3rd Singapore Physics League

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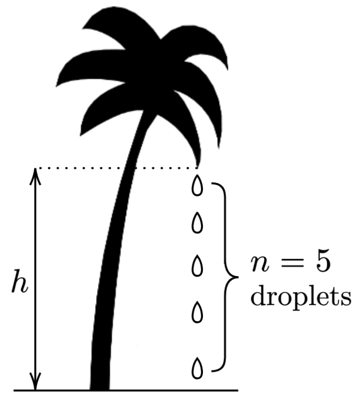
Problem 1: Wet Tree (3 points)

A tree is wet after a rain and slowly drips water, with one droplet falling from rest every $t = 1$ s. At any time, exactly $n = 5$ droplets can be observed mid-air. Determine the height h of the tree. Neglect air resistance.

Leave your answer to 2 significant figures in units of m.

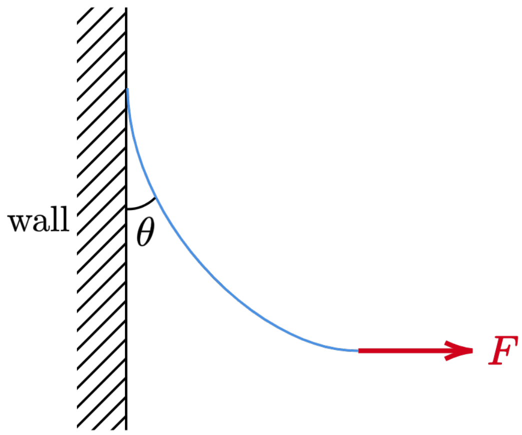


Problem A: Mysterious Rope (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

A rope is connected to a vertical wall at one end, and a horizontal external force $F = 15.0$ N pulls on the other end. The rope is in equilibrium and makes an angle $\theta = 25.0°$ with the wall. What is the weight W of the rope?

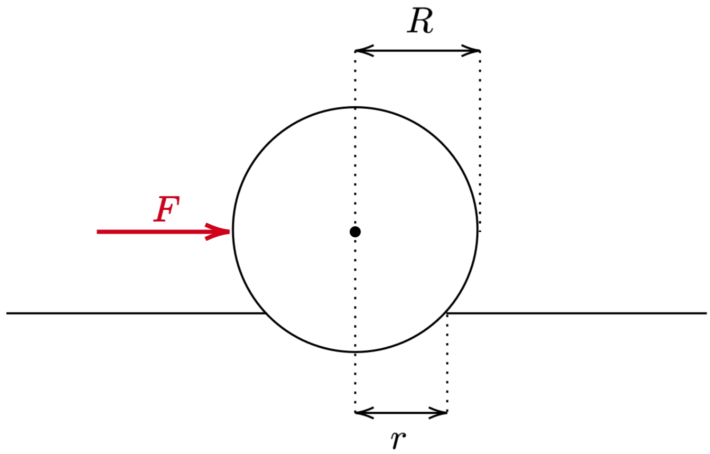
Leave your answer to 3 significant figures in units of N.



Problem B: Stuck Sphere (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

A uniform sphere of mass $m = 1$ kg and radius $R = 5$ cm is partially lodged within a circular hole of radius $r = 3$ cm on a flat horizontal surface. We apply a constant horizontal force F towards the sphere's centre. What is the minimum F required to remove the sphere from the hole? Assume that all surfaces are sufficiently rough such that the sphere never slips.



Leave your answer to 2 significant figures in units of N.

Problem C: Underwater Lamp (3 points)

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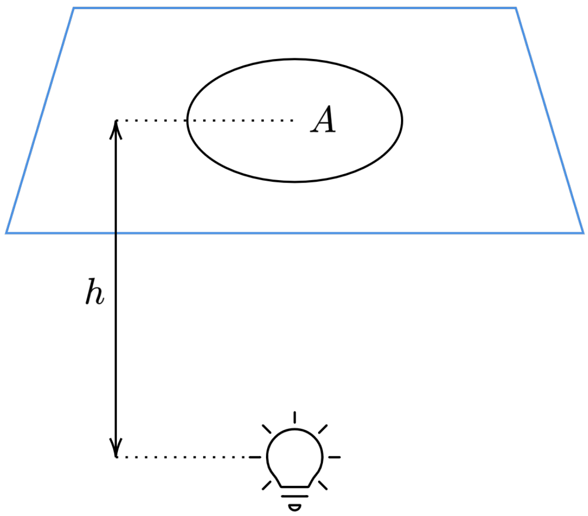
A lit lamp is immersed in water of refractive index $n = 1.33$ on a dark night. It is placed a height $h = 5.0$ m below the horizontal water surface. When viewed from directly above, a bright circular patch of area A is visible on the water surface. Find A. Assume that the lamp emits light in all directions.

Leave your answer to 3 significant figures in units of m^2.



Problem 2: Relative Work

A block of mass $m = 1.0$ kg is initially at rest on horizontal frictionless ground. A horizontal non-constant force $F(t)$ is exerted on the block for some time, after which the block has a final speed $v = 4.0 \text{ m s}^{-1}$ relative to the ground.

(a) From the perspective of an observer that is stationary relative to the ground, find the net work done W by force F on the block.

Leave your answer to 2 significant figures in units of J. (2 points)

(b) From the perspective of another observer that travels at constant horizontal speed u relative to the ground, the net work done W' by force F on the block is zero. Find u.

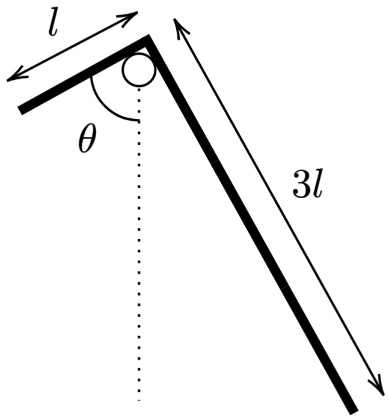
Leave your answer to 2 significant figures in units of m s^{-1}. (2 points)

Problem D: L (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

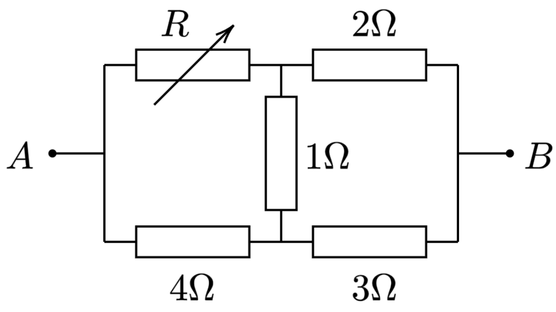
A thin uniform L-shaped bar with arm lengths l and $3l$ is hung on a frictionless pin of negligible radius at its right-angled corner as shown. What is the angle θ that the shorter arm makes with the vertical?

Leave your answer to 2 significant figures in units of degrees.



Problem 3: Variable Resistor

Chris creates an arrangement of 5 resistors as shown in the diagram, one of which is a variable resistor R. Here, we let the effective resistance across A and B be R_{AB}.



(a) What value does R_{AB} approach as the value of R approaches 0?

Leave your answer to 2 significant figures in units of Ω. (2 points)

(b) What value does R_{AB} approach as the value of R approaches ∞?

Leave your answer to 2 significant figures in units of Ω. (2 points)

Problem 4: A Sinking Feeling (3 points)

A worker uniformly mixes two materials, one with density $\rho_1 = 600\ \text{kg m}^{-3}$ and the other with density $\rho_2 = 1900\ \text{kg m}^{-3}$, and shapes them into a cube.

He then places the cube underwater, such that it is fully submerged with its top surface at a depth $d = 4.4$ m below the surface of the water, and releases it from rest. Surprisingly, the cube stays in position there.

Find η, the proportion (by volume) of the cube that is made with the material of density ρ_1.

Leave your answer to 2 significant figures.

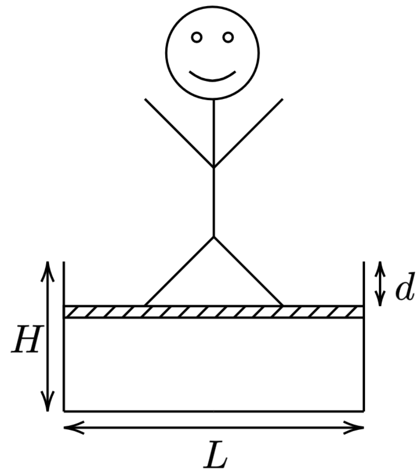
Your answer should be between 0 and 1.

Problem E: Gas Weighing Scale (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

Physicist S has designed a weighing scale that uses gas pressure. It consists of a cuboidal container, which has a square base of side length $L = 20$ cm. The top face of the container is light and able to freely slide up and down without resistance. The container is filled with an ideal gas such that the top face is at initial height $H = 10$ cm. When Physicist S steps onto the top face of the container, it lowers by $d = 1.3$ cm. Assuming that the container is a perfect thermal conductor and airtight, find her mass m.

