$ m and spring constant $k = 40$ N m^{-1}. You grab the right end of the spring, and start pulling it with constant velocity v towards the right. The ground is horizontal and frictionless.

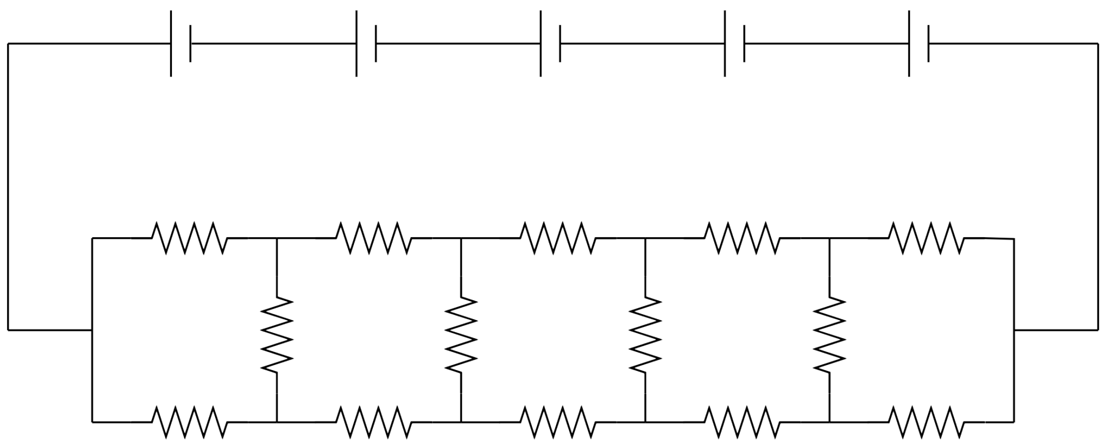


If v is sufficiently large, it is possible for the mass to "catch up" and make contact with your hand at some point during its subsequent motion. Find the minimum value of v required for this to happen.

Leave your answer to 2 significant figures in units of m s^{-1}.

Problem 10: Resistor Squares

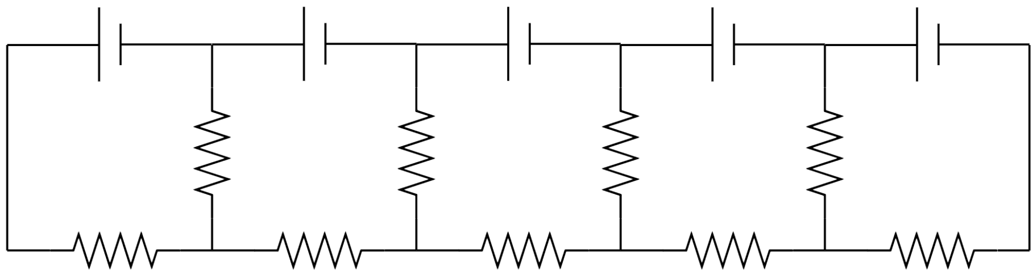
(a) 5 identical cells, each with emf $\varepsilon = 12$ V and negligible internal resistance, are connected to a network of 5 squares comprising 14 identical resistors each with resistance $R = 8.0\ \Omega$, as shown below.



What is the total power supplied by the cells?

Leave your answer to 2 significant figures in units of W. (3 points)

(b) The circuit is now rearranged, with each of the 5 cells replacing each of the 5 resistors on the top branch, as shown below.



What is the total power supplied by the cells?

Leave your answer to 2 significant figures in units of W. (3 points)

Problem 11: Self-Propelled Flashlight (3 points)

Theoretically, it is possible for a flashlight to propel itself when turned on. For a flashlight of mass $m = 0.8$ kg on a frictionless horizontal surface shining light horizontally, determine the minimum operating power P required for the flashlight to self-propel with acceleration $a = 1.0 \text{ m s}^{-2}$.

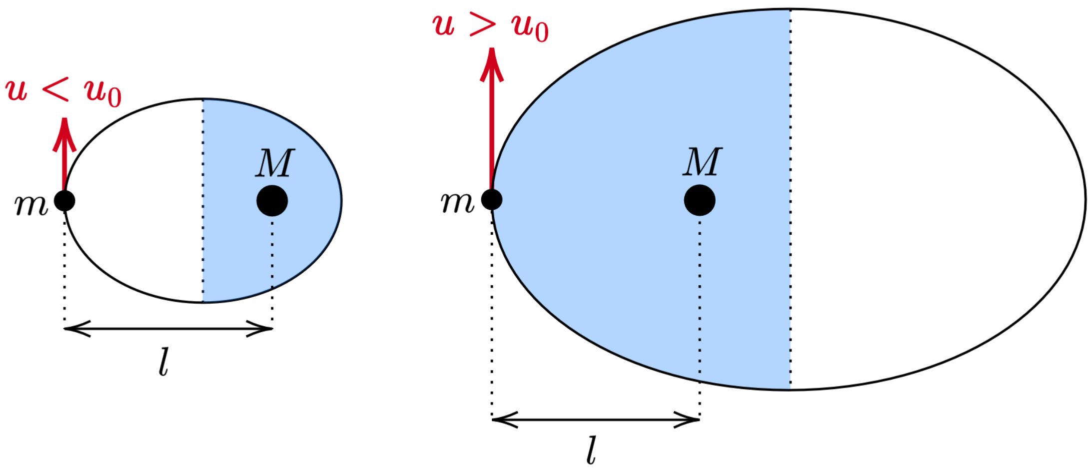
Leave your answer to 2 significant figures in units of MW.

Problem 12: Ellipse Halves (3 points)

At distance $l = 4 \times 10^7$ m to the left of a stationary planet with mass $M = 10^{22}$ kg, there is a small projectile of mass $m \ll M$. It is launched tangentially with velocity u, such that the resulting trajectory of its orbit takes the shape of an ellipse.

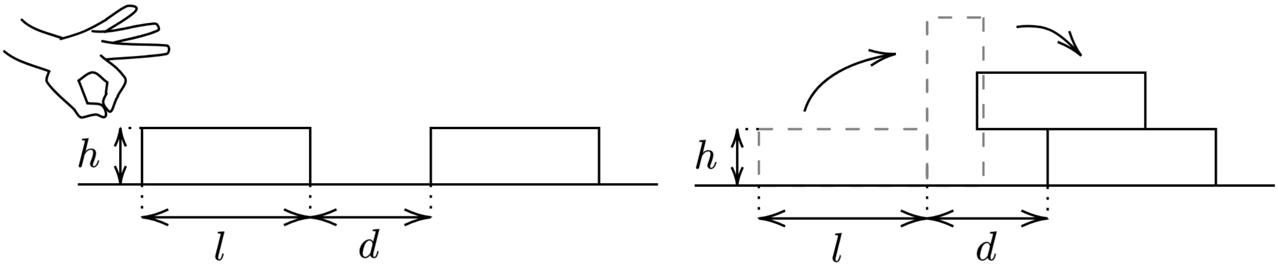
It is observed that for $u < u_0$, the planet is always located in the right half of the elliptical orbit, whereas for $u > u_0$, the planet always lies in the left half of the elliptical orbit. Determine the value of u_0.

Leave your answer to 2 significant figures in units of m s^{-1}.



Problem 13: Country Erasers

A game of "Country Erasers" is played with two identical square-shaped erasers of length $l = 3.0$ cm, thickness $h = 0.5$ cm and uniform mass $m = 20$ g. They initially lie flat beside each other on a horizontal table, with their ends a distance $d > h$ apart. In a winning move, a player flicks one eraser towards the other, such that it rotates vertically and eventually lands resting flat on the other eraser, as shown below. Assume that neither of the erasers slip at any point of contact with the table or with each other throughout their motion.



(a) Find the minimum energy E_0 delivered by the flick such that this winning move can be performed.

Leave your answer to 2 significant figures in units of mJ. (2 points)

(b) Find the minimum energy E_1 dissipated in the process after the eraser is flicked.

Leave your answer to 2 significant figures in units of mJ. (2 points)

(c) Find the maximum value of d such that this winning move is physically possible.

Leave your answer to 2 significant figures in units of cm. (3 points)

Problem 14: Simp (4 points)

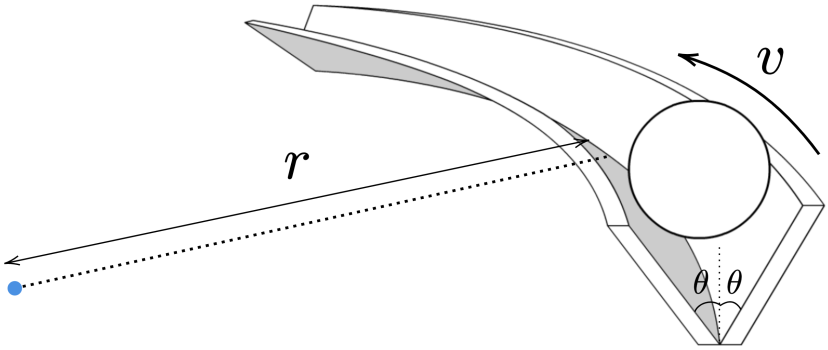
Amy has a crush on Jake. Jake runs in a constant direction with speed $v_J = 7$ m s^{-1} relative to the ground. Amy chases him with a speed of $v_A = 2$ m s^{-1}. Jake's and Amy's eye levels are at height $h_J = 2.0$ m and $h_A = 1.5$ m from the ground respectively. Assume that Earth is a perfect sphere with radius $R = 6400$ km, and that there are no obstructions. Given that they start running from the same point at the same time, find t, the time taken for Jake to disappear from Amy's view.

Leave your answer to 3 significant figures in units of min.

Problem 15: Bending Groove (4 points)

A sphere slides without rolling through a fixed frictionless V-shaped groove. Each side of the groove makes angle $\theta = 30°$ with the vertical. The path traced by the groove is horizontal and bends with radius of curvature $r = 0.15$ m. Determine the maximum speed v of the sphere such that it can traverse the bend without straying from the path of the groove.

Leave your answer to 2 significant figures in units of m s^{-1}.

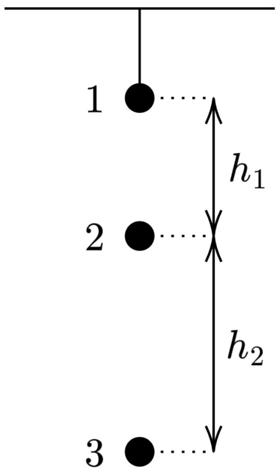


Problem 16: Decorating the Future (4 points)

Three identical small spherical ornaments are used to make a futuristic decoration. All three ornaments have charges of equal magnitude and are perfectly insulating.

The first ornament is suspended from the ceiling by a light string. The second ornament is placed a distance h_1 directly below it, and the third is placed a distance h_2 directly below the second. Given that the system is in equilibrium, find h_1/h_2.

Leave your answer to 3 significant figures.