Leave your answer to 2 significant figures in units of kg.

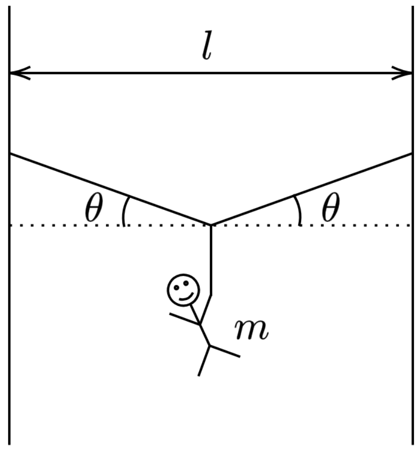


Problem F: Help! (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

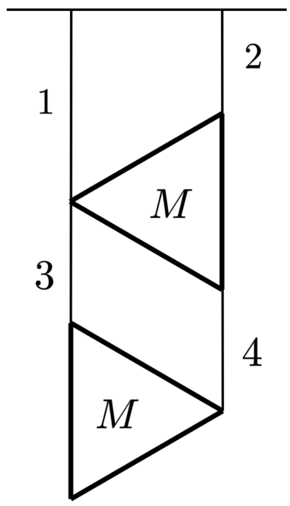
Roger is trying to learn how to tightrope walk, on a light horizontal elastic rope with force constant $k = 650.0\ \mathrm{Nm}^{-1}$ and unstretched length $l = 20.0$ m that is connected to two walls a distance l apart. Suddenly, he slips and falls halfway along the rope. Luckily, he grabs onto the rope and manages to hang there. The rope reaches an equilibrium position bent at an angle $\theta = 20.0°$ from the horizontal. Calculate Roger's mass m.

Leave your answer to 3 significant figures in units of kg.



Problem 5: Suspended Triangle

Two identical uniform equilateral triangles of mass M are suspended by 4 vertical strings (as shown in the diagram). Strings 1 and 2 can withstand twice the amount of tension as strings 3 and 4 before breaking.



(a) Which string will break first as M increases? (2 points)

(1) String 1

(2) String 2

(3) String 3

(4) String 4

(5) All will break at the same point.

(b) Let T_n be the tension in string n. If $M = 5.0$ kg, find the value of $T_2 + T_4$.

Leave your answer to 2 significant figures in units of N. (3 points)

Problem 6: Video Misinformation (3 points)

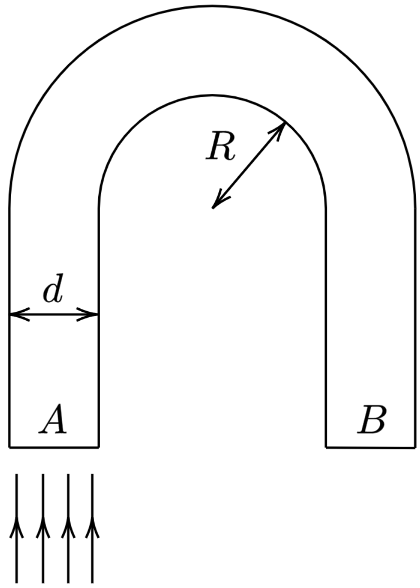
A fan blade rotates clockwise at a constant angular velocity ω. A fixed camera records a video of it with a frame rate of $f = 30$ frames per second. In that video, the fan blade *appears* to be rotating clockwise at angular velocity $\omega' = 10$ rad s^{-1}. However, the actual value of ω differs from ω'. Find the smallest possible value of ω.

Leave your answer to 2 significant figures in units of rad s^{-1}.

Problem 7: YouTube (3 points)

A glass U-tube with refractive index $n = 1.52$ is constructed by bending a glass rod into a U-shape, where the curved parts of the tube are two arcs of a semicircle. A collimated beam of light falls perpendicularly on the flat surface A. Determine the minimum value of the ratio $\frac{R}{d}$ for which all light entering the glass through surface A will emerge from the glass through surface B.

Leave your answer to 3 significant figures.

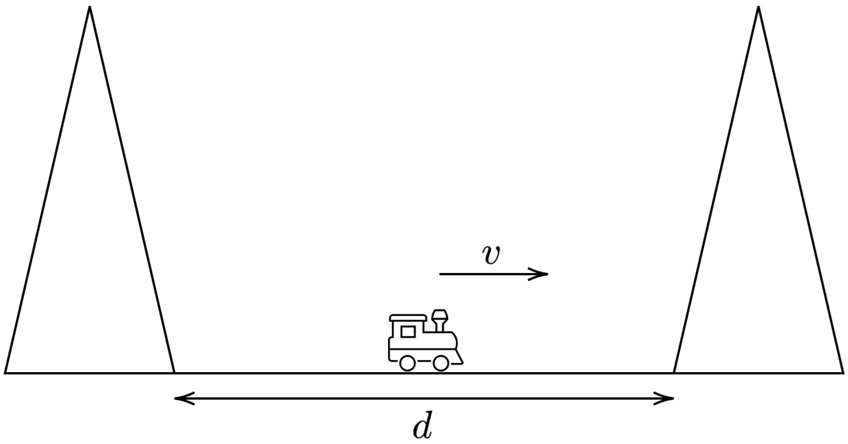


Problem 8: Choo Choo (3 points)

Thomas the Train is parked at the midpoint of two steep mountains of distance $d = 4000$ m apart. He blows his horn and immediately begins travelling with constant velocity v towards one of the mountains. Given that Thomas hears the echo of the horn off the two mountains with a time difference of $\Delta t = 2.0$ s, find v.

Take the speed of sound in air to be $v_s = 340$ m s^{-1}.

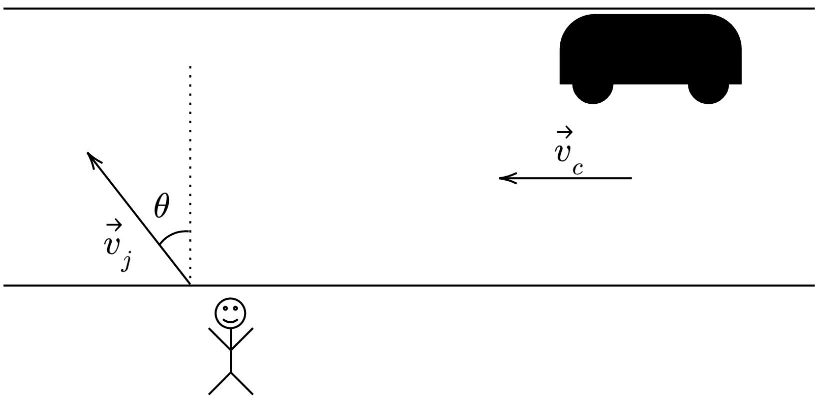
Leave your answer to 2 significant figures in units of m s^{-1}.



Problem 9: Jaywalking (3 points)

Jay stands at the south edge of a road which runs from east to west, intending to cross. He notices that a car is travelling west at $v_c = 30$ km h^{-1}, keeping to the opposite edge of the road. Given that Jay can run at a speed $v_j = 10$ km h^{-1}, what angle θ (measured counterclockwise from north) should he run at to maximise the distance between him and the car upon reaching the other side of the road? You can assume that the car has not passed Jay when he reaches the other side of the road.

Leave your answer to 3 significant figures in units of degrees.



Problem 10: Unfreezable (4 points)

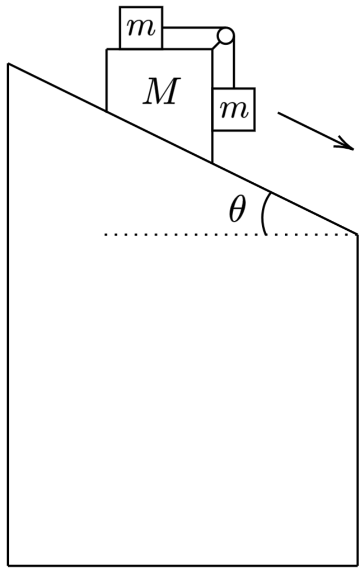
Consider an isolated system of pure water that is supercooled to a temperature of $T_i = -15°\text{C}$, such that it remains completely liquid. When the supercooled water is disturbed, some but not all of the water freezes. Determine the percentage by mass of water that gets frozen.

Assume the specific heat capacity of ice to be equal to the specific heat capacity of water, which can be treated to be constant at $c_w = 4.19 \times 10^3$ J kg^{-1} K^{-1}. The specific latent heat of fusion of water can also be treated to be constant at $l_f = 3.34 \times 10^5$ J kg^{-1}.

Leave your answers to 2 significant figures as a percentage. (For example, if you think the final answer should be 51%*, input your answer as* 51*)*

Problem 11: Falling Chimney

A chimney-shaped block of mass $M = 3.0$ kg slides down a rooftop (beginning from rest) tilted at an angle θ, with a slanted base and its top face perfectly horizontal. A massless pulley is attached to a corner of the block and a light inextensible string is run over the pulley, with a smaller block of mass $m = 1.0$ kg attached to each side of the string. Take all surfaces to be frictionless.



(a) Find the value of θ such that the blocks slide down sticking together (i.e. they do not have any movement relative to each other).

Leave your answer to 3 significant figures in units of degrees. (2 points)

(b) Suppose now that $\theta = 0°$. Find A, the magnitude of the initial acceleration of the large block of mass M when the blocks are released from rest.

Leave your answer to 2 significant figures in units of m s^{-2}. (4 points)

Problem 12: Spring Collision

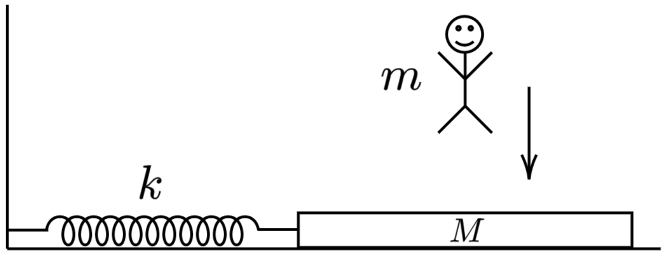
A playground ride is made of a platform of mass M connected to a light spring of force constant k, that oscillates in simple harmonic motion with amplitude x_0. When the platform is at maximum displacement from equilibrium, Roger of mass $m = \frac{M}{3}$ jumps such that he lands perfectly vertically on the platform. Subsequently, Roger sticks on and remains at rest relative to the platform, and the combined body oscillates in simple harmonic motion with amplitude x_1.

(a) What is $\frac{x_1}{x_0}$?

Leave your answer to 3 significant figures. (2 points)

(b) Now consider the case where Roger jumps onto the platform when the platform is at displacement $x = \frac{3}{4}x_0$ instead. The combined body then oscillates with amplitude x_2. What is $\frac{x_2}{x_0}$?

Leave your answer to 3 significant figures. (3 points)



Problem 13: Balanced Plates (4 points)

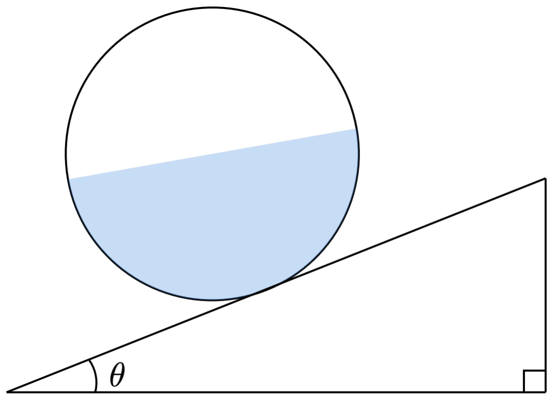
A capacitor consists of one plate of area $A = 2.0\ \mathrm{m}^2$ and mass $m = 2$ g attached to the ceiling, and a second plate of identical mass and dimensions floating freely at distance $d = 2.0$ cm below the top plate. What voltage V should be applied across the capacitor such that the bottom plate remains stationary? You may neglect edge effects.

Leave your answer to 2 significant figures in units of V.

Problem 14: Strange Sphere (4 points)

A sphere is made up of two uniform solid hemispheres of different densities joined together. It is placed on a slope inclined at angle θ from the horizontal. Given that the hemispheres' densities and the sphere's rotation can be freely varied, what is the maximum angle θ for which the sphere can rest in equilibrium on the slope? Assume the coefficient of static friction is sufficiently large for the sphere to remain in translational equilibrium.

Leave your answer to 3 significant figures in units of degrees.



Problem 15: Nuclear Fusion

(a) In a fusion power experiment, a mobile deuterium nucleus is fired at a stationary deuterium nucleus such that they have relative velocity v_1 right before collision. The two undergo nuclear fusion to form **only** a helium-4 nucleus. There are no other particles in the reaction chamber for them to interact with. Find v_1.

Leave your answer to 2 significant figures in units of km s^{-1}. (2 points)

If there exists either zero or multiple possible values of v_1, *input your answer as* -1.

(b) In another fusion power experiment, a mobile helium-4 nucleus is fired at a stationary helium-4 nucleus such that they have relative velocity v_2 right before collision. The two undergo nuclear fusion to form **only** a beryllium-8 nucleus. There are no other particles in the reaction chamber for them to interact with. Find v_2.

Leave your answer to 2 significant figures in units of km s^{-1}. (3 points)

If there exists either zero or multiple possible values of v_1, *input your answer as* -1.

You should assume that all nuclei involved, including the nuclei produced by the reaction, are in their ground states.

Data:
Rest mass of deuterium nucleus: $m_D = 2.01410178$ u
Rest mass of helium-4 nucleus: $m_{He} = 4.00260325$ u
Rest mass of beryllium-8 nucleus: $m_{Be} = 8.00530510$ u

Problem 16: Public Nuisance (4 points)

Bobbins is trying to set a football on a travelator that is moving leftward at constant velocity $u = 5.0$ cm s^{-1}, such that the ball appears to be rolling on the spot to a stationary observer.

As such, he imparts an initial velocity v (relative to himself) to the ball, without imparting any rotation, as he places it on the travelator. Modelling the ball as a uniform solid sphere, find v such that he succeeds in his goal.

Leave your answer to 2 significant figures in units of cm s^{-1}.

Leave your answer as positive if you think the velocity should be rightward, and as negative if you think the velocity should be leftward.

Problem 17: Unknown Motion (3 points)

A charged particle is in a magnetic field within a three-dimensional space described by the standard Cartesian axes x, y, z. The magnetic field strength and direction varies with time and space. Initially, the particle has velocity $v_x = 200$ m s^{-1} and $v_y = 210$ m s^{-1}. After travelling in the magnetic field for some time, the particle has travelled a distance of $d = 500$ m and has $v_z = 290$ m s^{-1}. Find $|\langle \vec{a} \rangle|$, the magnitude of the average acceleration of the particle. Assume there is no gravity.

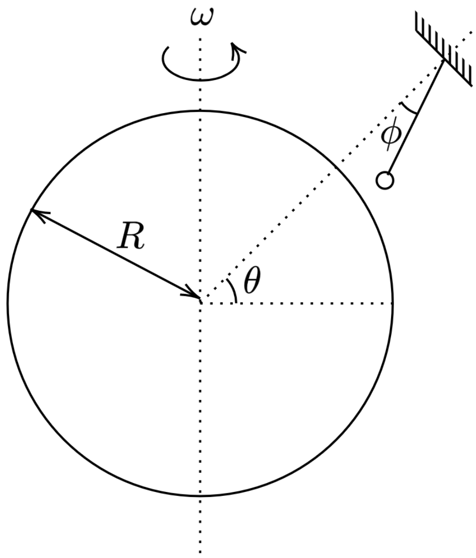
Leave your answer to 3 significant figures in units of m s^{-2}.

Problem 18: Curious Pendulum (4 points)

Guangyuan is standing at a point on Earth with latitude $\theta = 45°$. He suspends a simple pendulum at rest from a ceiling. Surprisingly, he claims that the pendulum does not hang completely vertical but instead is offset by an angle ϕ from the vertical, where the vertical axis is defined normal to the ground. Calculate ϕ.

Take the radius of Earth to be $R = 6370$ km, and the period of Earth's rotation to be $T = 24.0$ h.

Leave your answer to 3 significant figures in units of degrees.



Problem 19: Rainbow

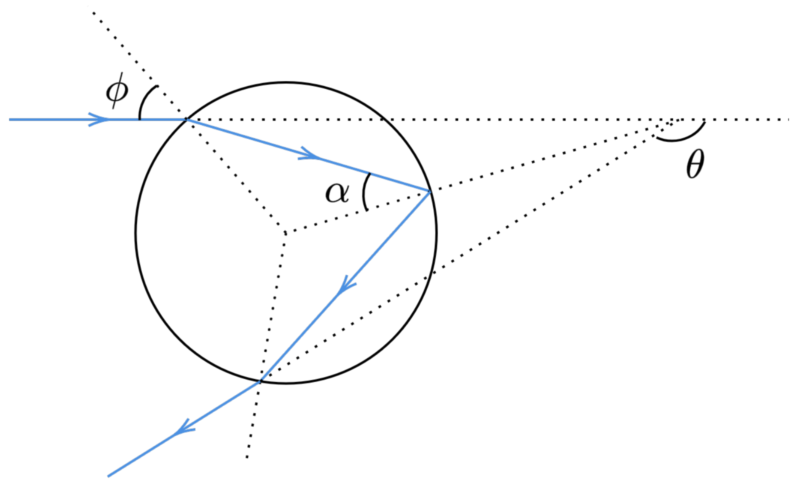
A ray of light in air enters a spherical drop of water of index $n = 1.33$ at an angle $\phi = 50°$ to the normal of the water surface.

(a) What is the angle of incidence α of the ray on the droplet's back surface?