Problem 17: Useless Pipe (4 points)

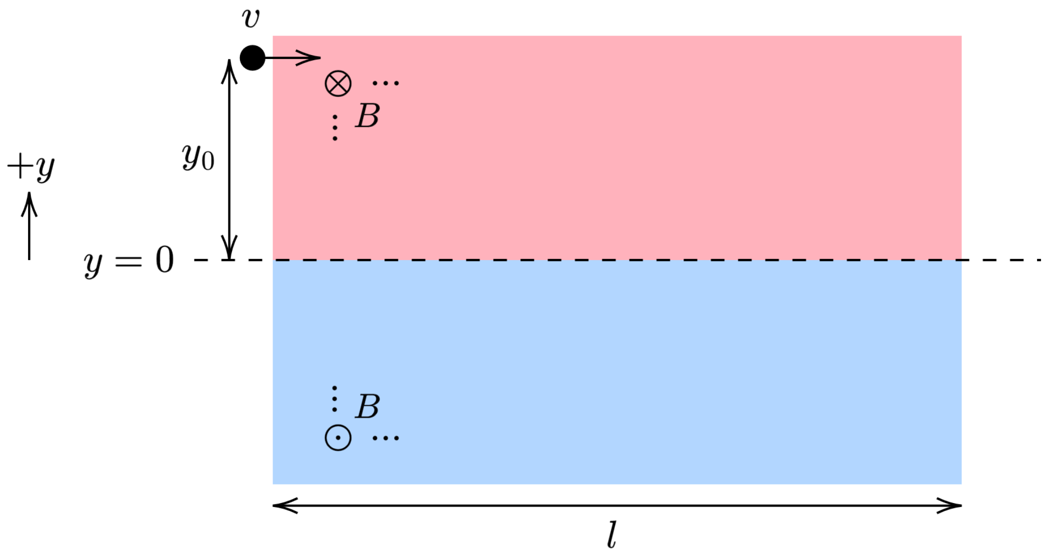
A light U-shaped pipe lies on a horizontal floor. The pipe comprises two straight sections that are affixed to a vertical wall and connected by a semicircular section with radius of curvature $r = 1.00$ m. The pipe has a circular cross-section with an inner diameter of $d = 0.200$ m. Water flows in one end at a constant rate of $Q = 0.100\ \text{m}^3\ \text{s}^{-1}$ and out the other. What is the total force F that the semicircular section of the pipe exerts on the straight sections of the pipe?

Leave your answer to 2 significant figures in units of N.

Problem 18: A Swimming Electron (4 points)

Consider a magnetic field which extends infinitely along the vertical y-axis, but has a finite length $l = 900$ nm along the horizontal x-axis. The magnetic field has uniform magnitude B, and points into the page for $y > 0$, but out of the page for $y < 0$.

An electron is initially located at y-coordinate $y_0 = +170$ nm. It is propelled horizontally into the magnetic field with a velocity $v = y_0 eB/m_e$, where e is the elementary charge, and m_e is the mass of an electron.

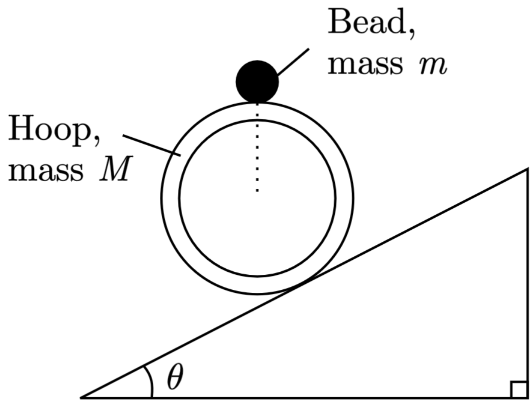


Find the y-coordinate of the electron, y_1, when it exits the field. Neglect gravity.

Leave your answer to 2 significant figures in units of nm.

Problem 19: Defying Gravity

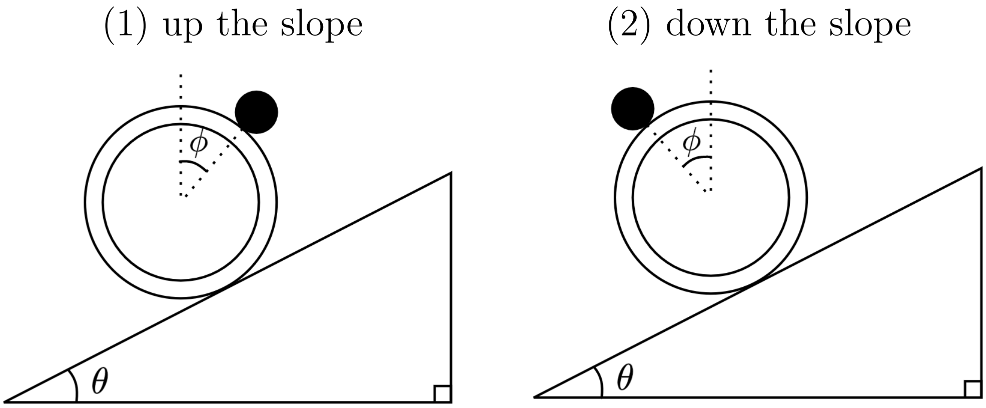
A thin uniform hoop of mass $M = 50$ g has a small bead of mass $m = 150$ g glued to a point on its rim. The hoop is placed on a slope inclined at angle $\theta = 30^\circ$ above the horizontal, and is initially oriented such that the bead is directly above the hoop's centre.



Then, the hoop's position is slightly adjusted by rotating it ____________ by some angle $0^\circ < \phi < 180^\circ$. This way, when the hoop is released from rest, it begins rolling **up** the slope. Assume that the slope is sufficiently rough such that the hoop never slips.

(a) Which of the following options correctly fills in the blank above? (You may refer to the diagram below to visualise the physical setup illustrated by each option.)

(1 point)



(b) The hoop rolls up upon release from rest if and only if $\phi_1 < \phi < \phi_2$, where ϕ_1 and ϕ_2 are constants to be determined. Find $\phi_2 - \phi_1$.

Leave your answer to 2 significant figures in units of degrees. (3 points)

Problem 20: Water Cube (4 points)

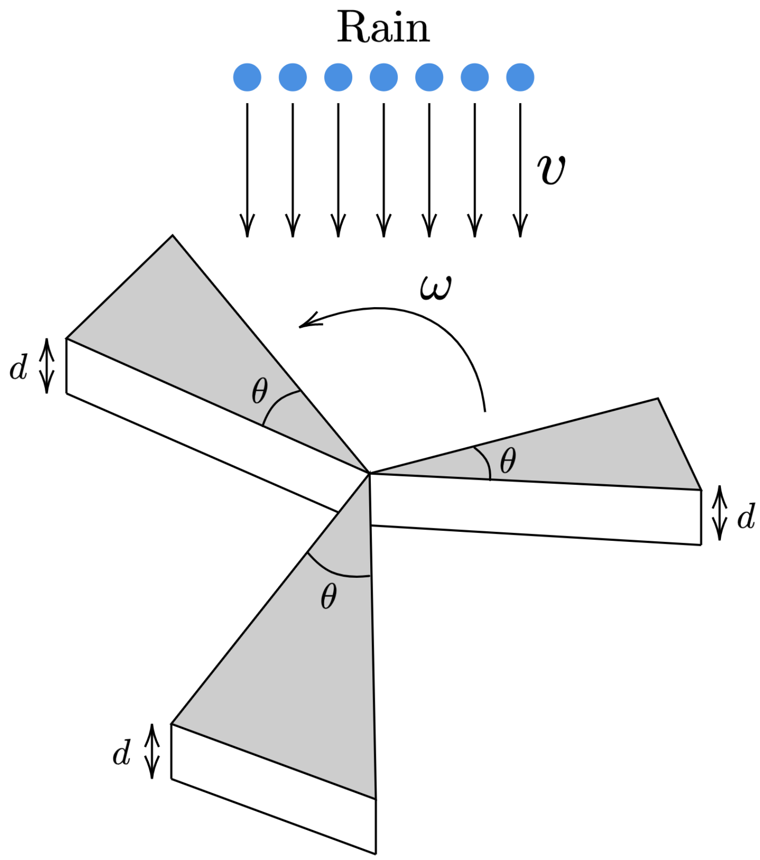
What is the work required to deform a spherical water droplet of radius $r = 1.0$ cm into the shape of a cube? Take the surface tension of water as $\gamma = 7.2 \times 10^{-2}$ N m^{-1}. Neglect gravity.

Leave your answer to 2 significant figures in units of μJ.

Problem 21: Ceiling Fan (3 points)

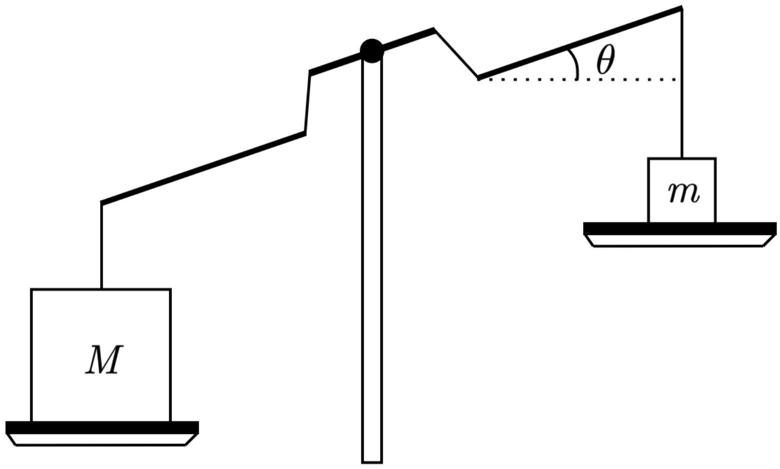
A ceiling fan comprises three horizontal triangular rigid blades arranged symmetrically, as drawn below. It rotates with a constant angular speed ω. Each blade has thickness $d = 0.1$ m and subtends an angle $\theta = \pi/6$ rad. Raindrops from above fall through the ceiling fan with **constant** velocity $v = 8$ m s^{-1} vertically downwards. What is the minimum ω required for every raindrop to be hit by the fan? Assume that the fan rotates about a fixed vertical axis without wobbling.

Leave your answer to 3 significant figures in units of rad s^{-1}.



Problem 22: Balance Scale (4 points)

The first instrument invented for measuring mass was the balance scale, as drawn below. It comprises a beam of negligible weight (as shown in the diagram below) whose point of symmetry is held by a frictionless pivot, with two identical pans attached to its ends. When the two pans contain different masses, the beam deflects towards the heavier end.



Let M and m be the masses of each pan and its respective contents. When $M/m = 2$, the equilibrium angle of the beam from the horizontal is $\theta = 25°$. If this ratio is doubled to $M/m = 4$, what is the new value of θ?

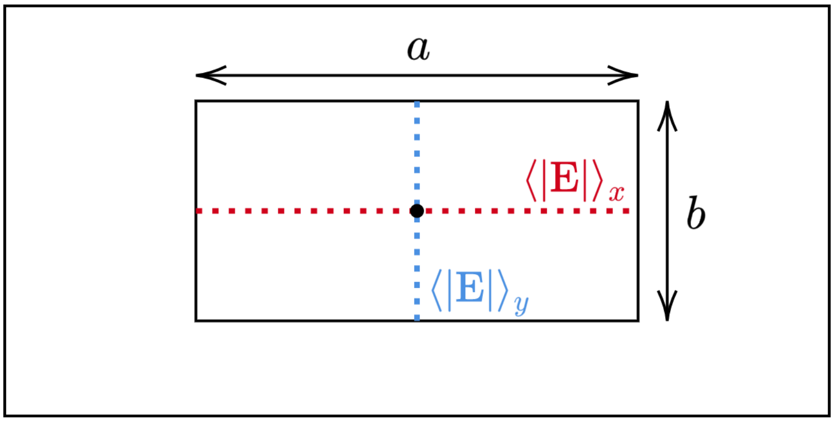
Leave your answer to 2 significant figures in degrees.

Problem 23: Rectangular Cavity (3 points)

Consider a thin charged rectangular plate made of conducting material. A rectangular cavity of horizontal length $a = 12$ cm and vertical length $b = 5$ cm is made at the centre of the plate.