Leave your answer to 3 significant figures in units of degrees. (2 points)

(b) The light ray is partially reflected off the back surface, before exiting the drop at some angle of deflection θ from its initial direction. For such a light ray that reflects exactly once, find the angle ϕ that minimises θ. (3 points)

Leave your answer to 3 significant figures in units of degrees.



Problem 20: Rolling Ring

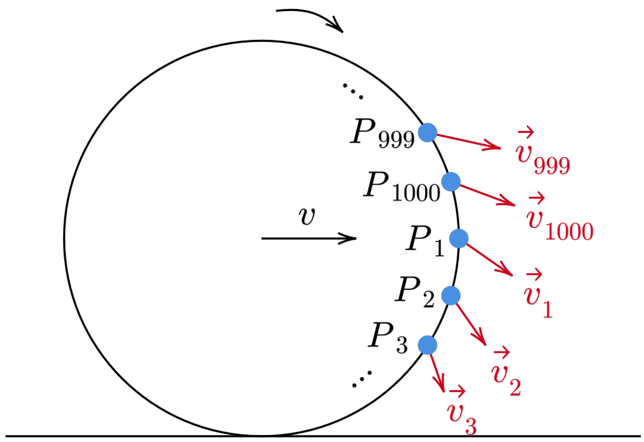
A rigid ring rolls without slipping along horizontal ground, with constant translational velocity $v = 5.0$ m s^{-1} towards the right. Consider 1000 points $P_1, P_2, \ldots, P_{1000}$ on the ring, evenly spaced across the ring's full circumference, with the first point P_1 taken to be the ring's rightmost point. Denote their velocities as $\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_{1000}$ respectively.

(a) Find $|\vec{v}_1|$, the magnitude of point P_1's velocity.

Leave your answers to 2 significant figures in units of m s^{-1}. (2 points)

(b) Find $|\vec{v}_1 + \vec{v}_2 + ... + \vec{v}_{1000}|$, the magnitude of the sum of velocities of all 1000 points.

Leave your answers to 2 significant figures in units of m s^{-1}. (2 points)



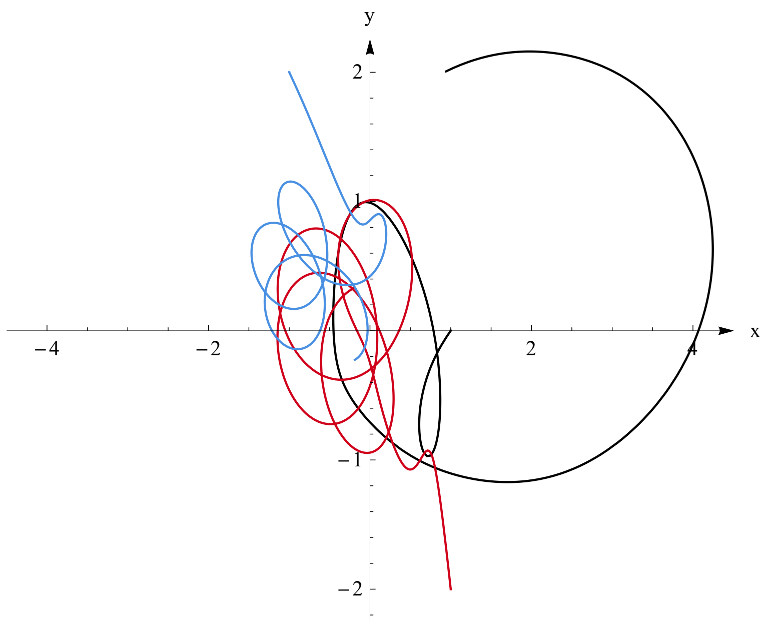
Problem 21: Three-Body Problem (4 points)

Three planets of mass m_1, m_2 and m_3 begin at rest with position vectors $\vec{r}_1$, $\vec{r}_2$ and $\vec{r}_3$ respectively. The masses are then released, and move only in the 2D plane of the diagram below in a chaotic manner. After some time t, the position vectors of m_1 and m_2 are $\vec{r}_1'$ and $\vec{r}_2'$ respectively. Find $|\vec{r}_3'|$, the distance from m_3 to the origin at that time.

<u>Data:</u>
$m_1 = 1.00 \times 10^{24}$ kg
$m_2 = 2.00 \times 10^{24}$ kg
$m_3 = 3.00 \times 10^{24}$ kg
$\vec{r}_1 = (1.00, 0.00)$
$\vec{r}_2 = (1.00, -2.00)$
$\vec{r}_3 = (-1.00, 2.00)$
$\vec{r}_1' = (3.93, -0.20)$
$\vec{r}_2' = (-1.06, 0.59)$

All position vectors are given in units of AU.

Leave your answer to 2 significant figures in units of AU.

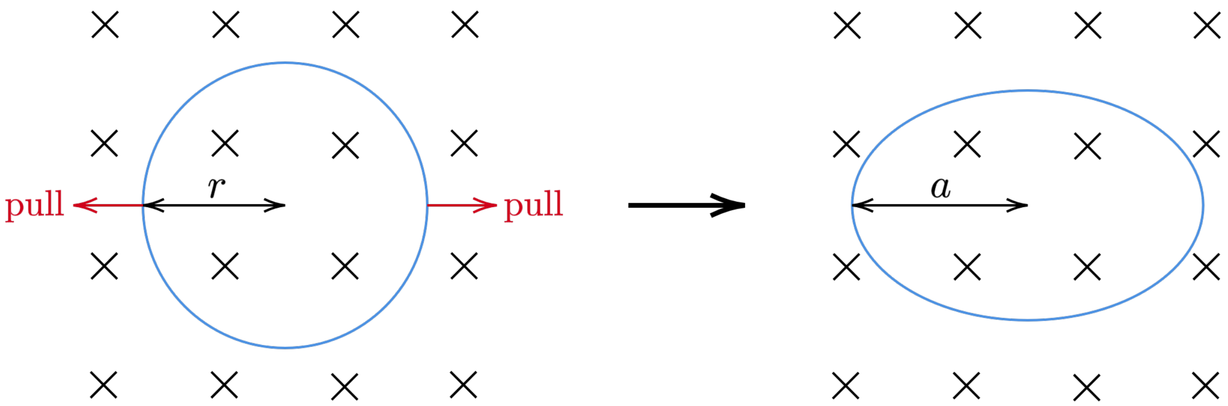


Problem 22: Wire Distortion (4 points)

A circular loop of wire with radius $r = 10.0$ cm and resistance $R = 5.00$ mΩ is placed in a uniform magnetic field $B = 3.00$ mT perpendicular to the plane of the loop. The wire is pulled at opposite ends outwards such that it now forms an ellipse with semi-major axis $a = 12.0$ cm. How much charge Q flows through the wire during this process?

Leave your answer to 3 significant figures in units of C.

Use the formula $p \approx 2\pi\sqrt{\frac{a^2+b^2}{2}}$ for the perimeter p of an ellipse with semi-major axis a and semi-minor axis b.



Problem 23: Interplanetary Bridge

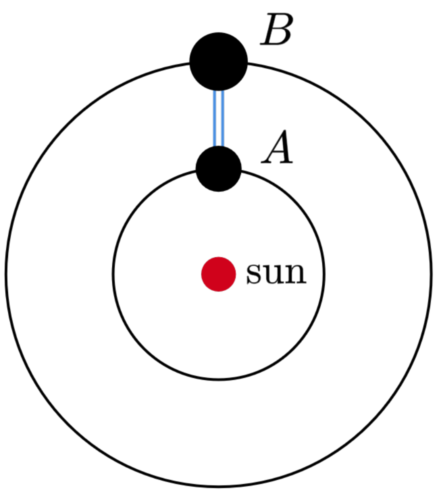
Consider two planets, A and B, of masses $M_A = 1.0 \times 10^{24}$ kg and $M_B = 2.0 \times 10^{24}$ kg, in circular orbits of radius $R_A = 1.0 \times 10^{11}$ m and $R_B = 1.3 \times 10^{11}$ m around a common star of mass $M_s = 2.0 \times 10^{30}$ kg. The two planets are orbiting in the same direction. You may assume that the planets are point masses and have no rotation about their own individual axes.

(a) The planets have angular momentum L_A and L_B about the star. Determine the ratio L_A/L_B.

Leave your answer to 2 significant figures. (2 points)

(b) The inhabitants of the planets have constructed a light indestructible link bridge, which they connect when planets A and B are closest to each other. This joins the two planets instantaneously. Determine the angular velocity ω of the two planets about their centre of mass at the instant just after the connection is made.