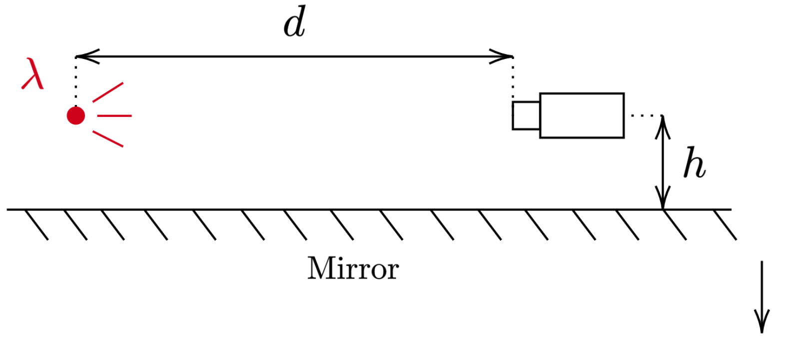
A probe measures the magnitude $|\mathbf{E}|$ of the electric field within the cavity at every point along the horizontal through the cavity's centre. The average magnitude measured is $\langle|\mathbf{E}|\rangle_x$. Then, the probe measures the magnitude of the electric field within the cavity at every point along the vertical through the cavity's centre. The average magnitude measured is $\langle|\mathbf{E}|\rangle_y$. Determine the ratio $\langle|\mathbf{E}|\rangle_y/\langle|\mathbf{E}|\rangle_x$.

Leave your answer to 2 significant figures.

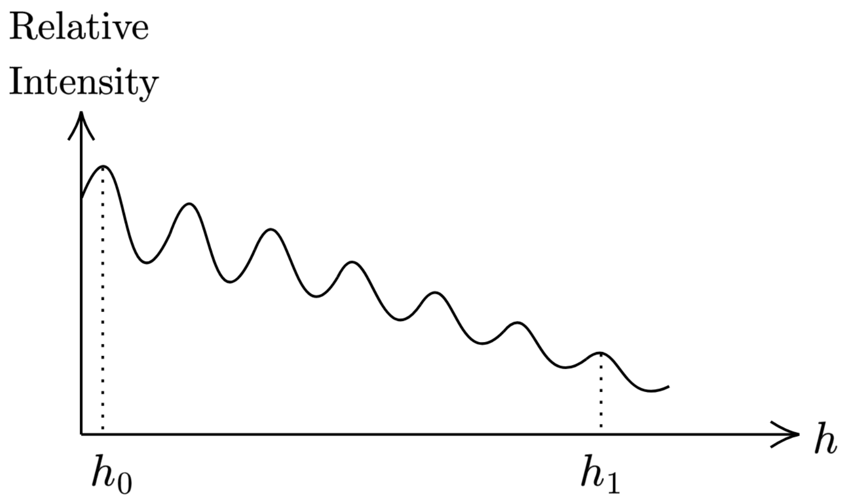


Problem 24: Uncomfortably Close Mirror (4 points)

A point source that emits monochromatic light of unknown wavelength λ and a photometer are placed a distance $d = 30$ cm apart and at some common perpendicular distance h from an adjacent mirror, as shown in the diagram below.



The mirror is then slowly moved from a distance of $h_0 = 1.0$ mm to a distance of $h_1 = 1.2$ mm from the source and photometer, while the resulting light intensity is recorded by the photometer. The graph of relative intensity recorded by the photometer is plotted below.

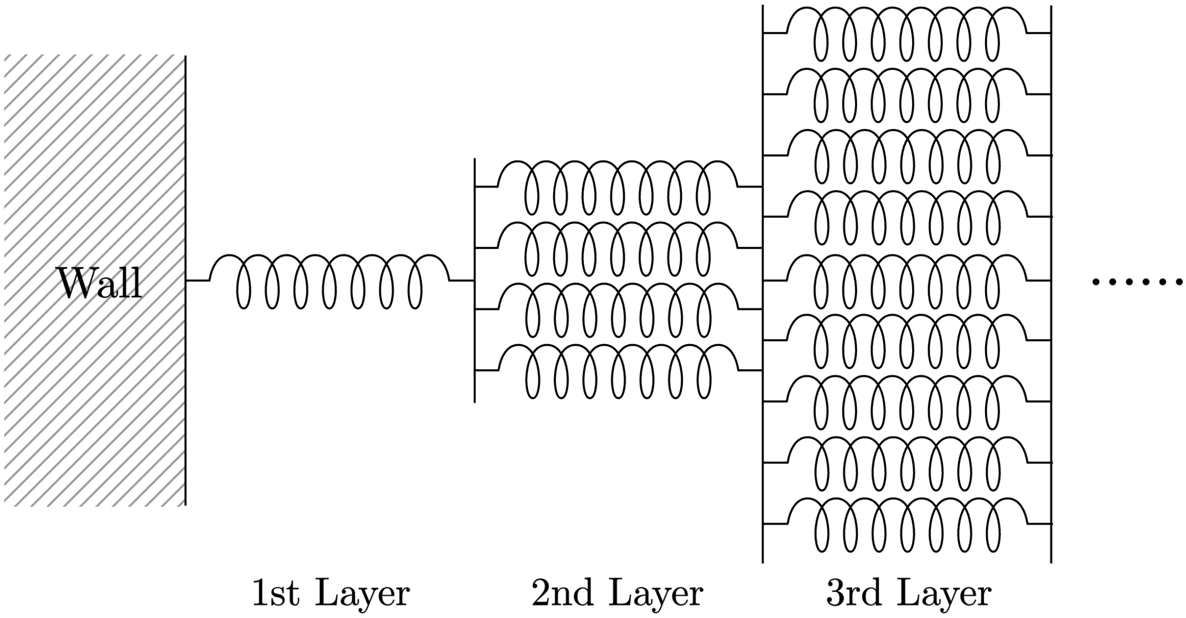


Deduce the wavelength λ of the source.

Leave your answer to 2 significant figures in units of nm.

Problem 25: To Infinity and Beyond (4 points)

Benson has many light and ideal springs. These springs are identical, each having spring constant $k_0 = 1\ \text{N m}^{-1}$. One day, as Benson was playing around, he accidentally arranged an *extremely large* number of them on a table in the following manner:



The arrangement comprises layers of springs separated by light plates. In the i^{th} layer, there are $n_i = i^2$ springs joining the $(i-1)^{\text{th}}$ plate to the i^{th} plate. The spring in the 1^{st} layer joins an immovable wall to the 1^{st} plate.

Benson now attaches a mass $m = 10$ kg to the last plate. He gives the mass a push so that it begins to oscillate. What would be the resultant period T of oscillation? Neglect damping.

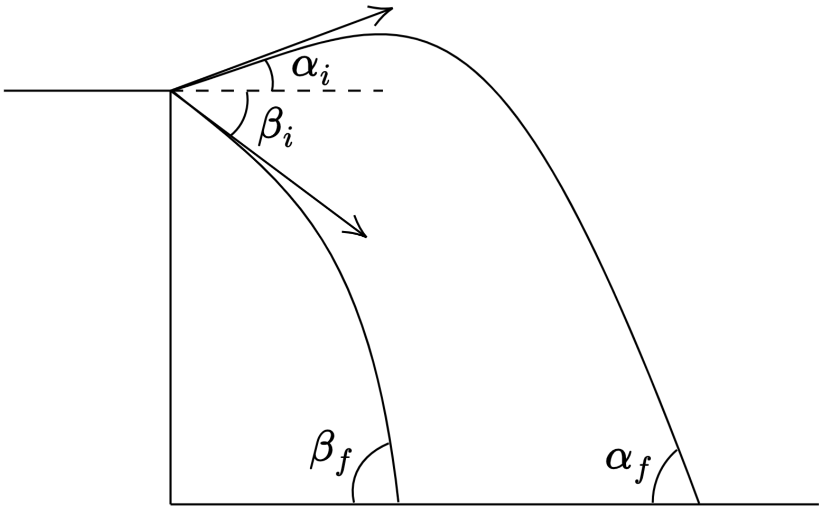
Leave your answer to 2 significant figures in units of s.

Problem 26: Landing Angles (4 points)

A projectile is launched from the edge of a cliff at angle $\alpha_i = 30°$ above the horizontal. At landing, it makes angle $\alpha_f = 60°$ with the flat horizontal ground below the cliff.

A second projectile is launched from the same point and at the same speed as the first projectile, but now at angle $\beta_i = 45°$ below the horizontal. At landing, what angle β_f does the second projectile make with the ground?

Leave your answer to 3 significant figures in units of degrees.



Problem 27: Take a Chance on Me (3 points)

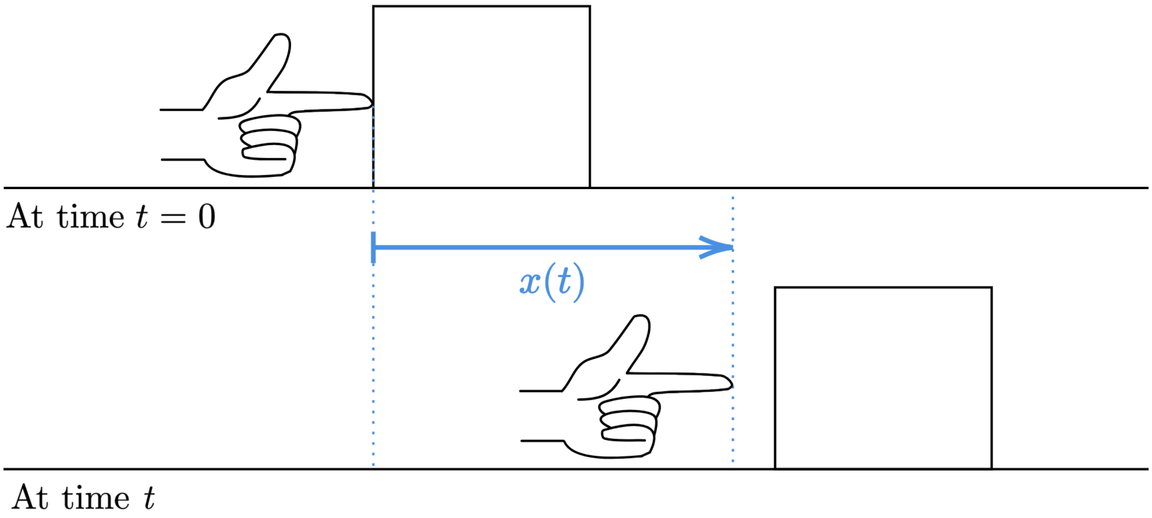
What is the probability that a sample of uranium-235 with mass $M = 15\ \mu\text{g}$ has at least 1 decay within time $T = 0.01$ s? You may take the half-life of uranium to be $t_{1/2} = 703.8$ million years.

Leave your answer to 2 significant figures.

Your answer should range between 0 and 1.

Problem 28: Pushing a Block

A block is placed on a frictionless horizontal table. A finger begins to push the block from its left side. The displacement of the finger at time t (in s) is x (in cm), where x is given by $x = t^2 \sin t$. (The argument of the sin function is taken to be in units of radians.) We set the origin $x = 0$ at the initial position of the block, with x increasing rightwards.



(a) What is the position x_0 of the block when it first loses contact with the finger?

Leave your answer to 2 significant figures in units of cm. (3 points)

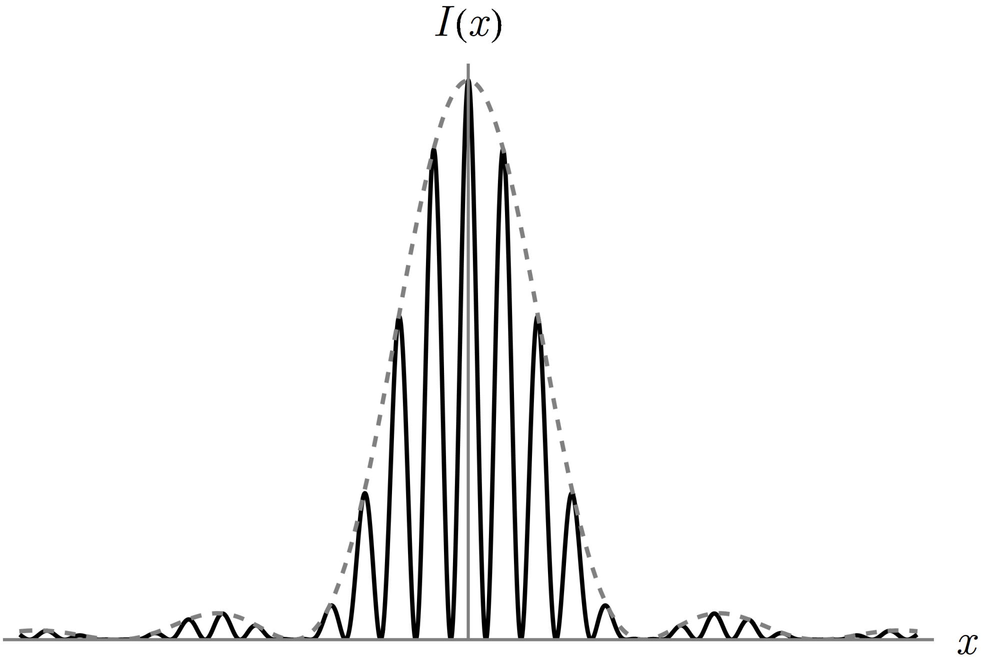
(b) What is the position x_1 of the block when it first regains contact with the finger?

Leave your answer to 2 significant figures in units of cm. (3 points)

If you think that the block does not regain contact with the finger, submit your answer as 0.

Problem 29: Intensity Profiles (3 points)

In a far-field double-slit interference experiment, the two slits are separated by distance D between their centres, and each slit has finite width $D/5$. The resulting intensity profile across the screen $I(x)$ is plotted against position x along the screen, as shown below.



Another interference experiment is conducted under identical conditions, except that the separation between the centre of the slits is now $4D/5$. The width of each slit is unchanged. The new intensity profile obtained is given by $i(x)$.

Calculate the ratio

$$\frac{\int_{-\infty}^{\infty} i(x)\,\mathrm{d}x}{\int_{-\infty}^{\infty} I(x)\,\mathrm{d}x}$$

Leave your answer to 2 significant figures.

Problem 30: A Water Bubble (4 points)

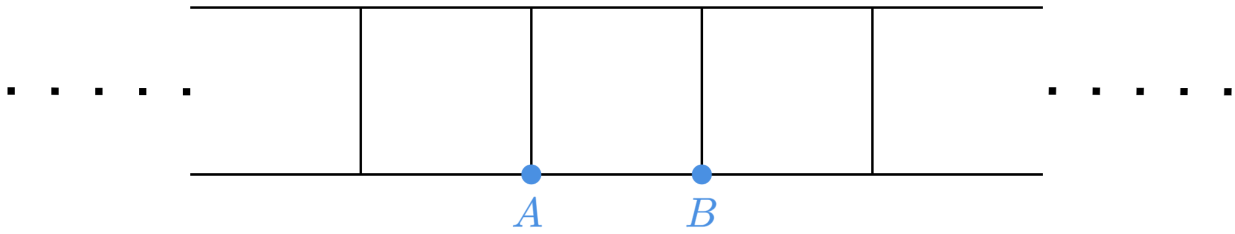
Conventional wisdom suggests that bubbles float. However, under certain conditions, it turns out that bubbles can, in fact, sink!

A bubble contains an ideal gas with molar mass $\mu = 100$ g mol^{-1} and temperature $T = 300$ K. Find the minimum depth h of the bubble below sea level such that it sinks. Assume that Earth's gravitational field is uniform with magnitude $g = 9.81$ m s^{-2}, and that the density of water is uniform at $\rho_w = 1000$ kg m^{-3}. Neglect any change in state of the gas, and take the bubble's radius to be much smaller than h.

Leave your answer to 3 significant figures in units of m.

Problem 31: Do You Prefer 2D or 3D?

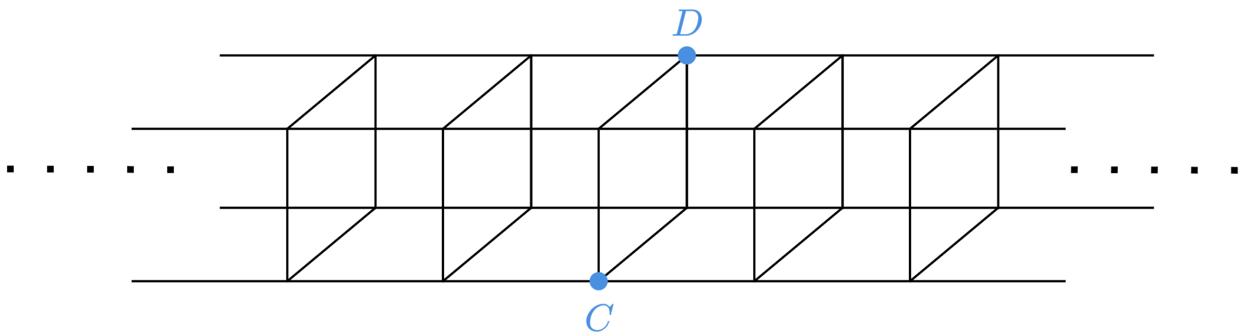
(a) Consider the planar infinitely-extending circuit as shown below, where each edge represents a resistor with resistance $R = 1\ \Omega$.



Let R_{AB} denote the effective resistance between the points A and B. Find R_{AB}.

Leave your answer to 3 significant figures in units of Ω. (4 points)

(b) Now, consider a similar infinitely-extending circuit that is cuboidal rather than planar as shown below, with each edge also having resistance $R = 1\ \Omega$.

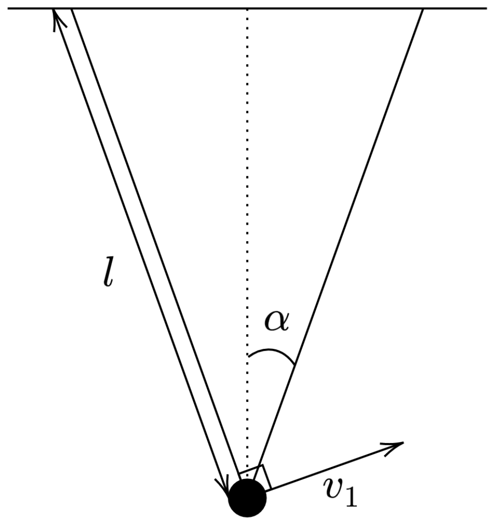


Let R_{CD} denote the effective resistance between the points C and D. Find R_{CD}.

Leave your answer to 3 significant figures in units of Ω. (4 points)

Problem 32: Strange Pendulum

A mass is suspended from the ceiling by two light inextensible strings of equal lengths $l = 3$ m arranged in a "V" shape. The angle between either string and the vertical is $\alpha = \pi/18$ rad. The mass is given a push that causes it to swing rightwards with an initial velocity $v_1 = 0.5$ m s^{-1} perpendicular to the left string. Assume that the small-angle approximation is true, and that all motion takes place in the plane of the figure below.



(a) Find the time taken for the mass to first return to its starting position, t_1.

Leave your answer to 2 significant figures in units of s. (3 points)

(b) Find the time taken for the mass to complete two oscillations, t_{total}.

Leave your answer to 2 significant figures in units of s. (4 points)

Problem 33: Don't Change My Field! (4 points)

A circular loop of radius $r = 1.00$ m and resistance $R = 5.00 \times 10^{-4}\ \Omega$ is fixed and prevented from moving or rotating. It is placed in an external uniform magnetic field B_{ext} that is directed perpendicular to the plane of the loop.

This external field B_{ext} increases with time t as given by $B_{\text{ext}}(t) = B_0 e^{\alpha t}$. As such, the net magnetic field at the loop's centre $B_{\text{loop}}(t)$ also increases over time, but at a slower rate due to Lenz's law. In fact, the ratio $B_{\text{loop}}(t)/B_{\text{ext}}(t)$ is always constant for all time t. Find this ratio.

Take $\alpha = 1.00\ \text{s}^{-1}$; you may treat this to be sufficiently small for the quasistatic approximation to be valid.

Leave your answer to 3 significant figures.

Problem 34: I Thought Solutions were Unique

Suppose that we have an exotic gas that has the strange property of “reverse drag” – it increases the velocity of anything in it. A particle of mass $m = 1$ kg is placed in this gas. We denote its velocity to be $v(t)$ at time t after entering the gas. The force exerted by the gas on the particle is given by $k\sqrt{|v(t)|}$, where $k = 1$ N m$^{-1/2}$ s$^{1/2}$.

(a) Given $v(0) = 1$ m s^{-1} and $v(2) = 4$ m s^{-1}, find $v(12)$.

Leave your answer to 2 significant figures in units of m s^{-1}. (2 points)

(b) Given $v(0) = 0$ m s^{-1} and $v(2) = 1$ m s^{-1}, find $v(12)$.

Leave your answer to 2 significant figures in units of m s^{-1}. (2 points)

(c) Given $v(0) = 0$ m s^{-1} and $v(8) = 4$ m s^{-1}, find $v(12)$.

Leave your answer to 2 significant figures in units of m s^{-1}. (3 points)

Problem 35: Mirror Mirror on the Wall

The curator of the museum *Texallate* wishes to create a room with mirrors as walls for a museum installation. The room is a polygon with straight vertical walls of equal length. The curator stands in the centre of the room and shines his flashlight centered along a line of symmetry, whose beam can be modelled as a circular sector with beam angle α. You may take the system to be 2D – the flashlight is shone parallel to the floor.

(a) The first room the curator makes is a rhombus, with an interior angle of $\theta = 60°$. As the mirrored walls aren't polished yet, the light from the curator's flashlight can be reflected 1 time (totalled across all mirrors) before being fully dissipated. What is the minimum beam angle α of his flashlight so that light reaches every part of the room?

Leave your answer to 3 significant figures in units of degrees. (3 points)

(b) To save costs, the museum's director suggests that the curator build a rectangular room instead. The room has length $l = 2$ m and width $w = 1$ m. With the walls now polished, each light ray can be reflected $n = 100$ times (totalled across all mirrors) before being fully dissipated. What is the minimum beam angle α of his flashlight so that light reaches every part of the room?

Leave your answer to 3 significant figures in units of degrees. (3 points)

Problem 36: Oscillating E (4 points)

A few students are given an electrical source that supplies AC voltage at a fixed amplitude and frequency $f = 50.0$ Hz. They are tasked to set up an oscillating electric field whose amplitude is as large as possible. The electric field is to be set up between the plates of a capacitor with capacitance $C = 100\ \mu$F.

Shuan says, "The best way to do this would be to directly connect the plates of the capacitor to the AC source." This way, he achieves amplitude E for the electric field.

But Chris objects, "No, an increased amplitude E' for the electric field could be achieved by connecting an inductor in series with the capacitor!"

Surprisingly, Chris is right. If he uses an inductor with self-inductance $L = 80.0$ mH, what is the ratio E'/E? You may neglect any resistance present in the circuit.

Leave your answer to 3 significant figures.

Problem 37: Weird Flux But Ok

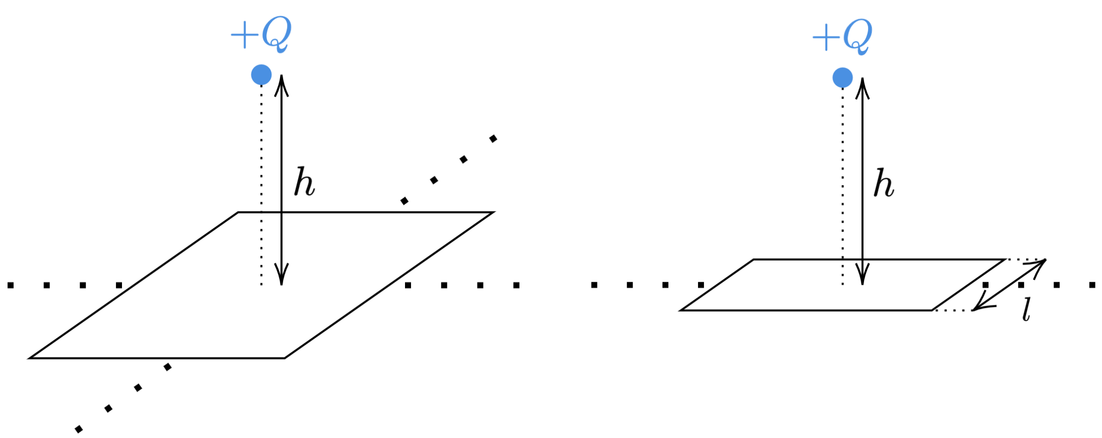
A point charge $Q = +2.5$ nC is placed at vertical distance $h = 0.15$ m above the centre of a horizontal rectangular insulating plate. Determine the total electric flux through the plate if...