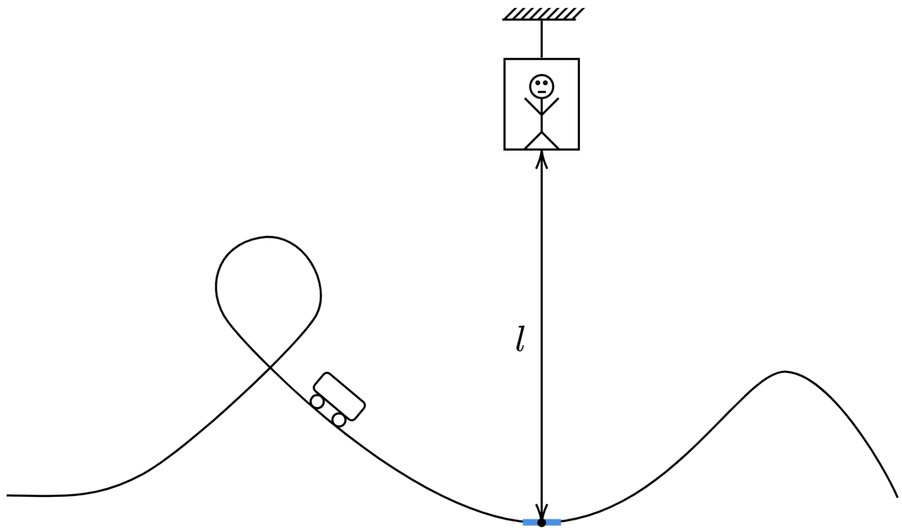
Leave your answer to 2 significant figures in units of rad year^{-1}. (3 points)



Problem 24: Killer Coaster (3 points)

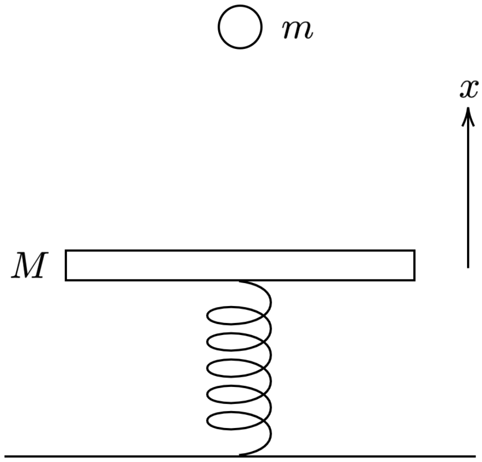
Roger is a worker at a theme park who dislikes his boss. His job is to maintain a roller coaster with carts of mass $m = 100$ kg. One day, he paints a section of the roller coaster near the bottom of its loop with reflective paint. At midday, his boss (who sits in an office $l = 75$ m above the bottom of the roller coaster) was fried to a crisp, bringing Roger great joy. If the roller coaster carts have a velocity of $v = 35$ m s^{-1} when they travel across that section, what is the normal force N exerted by the track on the cart at the bottom? Assume that the rays from the sun are normal to the track surface at the point where it is painted.

Leave your answer to 2 significant figures in units of N.



Problem 25: Boing Boing (5 points)

Roger watches a video of table tennis players juggling a ball on their bats, seemingly indefinitely. Inspired, Roger tries his luck at replicating this trick with a physical model.



A large plate of mass $M = 0.50$ kg is mounted on top of a spring of spring constant k as shown in the diagram. The position $x = 0$ is defined as the equilibrium position of the spring, with x being positive upwards. The spring is stretched by a distance $x_0 = +0.070$ m and released. Assume that the damping of the spring oscillations is negligible.

At a certain time $t = t_0$, a ball of mass $m = 0.050$ kg is dropped onto the centre of the plate and collides elastically with the plate while the plate is at position $x_1 = +0.040$ m. To Roger's delight, the ball continues to collide with the plate indefinitely, each collision appearing identical to the last. Determine the smallest possible value of k.

Leave your answer to 3 significant figures in units of N m^{-1}.

Problem 26: Pulling a Rope

A uniform inelastic rope of linear mass density $\lambda = 0.120$ kg m^{-1} and length $l = 2.00$ m is hung over a pole of negligible radius, such that both ends start at the same height. Roger pulls one end downwards with a constant velocity $v = 0.500$ m s^{-1}. The pole exerts a constant frictional force $f = 3.00$ N as the rope slides over the pole.

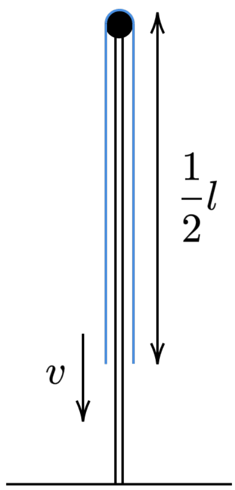
(a) Find the acceleration a_{cm} of the centre of mass of the rope.

Leave your answer to 3 significant figures in units of m s^{-2}. (3 points)

If you think the the centre of mass of the rope is not accelerating, input your answer as $a_{cm} = 0.00$ *m s*$^{-2}$.

(b) Find the pulling force F exerted by Roger when the end being pulled is a vertical distance $h = 0.400$ m below the other end.

Leave your answer to 2 significant figures in units of N. (2 points)



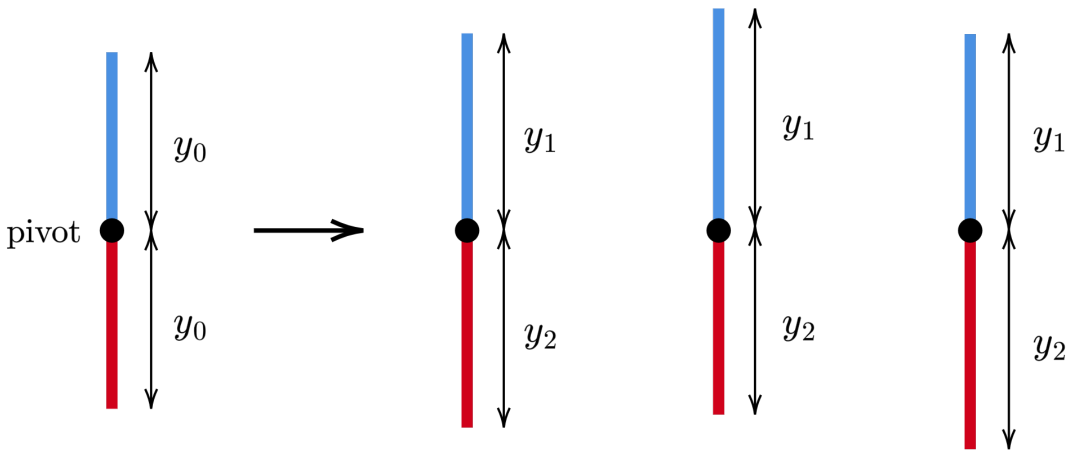
Problem 27: Vertical Expansion

A uniform vertical rod is pivoted at its centre. This way, the top of the rod is at distance y_0 above the pivot, while the bottom of the rod is at the same distance y_0 below the pivot.

We now apply a small amount of heat uniformly across the rod, causing it to expand. As such, the top of the rod is now at distance y_1 above the pivot, while the bottom of the rod is at distance y_2 below the pivot.

Assume that there is no heat flow within the rod and no heat loss to the surroundings.

(a) Select the correct relation between y_1 and y_2. (You may refer to the diagram below to visualise the physical setup illustrated by each option.) (1 point)



(1) $y_1 = y_2$ (2) $y_1 > y_2$ (3) $y_1 < y_2$

(b) The rod has specific heat capacity $c = 50$ J kg^{-1} K^{-1}, linear expansion coefficient $\alpha = 8.0 \times 10^{-3}$ K^{-1}, and $y_0 = 10$ m. Calculate the ratio $\dfrac{y_1 - y_0}{y_2 - y_0}$.

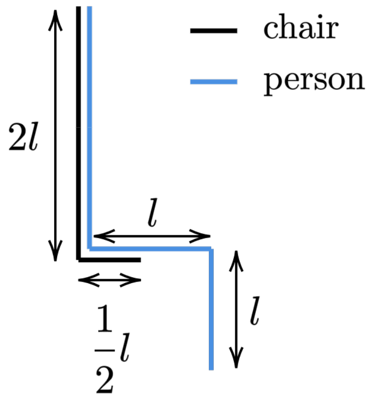
Leave your answer to 3 significant figures. (4 points)

Problem 28: Bad Driver (4 points)

Paul is sitting on a fixed bus seat with his feet off the floor. Here, he is modelled as 3 thin, rigid rods of uniform mass density joined to each other, with dimensions as shown in the figure. His seat consists of a base and a backrest. The coefficient of static friction between Paul and the seat is $\mu = 0.5$. The bus driver suddenly brakes at a constant deceleration and Paul finds himself crashing into the seat in front of him. Fuming, Paul decides to calculate the maximum bus deceleration a at which he would have remained stationary. What is the value of a?

Leave your answer to 2 significant figures in units of m s^{-2}.