(a) the plate has infinite length and breadth.

Leave your answer to 2 significant figures in units of V m. (2 points)

(b) the plate has infinite length and finite breadth $l = 0.10$ m.

Leave your answer to 2 significant figures in units of V m. (4 points)



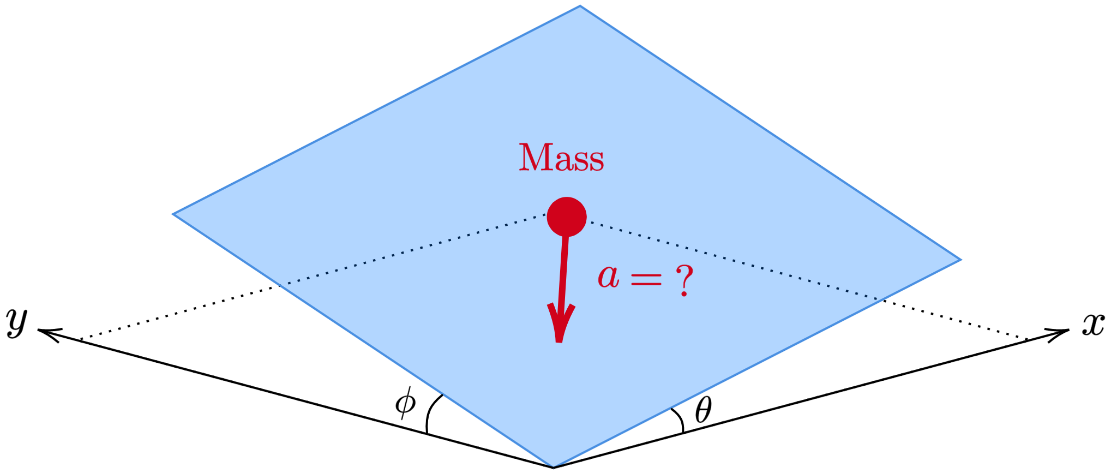
(a) spans infinitely in all directions

(b) spans infinitely in length but not in breadth

Problem 38: Inclined Plane (5 points)

A fixed frictionless plane is inclined at angle $\theta = 30^\circ$ above the x-axis and angle $\phi = 60^\circ$ above the y-axis, where the xy-plane is horizontal. Find the acceleration a of a mass that slides down the inclined plane.

Leave your answer to 2 significant figures in units of m s^{-2}.



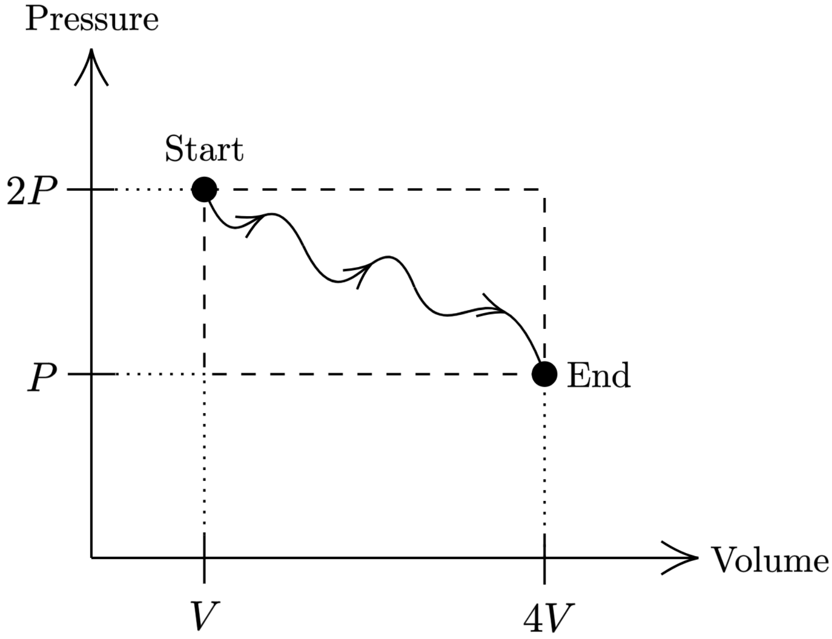
Problem 39: Strategic Syringe (5 points)

Ideal monatomic gas is contained within a sealed syringe. The piston in the syringe can be moved such that the volume can be freely varied between V and $4V$. Simultaneously, valves in the syringe allow the pressure to be freely controlled between P and $2P$.

The gas starts out at pressure $2P$ and volume V. It is to be brought to final pressure P and volume $4V$. Depending on how the piston and the valves are controlled over time, the heat absorbed by the gas in the process can range between Q_{min} and Q_{max}. Find the ratio $Q_{\text{max}}/Q_{\text{min}}$.

Assume that the piston and the valves are controlled such that the process undergone by the gas is reversible, and that the path traced by the process on the pressure-volume graph does not self-intersect.

Leave your answer to 2 significant figures.



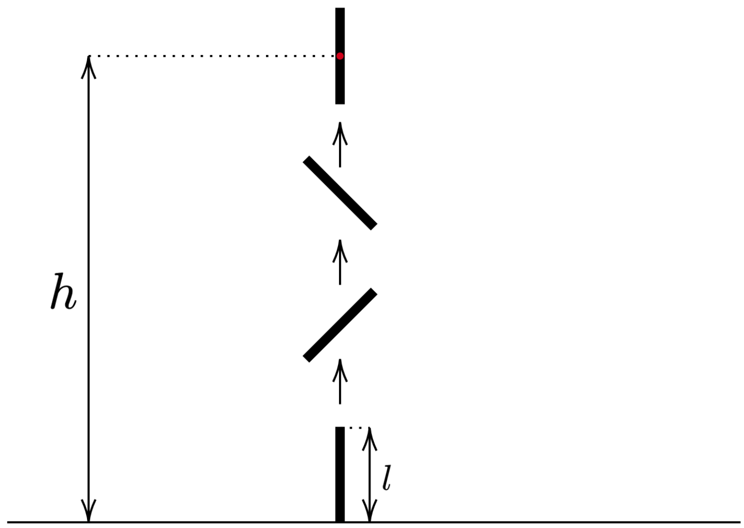
Problem 40: Optimised Backflip (5 points)

Let's investigate the most efficient backflip physically possible. In a perfect backflip, the person starts out standing straight on his feet. He jumps, and his body rotates mid-air such that by the time he returns to the ground, he lands vertically on his feet.

As a toy model of a backflip, consider the person's body to be a uniform rigid stick of length l that initially stands on the ground with its axis vertical. Suppose that the stick is imparted the theoretical minimum energy required to perform a perfect backflip. As a result, the stick's centre reaches maximum height h above the ground. Find the ratio h/l.

Neglect air resistance.

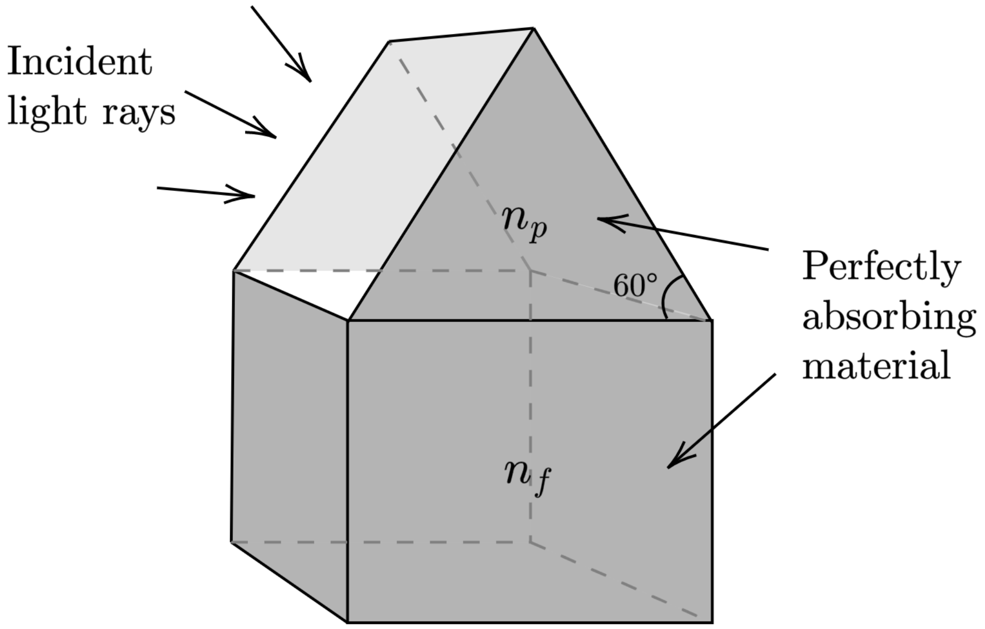
Leave your answer to 3 significant figures.



Problem 41: Stop Blocking My View (5 points)

A silicon carbide prism of refractive index $n_p = 2.63$ is placed above a tank. The tank is in the shape of a cuboid, open at the top, and is completely filled with fluid. The cross section of the prism is in the shape of an equilateral triangle. The outer surface of the prism-and-tank setup is covered entirely by perfectly absorbing material, except for one of the slanted faces of the prism. What is the minimum refractive index n_f that the fluid must have such that any observer outside the set-up is unable to see light from inside the fluid?

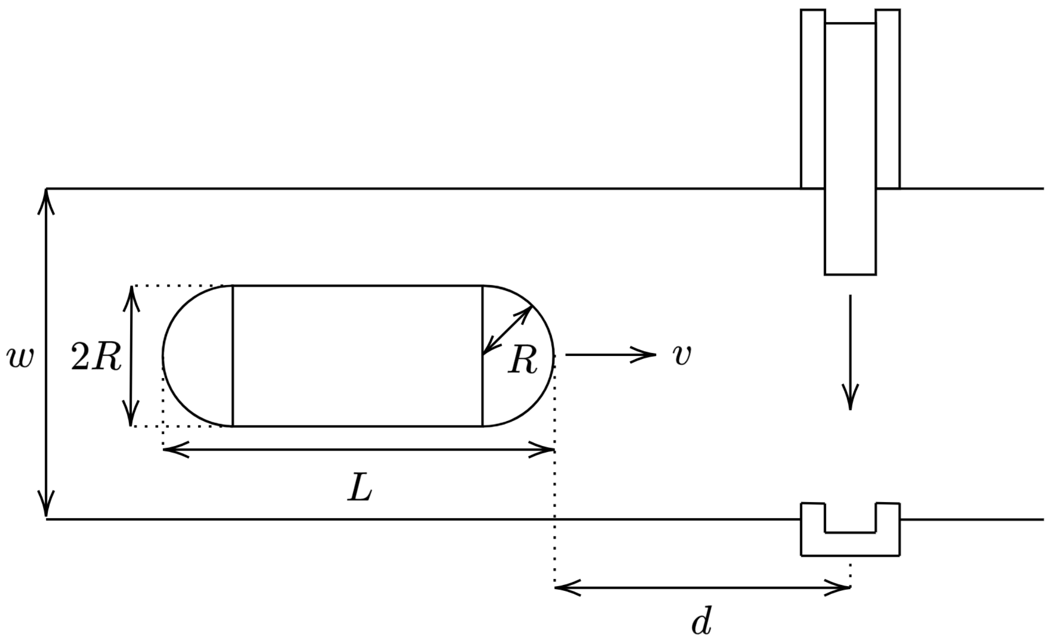
Leave your answer to 3 significant figures.



Problem 42: Closing Gate (5 points)

A car has semicircular bumpers on the front and back ends of an otherwise rectangular body. Each bumper has radius $R = 1.0$ m. Overall, the car has width $2R$ and length $L = 4.0$ m, as shown below. It drives at a steady speed along the center of a road of width $w = 10$ m towards a sliding gate at distance $d = 50$ m ahead of the car, which has just started closing at a constant rate. Given that the gate takes time $\Delta t = 30$ s to close fully, what minimum speed v must the car travel at in order to make it through the gate without contacting the gate?

Leave your answer to 3 significant figures in units of m s^{-1}.



Problem 43: Solar Eclipse (5 points)

Consider a parallel universe where, just like our own universe, an Earth month is $T_M \approx 30$ days long, an Earth year is $T_S \approx 365$ days long, the Moon has radius $r_M \approx 1,700$ km, and the Sun has radius $r_S \approx 700,000$ km. However, in this universe, the ratio m_E/m_S (where the Earth's mass is m_E and the Sun's mass is m_S) is different from ours. We do not know the precise value of this ratio.

But we do know one thing: While the Earth of this universe can sometimes observe *partial* solar eclipses, *total* solar eclipses can **never** happen. Using this fact, find the minimum possible value of $\ln(m_E/m_S)$.

You may assume the following properties of this parallel universe:

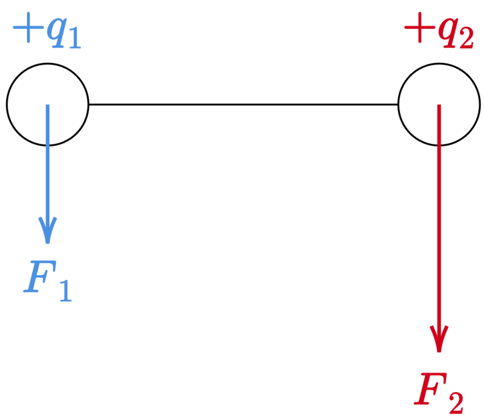
- The same laws of physics apply
- The mass of Moon is very small compared to the mass of Earth, and the mass of Earth is very small compared to the mass of Sun
- The radius of Earth is very small compared to the Sun-Earth distance and the Moon-Earth distance
- All orbits are circular
- The centres of the Sun, Earth and Moon always lie on the same plane

Leave your answer to 2 significant figures.

Problem 44: Unbalanced Fall (6 points)

Two point charges $+q_1$ and $+q_2$, each of equal mass $m = 1.0$ kg, are joined to opposite ends of a massless rigid rod. There is a uniform downward electric field that exerts constant downward forces $F_1 = 1.0$ N and $F_2 = 5.0$ N on charges q_1 and q_2 respectively. The charges are simultaneously released from rest with the rod initially horizontal. In its subsequent motion, find the maximum downward acceleration of charge q_1. Neglect gravity.

Leave your answer to 2 significant figures in units of m s^{-2}.



Problem 45: Leonard the Oiler (5 points)

Leonard proposes a machine with a frictionless piston in an infinitely extending cylinder, contained in which is an ideal gas.

The piston starts off at infinity, with work W_1 done on the gas to compress it isothermally until the gas reaches $P_0 = 120$ kPa and a finite volume $V_0 = 1$ m^3. The gas then undergoes an alternating series of reversible processes: first, an isobaric expansion to increase the volume by $\Delta V = +V_0$, followed by an isochoric reduction in pressure to restore it to its initial temperature. These iterations are performed until the piston returns to infinity, and the total work done by the gas during these iterations is W_2.

Determine the net work done **by** the gas throughout the entire procedure, $W = W_2 - W_1$.

Leave your answer to 2 significant figures in units of kJ.

Problem 46: Rebounding Particles

A neutral atom of unknown rest mass m_n is travelling rightwards, towards a positive ion of unknown rest mass m_p and charge $+e$ (where e is the elementary charge) that is travelling leftwards. There is also an electric field $E = 1.0$ kV m^{-1} pointing rightwards. Both particles collide with each other at the origin, with equal and opposite velocities of magnitude $v_0 = 0.50c$ at the instant of collision.

After the collision, they stick together. The resulting composite particle moves leftwards for a while, before reversing and returning to the origin at a time $\Delta t = 15.0$ ms after the collision. If this composite particle is determined to have total (relativistic) energy $E_T = 30.0$ GeV upon returning to the origin, find

(a) the original rest mass of the neutral atom m_n.

Leave your answer to 2 significant figures in units of u (atomic mass unit), where 1 $u = 1.66 \times 10^{-27}$ *kg.* (3 points)

(b) the original rest mass of the positive ion m_p.

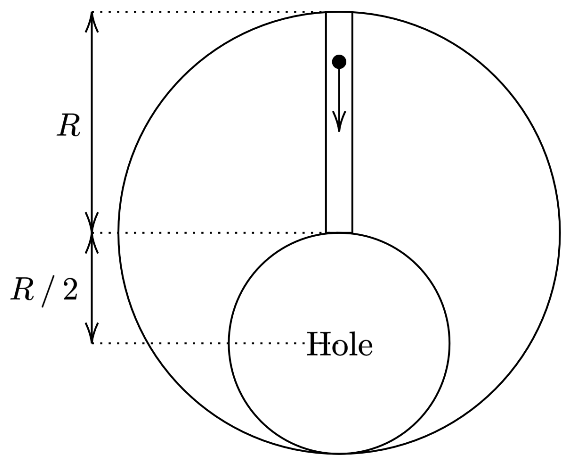
Leave your answer to 2 significant figures in units of u (atomic mass unit), where 1 $u = 1.66 \times 10^{-27}$ *kg.* (3 points)

Problem 47: Journey to the Centre of the Earth (8 points)

A large spherical hole with radius $R/2 = 3200$ km is drilled in the Earth such that it is tangent to the surface of the Earth. A tunnel is then drilled along the common axis of the Earth and the hole. A stone is dropped from rest into the tunnel from the end that is further from the hole. Find the maximum distance the stone will fall from its starting position.

Assume that the Earth is a perfect sphere of uniform density with radius $R = 6400$ km, and neglect effects due to friction and the rotation of the Earth.

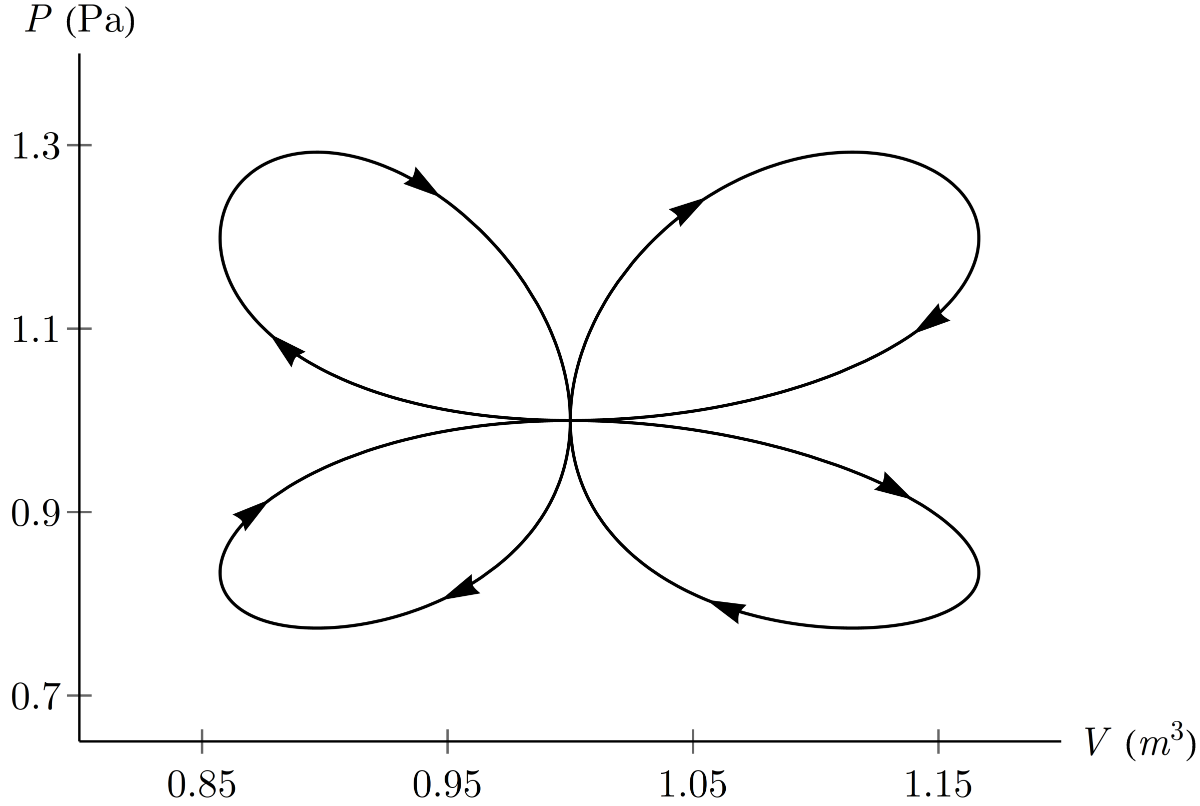
Leave your answer to 3 significant figures in units of km.



Problem 48: Entropic Star (7 points)

Consider n mol of ideal monatomic gas that undergoes a reversible thermodynamic cycle. Let P and V respectively denote the pressure and volume of the gas at any point in the cycle. The $P-V$ graph is given by the following parametric equation for $0 \leq t \leq 2\pi$:

$$(P, V) = \left(\left(e^{\frac{1}{3}\sin 2t \cos t}\right)\text{Pa}, \left(e^{\frac{1}{5}\sin 2t \sin t}\right)\text{m}^3\right)$$



Suppose that the maximum entropy reached by the gas throughout the cycle is $S_{\max}$, while the minimum entropy it reaches is $S_{\min}$. Letting R denote the molar gas constant, compute the dimensionless quantity:

$$\left(\frac{S_{\max} - S_{\min}}{nR}\right)^2$$

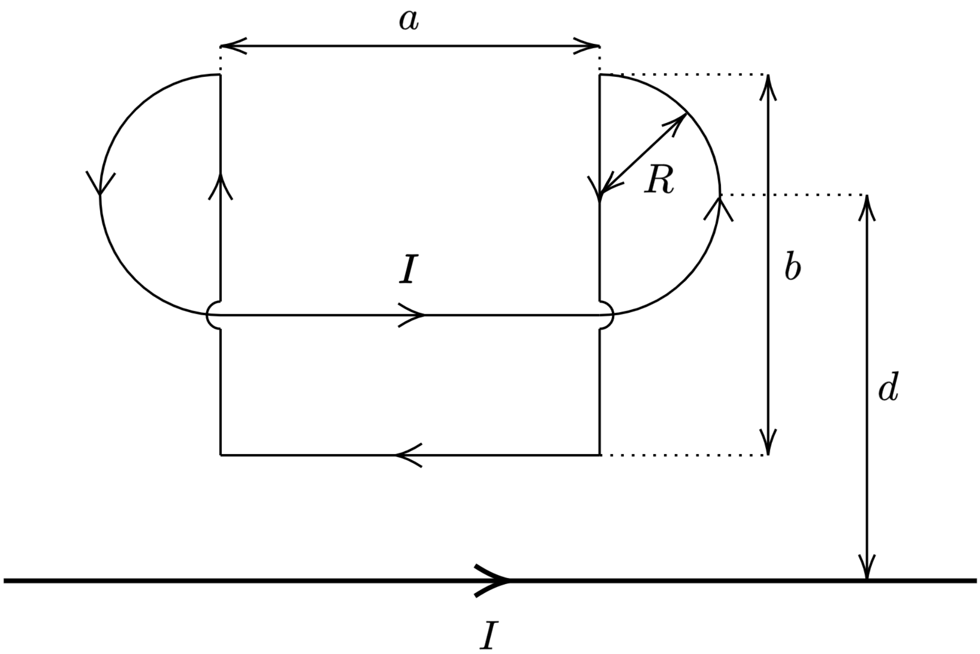
Leave your answer to 2 significant figures.