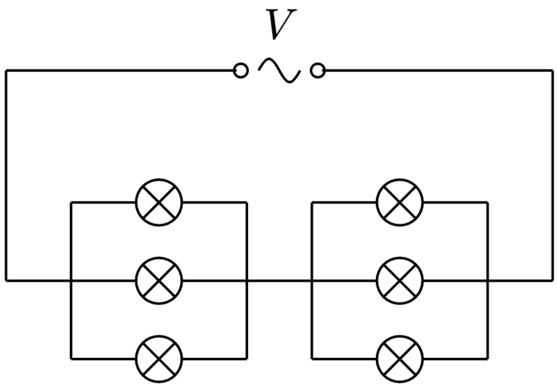
Your answer should be positive.



Problem 29: Power Saving Mode (4 points)

Paul is trying to save the Earth by replacing bulbs with diodes. In his setup, there are six bulbs, each of resistance R, connected to an external voltage source as shown in the diagram below. The voltage source is alternating, creating an electromotive force in the form $V = V_0 \cos \omega t$. The average power drawn by this setup over a long period of time is P_0. Now, one of the bulbs is replaced with an ideal diode, and the average power drawn over a long period of time is P_1. Determine the ratio $\frac{P_1}{P_0}$.

Leave your answers to 2 significant figures.



Problem 30: Infinite Energy

$2n$ point charges are arranged in a straight line, each separated from its neighbour by distance $l = 5.0$ mm. The charges have alternating signs, but the same magnitude $q = 1.5 \times 10^{-6}$ C. The total potential energy of this system of charges is denoted by U_n. Take the potential energy of the system to be zero when the charges are far apart.

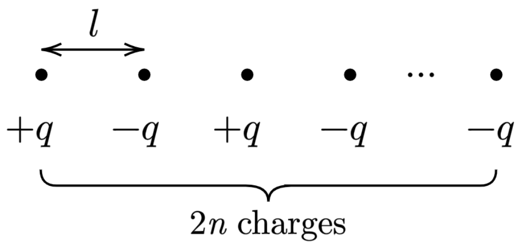
(a) Determine $\frac{U_n}{2n}$ for $n = 2$. This is the average potential energy contribution due to each of the $2n$ charges.

Leave your answers to 2 significant figures in units of J. (3 points)

(b) Determine $\lim\limits_{n\to\infty} \frac{U_n}{2n}$.

Leave your answers to 2 significant figures in units of J. (3 points)

If the limit tends towards positive or negative infinity, input your answer as 1000 *or* −1000 *respectively.*



Problem 31: High Level Golf (4 points)

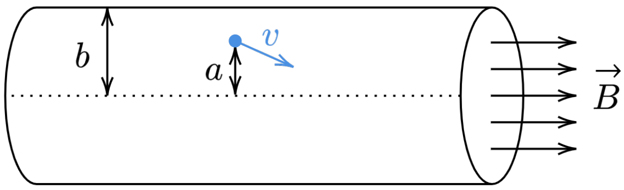
Brian and Chris are playing a game of golf. Brian's ball is at a distance d_1 from the hole and Chris's ball is at a distance d_2 from the hole. Aiming for the hole and hitting each of their balls simultaneously, they each drive their ball too high; Brian's ball flies with speed v_1 angled $\theta_1 = 45°$ above the ground, while Chris's flies with speed v_2 angled $\theta_2 = 60°$ above the ground. Miraculously, the two balls collide at a height $h = 1$ m perfectly above the hole. Given that $d_1 + d_2 = 10$ m, find $v_1 + v_2$.

Leave your answer to 3 significant figures in units of $\mathrm{m\ s^{-1}}$.

Problem 32: So Close Yet So Far (5 points)

A uniform magnetic field of field strength $B = 0.500$ T runs parallel to the axis of a long insulating cylindrical shell of radius $b = 35.0$ m. A charged particle with mass $m = 0.0500$ kg and charge $q = 0.100$ C is initially positioned at a distance $a = 10.0$ m away from the axis of the cylinder. The particle is launched with speed $v = 20.0$ m s^{-1} in an arbitrary direction. What is the minimum time taken t for the particle to reach the wall of the cylinder?

Leave your answer to 3 significant figures in units of s.



Problem 33: Problematic Proton (5 points)

Alice carries out experiments in a spherical charged gas cloud with radius $R = 5.0$ m and uniform volume charge density $+\rho$. She releases an electron at rest at a distance of $r_0 = 2.0$ m from the centre of the cloud, and notices that it performs oscillatory motion with period $T_1 = 0.60$ s.

However, one day she accidentally releases a proton from the same position, and notices that it reaches the surface of the gas cloud in a time T_2. Determine T_2.

Neglect any gravitational effects and collisions between the electron/proton and gas particles, and assume that the gas particles remain stationary.

Leave your answer to 2 significant figures in units of s.

Problem 34: Triple Bounce

(a) A ball is projected horizontally from the top of an edge of a square pit with side length ℓ. Consider the following scenarios:

A: an initial velocity v_A makes the ball bounce three times only on the base, before reaching the top of the opposite edge.

B: an initial velocity v_B makes the ball bounce exactly once on each of the three sides (including the base), before reaching the top of the opposite edge.

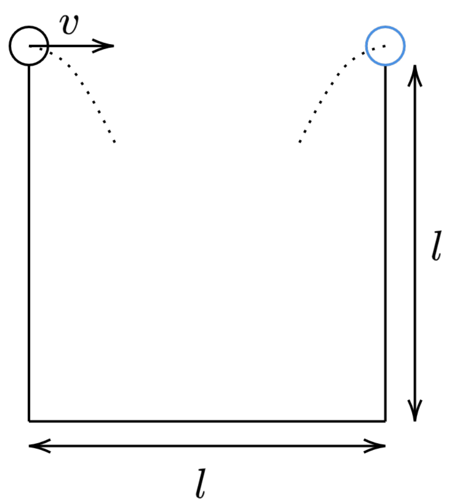
Find the ratio v_A/v_B.

Leave your answer to 3 significant figures. (3 points)

(b) Scenario B is now modified such that the ball is projected at an angle θ below the horizontal, bouncing off each of the three sides exactly once before reaching the top of the opposite edge. In addition, its maximum height during its journey exceeds its initial height by $\ell/2$. Find θ.

Leave your answer to 3 significant figures in units of degrees. (3 points)

Assume all collisions are elastic, and no spin is imparted to the ball.



Problem 35: Hard Work (5 points)

Bob is trying to pump a ball. Model the pump as a vessel with initial volume $V_0 = 2.5 \times 10^{-5}$ m^3 and the ball as a vessel with constant volume $V_1 = 7.0 \times 10^{-3}$ m^3.

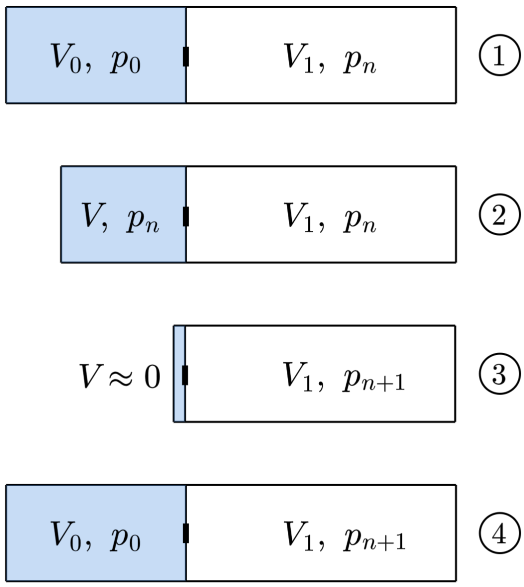
Initially, the pump is empty, and the ball is at atmospheric pressure p_0. During each pumping process,

1. Air from the atmosphere first fills up the pump to pressure p_0.
2. Bob then pushes down on the pump handle, causing the pump's volume to contract until the pressure in the pump is equal to the pressure in the ball.
3. A valve connecting the pump to the ball then opens, and Bob pushes down again until the volume of the pump is zero.
4. The valve is closed, and the pump volume is restored to V_0 by letting air from the atmosphere fill it up. The cycle then repeats itself.

Assume that air molecules in the atmosphere are diatomic, and all compressions are adiabatic. Determine the ratio $\frac{p_{10}}{p_1}$.

Leave your answers to 3 significant figures.

Hint: The pressure in the ball after the n-th cycle can be expressed as $p_n = p_0 a_n^\gamma$ where $a_n(n)$ is some function of n and $\gamma = 1.4$ is the heat capacity ratio of atmospheric air.

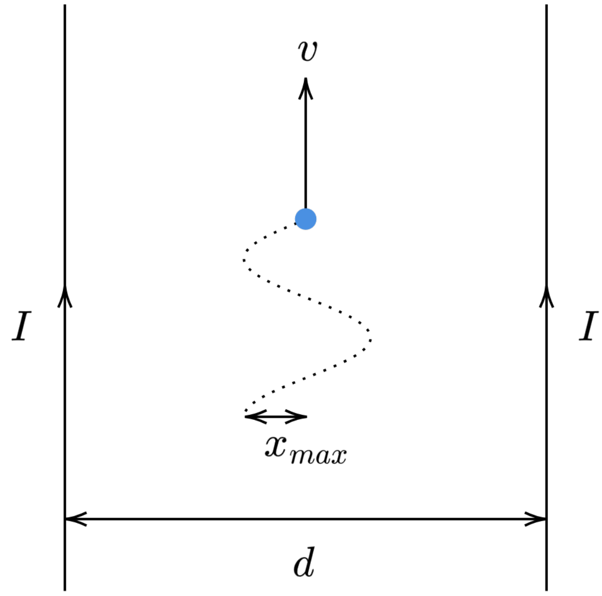


Problem 36: Approximate Magnetic Oscillation (5 points)

Two parallel, infinitely long wires carrying current $I = 3.40$ mA upwards are fixed at a large distance $d = 1.50$ m apart. An electron is in the same plane, halfway between the two wires, with velocity $v = 5.00$ m s^{-1} upwards. The electron is given a slight horizontal displacement such that it exhibits simple harmonic motion along the horizontal axis. Find the period of small oscillations, T.

Assume that both gravity and the vertical magnetic force are negligible. Additionally, as d is extremely large, assume the amplitude of oscillations $x_{\text{max}} \ll d$.

Leave your answer to 3 significant figures in units of s.



Problem 37: Daredevil Paul

Paul the daredevil is trying to impress his girlfriend with his newest stunt — running off a tall tower! Of course, he has no intention of dying, so he reassures her he will run fast enough such that he can make it around the Earth and come back without colliding with the ground. He stands on a tower of height $h = R$ above the surface of the Earth, where R is the radius of the Earth.

Take the mass of Earth to be $M = 5.97 \times 10^{24}$ kg and the radius of Earth to be $R = 6370$ km.

(a) Assuming that he runs off the tower horizontally, what is the minimum velocity u which he needs to run at to survive?

Leave your answer to 2 significant figures in units of m s^{-1}. (4 points)

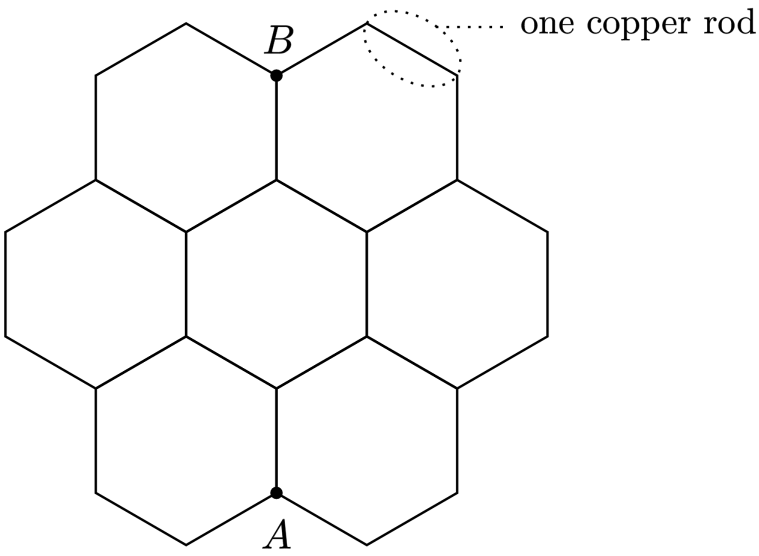
(b) Sadly, due to a skill issue, Paul only runs at $u' \equiv \eta u = 0.6u$. Compute v_n, the normal component of the velocity with which he hits the ground.

Leave your answer to 2 significant figures in units of m s^{-1}. (3 points)

Problem 38: Hexagonmania (5 points)

Roger is bored, so he decides to use his collection of uniform thin copper rods, each of resistance $R = 1.00\ \Omega$, to create a rigid compound shape shown below. The copper rods form seven regular hexagons. Calculate the effective resistance R_{AB} between points A and B.

Leave your answer to 3 significant figures in units of Ω.



Problem 39: Quantum Tunnelling (4 points)

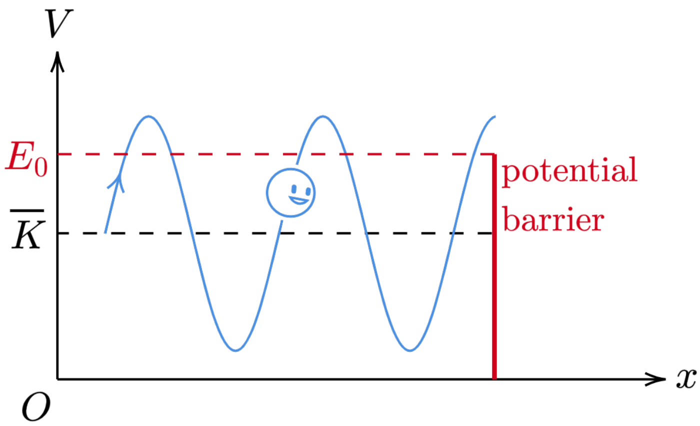
A particle of mass $m = 100$ g is projected at velocity v_0 towards a potential barrier of height $E_0 = 0.07000$ J. When the particle is near the potential barrier, Paul turns on his oscillator, and the particle begins to oscillate such that its velocity is given by $v(t) = v_0 + \epsilon\omega\cos(\omega t + \varphi)$, where $\omega = 440$ Hz and $\epsilon\omega = 0.5$ m s^{-1}.

We measure the average kinetic energy and find it to be $\bar{K} = 0.05625$ J. However, we cannot determine the exact kinetic energy of the particle when it hits the barrier, and thus cannot know for certain if it will pass through. What we can determine is the probability that the particle passes through the barrier, p, which you should give as your answer.

You may assume that ϵ is small, but you may not assume that $\epsilon\omega$ is. However, you may take $\epsilon\omega < v_0$ (the particle does not reverse direction due to Paul's oscillator).

Leave your answer to 2 significant figures.

Your answer should range between 0 and 1.



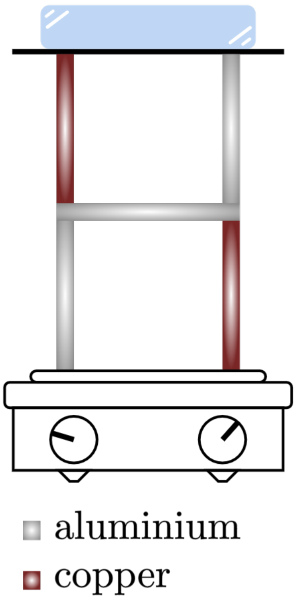
Problem 40: Crazy Electron (6 points)

An electron and a positron are separated by distance $r_0 = 100\ \mu$m in a region of uniform magnetic field $B = 1.00$ mT that is perpendicular to the line joining both charges. Given that the two charges are released simultaneously from rest, find the minimum distance $r_{\min}$ achieved between them throughout their motion.

Leave your answer to 3 significant figures in units of μm.

Problem 41: Thermal H Bar (5 points)

5 bars are joined together to form a H-shape (as pictured below). It comprises two metals: aluminium, of thermal conductivity $k_a = 200$ W m^{-1} K^{-1}, and copper, of thermal conductivity $k_c = 400$ W m^{-1} K^{-1}. Each bar has length $\ell = 0.10$ m, and has a square cross section with width $w = 0.0010$ m.



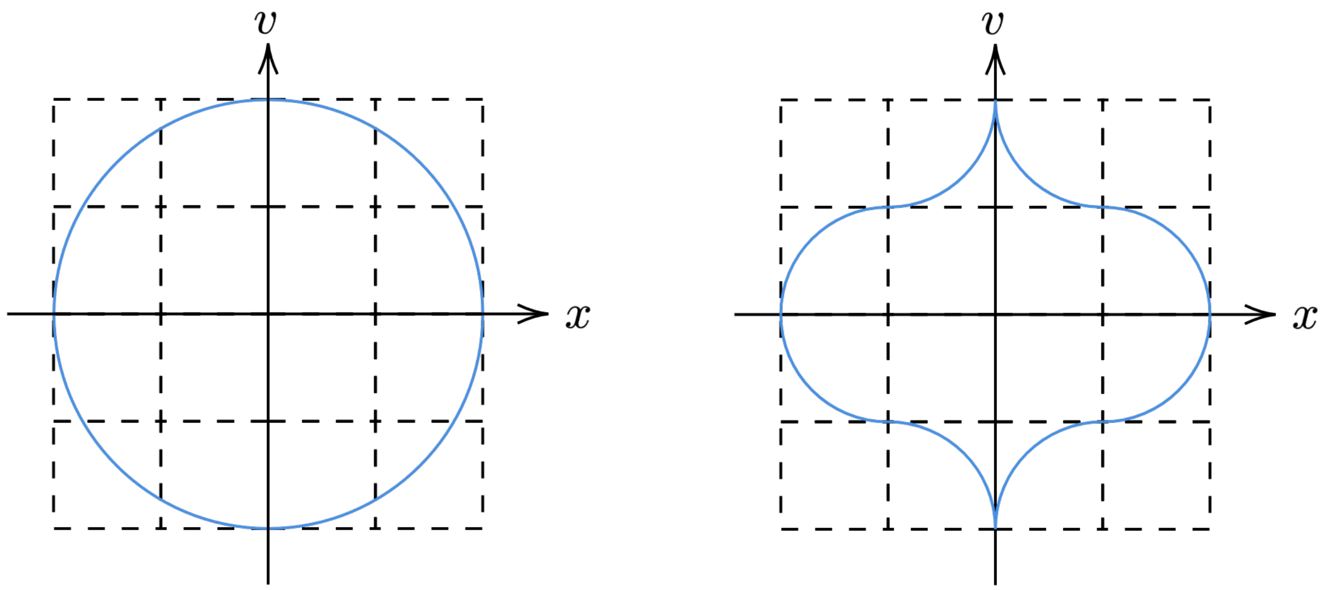
The system is placed on a hot plate of constant temperature $T_h = 100$°C. A thin conducting sheet with an ice block of constant temperature $T_c = 0$°C is then placed on top of it. What is the rate of heat flow from the hot plate to the ice?