Problem 49: Strangely Shaped Current Loop (7 points)

A rigid wire loop is made of two semicircular arcs of radius $R = 0.50$ m, and these arcs are connected by two horizontal segments of length $a = 1.00$ m and two vertical segments of unknown length b, as drawn below. (The loop appears to overlap itself in the diagram but small bends in the wire outside the plane of the loop ensure it does not self-intersect. The effect of these bends are negligible beyond preventing self-intersection, and the loop is otherwise planar.)

A fixed long straight wire coplanar with the loop is located parallel to and a distance d from a line segment joining the centers of the semicircular arcs. The same current $I = 200$ A is passed through both the wire and the loop, as shown below. If the loop settles into stable equilibrium when $d = 1.70$ m, find b. Neglect gravity.

Leave your answer to 3 significant figures in units of m.



Hint: The following integral might be useful, where $|\alpha| < 1$:

$$\int_0^\pi \frac{1}{1 + \alpha \cos\theta}\, d\theta = \frac{\pi}{\sqrt{1 - \alpha^2}}$$

Problem 50: Falling Hinges

A uniform rod of length $L = 4.20$ m is fixed to a pivot, A, around which it can freely rotate. Its other end is attached to the end of an identical rod at H, forming a freely rotating hinge. The remaining end of the second rod, B, carries a ball with the same mass as a single rod. The ball is constrained to only slide frictionlessly along the vertical line passing through A. The rods are initially held horizontal and are then released from rest. Let the angle formed by either rod with the horizontal be $\theta(t)$ where t is the time after release.

(a) At any time, the magnitude v of the velocity of the ball can be expressed as

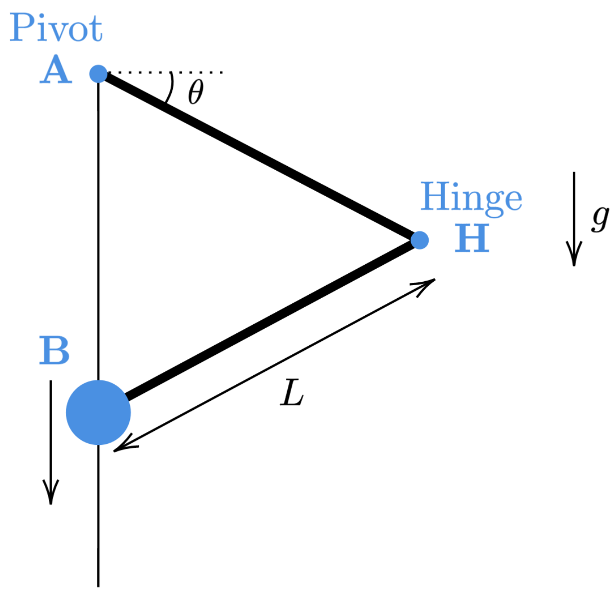
$$v = \alpha L \frac{\mathrm{d}\theta}{\mathrm{d}t} \cos\theta$$

where α is a positive numerical coefficient. Find α.

Leave your answers to 3 significant figures. (2 points)

(b) At what angle θ_* would v be maximal?

Leave your answers to 3 significant figures in units of degrees. (8 points)



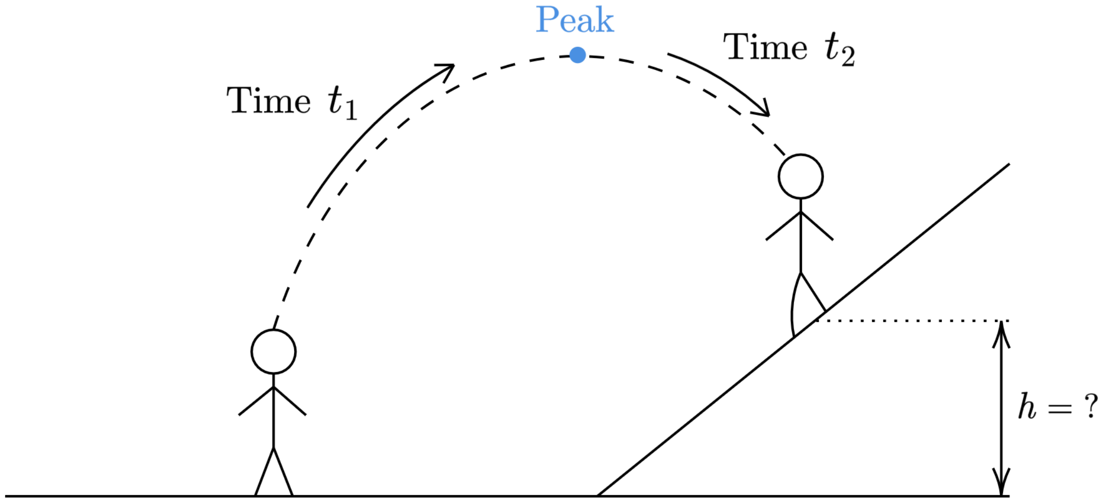
Half Hour Rush M1: I Have The High Ground (3 points)

In *Star Wars: Revenge of the Sith*, Obi-Wan and Anakin engage in a fierce duel on the planet Mustafar. When Obi-Wan reaches a position higher than Anakin, he exclaims, "It's over Anakin! I have the high ground!" In response, Anakin jumps towards Obi-Wan.

Based on the movie timestamps, he takes approximately $t_1 = 1.6$ s to reach the peak of his trajectory, and another $t_2 = 1.2$ s thereafter to reach Obi-Wan's position.

Use this information to determine the height difference h between Obi-Wan and Anakin's initial position. You may assume that the gravitational field on the surface of Mustafar is the same as that on Earth.

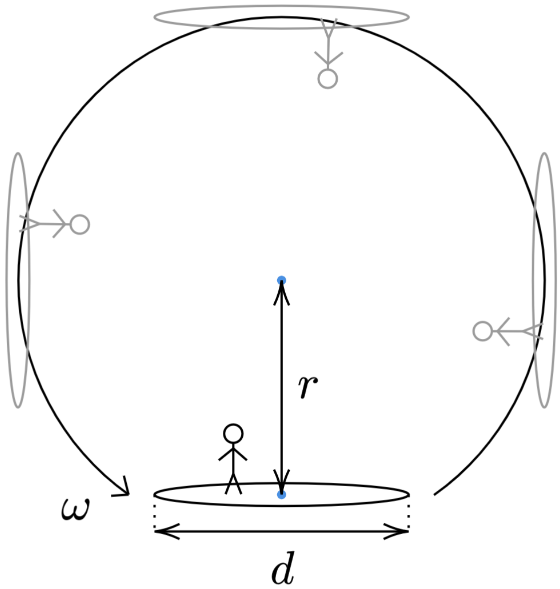
Leave your answer to 2 significant figures in units of m.



Half Hour Rush M2: Millennium Falcon (4 points)

In the *Star Wars* franchise, passengers in the Millennium Falcon appear to experience gravitational forces similar to those on Earth. While this is explained with fictional devices, artificial gravity is, in fact, possible in space, through rotation.

Consider the Falcon as a disc with diameter $d = 25.6$ m, which rotates with constant angular velocity about an axis parallel to the disc's flat surface at perpendicular distance $r = 50.0$ m away from the disc's centre, as illustrated below.



Luke stands on the flat surface of the disk and releases a ball from his position. Depending on where he stands, the initial magnitude of acceleration of the ball from his perspective differs. Denoting the maximum and minimum initial accelerations of the ball as a_{max} and a_{min} respectively, find the ratio $a_{\text{max}}/a_{\text{min}}$.

Assume there are no celestial bodies nearby, and that the ball's initial height above the floor is much smaller than r.

Leave your answer to 3 significant figures.

Half Hour Rush M3: Death Star (4 points)

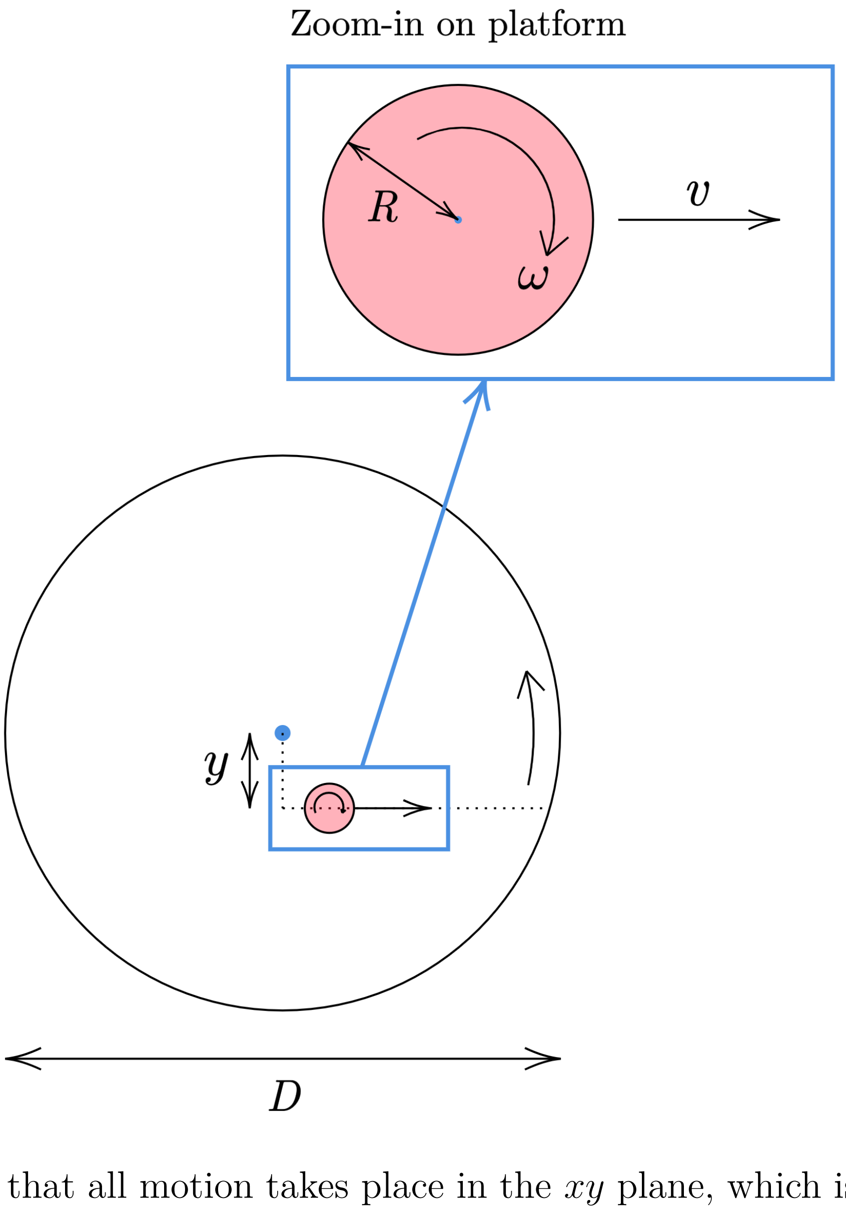
In the original 1977 *Star Wars* film, the Death Star, a space station of the Galactic Empire, annihilated a rebellious planet, Alderaan, in a matter of seconds using a superlaser, reducing it to a cloud of small asteroids scattered very far apart. Suppose that Alderaan was a uniform spherical planet of radius $R = 6000$ km and mass $M = 6.00 \times 10^{24}$ kg, similar to our Earth. If the Death Star had completed its deed within a time period of $\Delta t = 10.0$ s, what was the minimum average power P that the Death Star had to supply over this period to achieve this feat of destruction?

Leave your answer to 2 significant figures in units of $L_\odot$ *(solar luminosity), where* 1 $L_\odot$ *is the power output of a single Sun, or* 1 $L_\odot = 3.828 \times 10^{26}$ W.

Half Hour Rush M4: I Am The Senate (5 points)

In *Star Wars: Revenge of the Sith*, Yoda fights with Palpatine in the Senate chamber. Assume that the Senate is fixed in place and is circular with diameter $D = 100$ m.

Yoda stands $y = 20$ m south of the centre and hurls a spinning platform towards Palpatine in the x-direction as shown below. Assume the platform is a uniform disc of mass $M = 100$ kg and radius $R = 1$ m, with clockwise spin $\omega_0 = 1$ rad s^{-1} and speed $v_0 = 4$ m s^{-1}. However, he narrowly misses, and the platform collides inelastically with the walls of the Senate. After collision with the walls, the platform rolls along the walls of the Senate without slipping.



You may assume that all motion takes place in the xy plane, which is horizontal, and that there is no gravity. Find the time taken t for the platform to complete 1 full revolution around the Senate. Do not include the time before the collision.

Leave your answer to 3 significant figures in units of s.

Half Hour Rush E1: Cheap Christmas (3 points)

It's the most wonderful time of the year, and Paul needs to power all $n = 30$ of his identical Christmas lights via a constant voltage source. But Paul is broke, and electricity is expensive, so he seeks the cheapest way to do this!

Depending on how he wires the lights, the total light output may vary, and the total power they consume may range between P_{min} and P_{max}. Calculate $P_{\text{max}}/P_{\text{min}}$.

Leave your answer to 2 significant figures.

Half Hour Rush E2: Cutting Costs (3 points)

Paul needs to build a circuit that ordinarily uses a silver wire of length $L = 5.00$ m and resistance $R = 0.10\ \Omega$, but Paul switched the wire to copper instead to save on costs, as he is a very broke man. If the new wire still has the same length and resistance, how much money did Paul save with this move? Use the properties of silver and copper as listed below.

Metal	Cost c	Electrical Resistivity r	Density ρ
Silver	\$722.43 per kg	$1.63 \times 10^{-8}\ \Omega$ m	10490 kg m^{-3}
Copper	\$9.68 per kg	$1.68 \times 10^{-8}\ \Omega$ m	8960 kg m^{-3}

Leave your answer to 2 decimal places in units of dollars (\$).

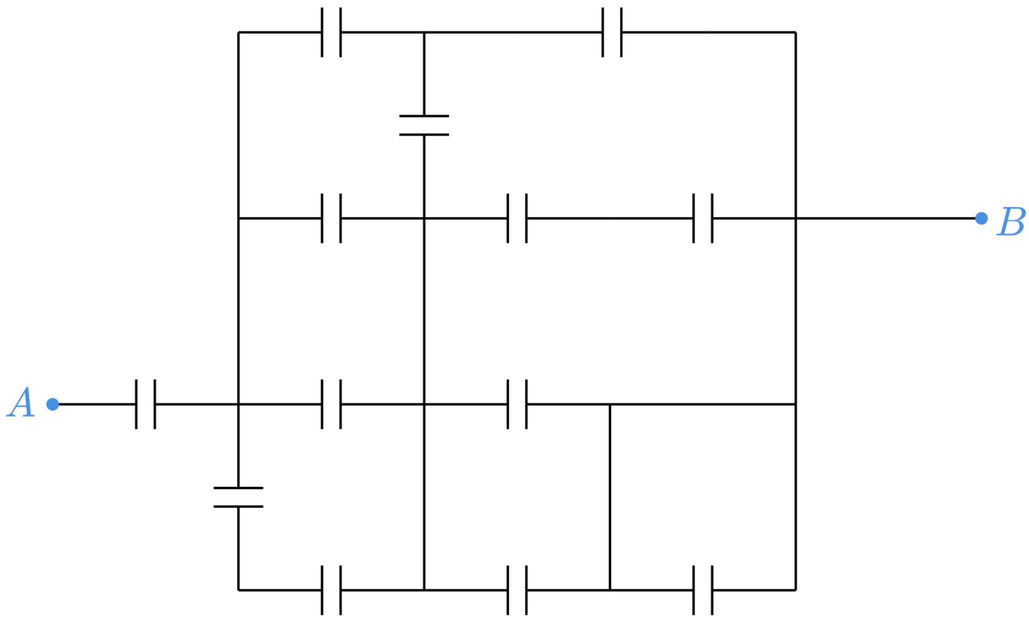
Half Hour Rush E3: Careful Construction (5 points)

Paul has a large number of $R = 1\ \Omega$ resistors, which he wants to arrange (using only series and parallel connections) into a network with an equivalent resistance of $R_{\text{eff}} = \frac{13}{19}\ \Omega$. To save money, he wants to use as few resistors as possible. What is the minimum number of $1\ \Omega$ resistors he needs? You may assume all wires are ideal.

Leave your answer as an integer.

Half Hour Rush E4: Capacitor Chaos (4 points)

The following circuit consists of identical capacitors, each of capacitance $C_0 = 2200$ F. It's extremely wasteful, and Paul wishes to be prudent. Hence, he replaces this with a single capacitor of capacitance C_{eq} between A and B. Find C_{eq}.



Leave your answer to 3 significant figures in units of F.

Half Hour Rush X1: Shrinking a Grape (3 points)

A spherical grape of radius $R = 2$ cm is dried in an oven. After a time $T = 24$ h in the oven, the grape has turned into a spherical raisin of radius $r = 1$ cm. Assuming that the missing volume is due entirely to evaporated water, and that all the energy from the oven is used to evaporate the water, what is the average power supplied by the oven?

Leave your answer to 2 significant figures in units of W.

Half Hour Rush X2: Throwing a Fish (3 points)

During lunch, Chris was mildly irritated to find that a piece of fish in his food had already become cold. He grabs the fish and throws it repeatedly against the table in an attempt to heat it up. If the piece of fish has mass $M = 300$ g and specific heat capacity $c = 1.67$ kJ kg^{-1} K^{-1} and it is thrown against the table uniformly at a speed of $v = 3.0$ m s^{-1} each time, what is the minimum number of throws N that Chris needs to heat the fish from the room temperature of $T_0 = 25°$C to a comfortably warm temperature of $T_1 = 60°$C? Assume that the fish remains intact throughout the throwing process.

Leave your answer to 2 significant figures.

Half Hour Rush X3: Cloudy With A Chance of Meatballs (3 points)

A meatball of mass $m = 60$ g and specific heat capacity $c = 1670$ J kg^{-1} K^{-1} is cooked by dropping it from height $h = 830$ m onto a plate. The drag force, F, is given by $F = kv^2$ where $k = 1.18 \times 10^{-3}$ kg m^{-1} and v is the velocity of the meatball.

In order to heat the meatball to $T = 175°\text{C}$ from an initial temperature $T_0 = 25°\text{C}$, what initial velocity, v_0, does the meatball have to be dropped with? You may assume that the meatball reaches terminal velocity quickly, and neglect heat loss to the surroundings. Assume that Earth's gravitational field is uniform with magnitude $g = 9.81$ m s^{-2}.

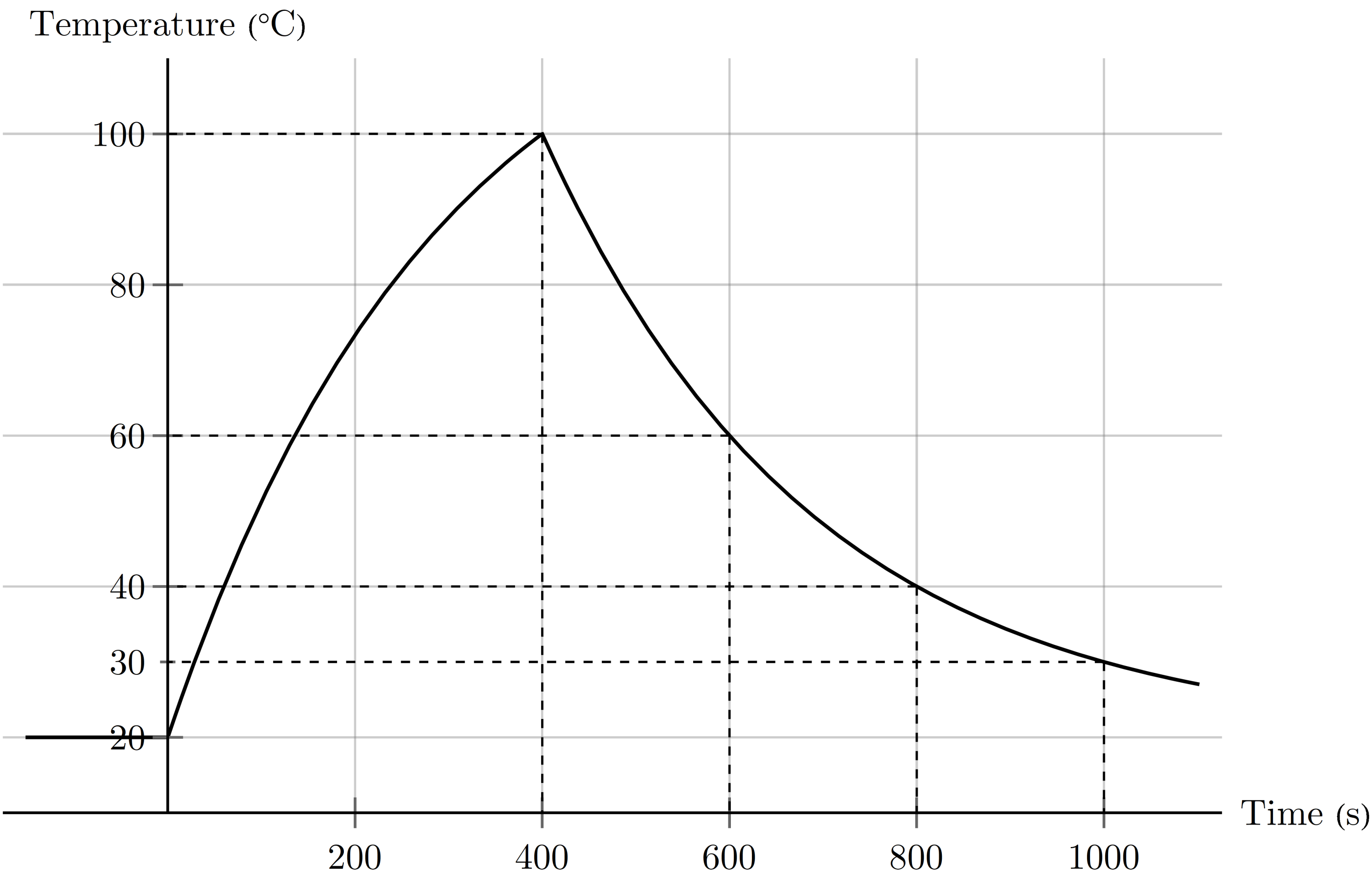
Leave your answer to 2 significant figures in units of m s^{-1}.

Half Hour Rush X4: Mala Hotpot (5 points)

Shuan is craving for some Mala soup, but he hasn't cooked in years, so he first checks that his stove is working. He fills a pot with mass $m = 1.0$ kg of water initially at room temperature $T_s = 20°\text{C}$. At time $t = 0$ s, he turns on the stove.

Later on, the moment he notices water coming to a boil, he switches off the stove. The graph of the water's temperature against time t is plotted below. Use this information to find the power P supplied by the stove to the water.

The stove's power and the mass of water in the pot may be assumed to be constant over time.



Leave your answer to 3 significant figures in units of W.