You may assume that no heat is transferred to the air and that the bar's width is negligible compared to its length.

Leave you answer to 2 significant figures in units of W.

Problem 42: Oscillating Particles

Consider the two oscillations shown in the figure below. Both graphs show the velocity v (in m s^{-1}) of the oscillating particle at position x (in m). Each small dotted square has a side length of 1 unit.



(a) The phase portrait on the left is a circle of radius 2 units, centred on the origin. What is the period of this oscillation?

Leave your answer to 3 significant figures in units of s. (2 points)

(b) The phase portrait on the right is made up of 8 quarter circles, each with radius of curvature 1 unit. What is the period of this oscillation?

Leave your answer to 3 significant figures in units of s. (4 points)

Problem 43: Falling Into a Plane (5 points)

A point charge $q = 3.00$ mC with mass $m = 5.00 \times 10^{-6}$ kg is held above a large grounded conducting plane at a distance $d_0 = 10.0$ m from it and released from rest. How much time t will it take for the point charge to reach the plane? Ignore gravity.

Leave your answer to 3 significant figures in units of ms.

Problem 44: Dying Photon (6 points)

Alex is doing a physics problem about solar sails. He notices that when photons are incident on a reflective object, the object gains kinetic energy, while the photons seem to be reflected with the same energy. Convinced that this will allow him to create a perpetual motion machine, he carries out an experiment.

A perfectly reflecting block of mass $m = 1.0 \times 10^{-21}$ kg is placed some distance away from a fixed perfectly reflecting wall. The wall is vertical while the ground is horizontal. A single photon is shot from a laser calibrated at wavelength $\lambda_0 = 1.0 \times 10^{-12}$ m. The photon then travels back and forth between the block and the fixed wall. The block moves only along the horizontal, frictionless ground. What is the velocity v of the block after $n = 5000$ collisions between the photon and the wall? Assume that $v \ll c$ and $\frac{h}{\lambda_0} \ll mc$.

Leave your answers to 2 significant figures in units of $\mathrm{m\ s^{-1}}$.

Problem 45: Broken Water Cooler

Roger places a large water bottle in a broken water cooler and fills it up to $V_0 = 800$ ml with $T_h = 90$°C water. Carelessly, he forgets to take back the bottle, and the broken water cooler continues to drip water at a constant rate $\frac{dV}{dt} = 1.00$ ml s^{-1}. The heat transferred per unit time between the water and the surroundings is proportional to their difference in temperature, with proportionality constant $k = 8.0$ J s^{-1} °C^{-1}. The surrounding temperature is $T_0 = 25$°C. Assume the surface area of water exposed to the surroundings during the entire process remains constant, and that the water bottle is large enough that it will never overflow.

(a) Find the equilibrium temperature T_f of the water after a long period of time.

Leave your answer to 3 significant figures in units of °C. (4 points)

(b) Find the time t taken for the water in the bottle to reach $T = 60$°C.

Leave your answer to 3 significant figures in units of s. (4 points)

Problem 46: Rotating Spring

One end of a massless spring of natural length $l = 0.100$ m and spring constant $k = 50.0$ N m^{-1} is fixed to a point O. The other end of the spring can freely rotate about O and is attached to a particle of mass $m = 1.00$ kg. It is also known that the spring breaks when its length exceeds $l_{\max} = 3l$. Initially, the spring is straight, at rest, and at natural length. An instantaneous impulse is then imparted to the particle such that it moves at an initial velocity $\vec{v}$ of arbitrary magnitude and direction. The spring-mass system is placed on a frictionless flat surface such that the particle's motion is constrained to a horizontal plane.

(a) Find $v_{\min}$, the minimum magnitude of $\vec{v}$ required to break the spring.

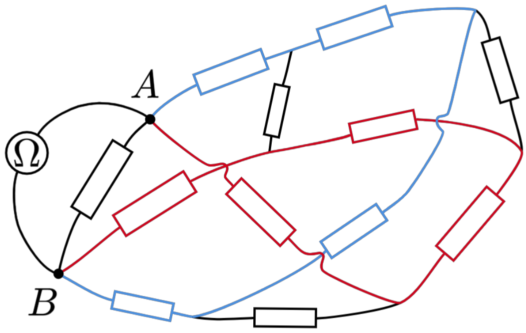
Leave your answer to 3 significant figures in units of m s^{-1}. (2 points)

(b) Find $v_{\max}$, the maximum magnitude of $\vec{v}$ for which the spring does not break.

Leave your answer to 3 significant figures in units of m s^{-1}. (3 points)

Problem 47: Möbius Strip (5 points)

Two wires are each strung with 4 resistors, along with another 4 resistors that bridge pairs of resistors across both wires. The wires are then twisted together to form a *Möbius strip*, as shown below.



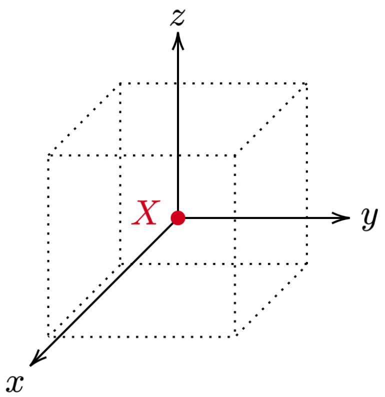
Every resistor has identical resistance $R = 1.0\ \Omega$. Determine the equivalent resistance R_{eq} between the points A and B in the Möbius strip.

Leave your answer to 2 significant figures in units of Ω.

Problem 48: Two-Dimensional Gas

Let us model a world as a three-dimensional space with three orthogonal axes x, y and z. There are two types of gas particles in this world, helium and **X**. The two types of particles have the same mass $m = 6.6465 \times 10^{-27}$ kg, and both may be assumed to display ideal gas behaviour. However, helium is able to move freely through all three dimensions, while the **X** particle is confined to the yz-plane ($x = 0$) and is unable to move along the x-axis. Assume that the laws of conservation of momentum and energy continue to hold true in this world, and that all collisions are elastic.

(a) An **X** particle is placed at $(0, 0, 0)$ with velocity $v = 3.00 \times 10^5$ m s^{-1} in a direction chosen uniformly at random in the yz-plane. It is contained within a heavy cube with side length $a = 1.00 \times 10^{-3}$ m centred at the origin, with axis-aligned edges.



The expected value of the time-averaged magnitude of force that the **X** particle exerts on the cube through collision with the sides is F, which is given as a numerical quantity in units of N. Find $\ln F$.

Leave your answer to 3 significant figures. (3 points)

(b) We now consider a collection of $n = 1000$ particles of **X**. The i^{th} **X** particle starts at a random position on the yz-plane and is given an initial velocity of $v_i = i$ m s^{-1} in a direction chosen uniformly at random in the yz-plane. The world is filled with helium gas at temperature T, which is allowed to interact with the collection of **X** particles. Calculate the value of T such that the expected total energy of the **X** particles stays constant over time.

Leave your answer to 3 significant figures in units of K. (3 points)

(c) We consider another collection of N particles of **X**, placed inside the cube from part (a). This collection stays in thermodynamic equilibrium with helium gas at temperature T, where T is the solution to part (b). The average pressure exerted on the cube by the **X** particles is $P = 3.00$ Pa. Find $\ln N$.

Leave your answer to 3 significant figures. (2 points)

Half Hour Rush M1: Clogged Bathtub (3 points)

Galen is taking a bath. His bathtub is shaped in the form of a cuboid, with length $L = 2.0$ m, width $W = 1.0$ m and vertical height $H = 1.0$ m. On the base of the bathtub is a drainage pipe, which is circular with inner radius $r = 0.050$ m. Much to Galen's dismay, there is a blockage at the pipe's entrance which requires a downward force of $F = 50$ N to be cleared. What is the required depth h of water in the bathtub to clear the blockage?

Leave your answer to 2 significant figures in units of m.

Half Hour Rush M2: Accidental Exposure (3 points)

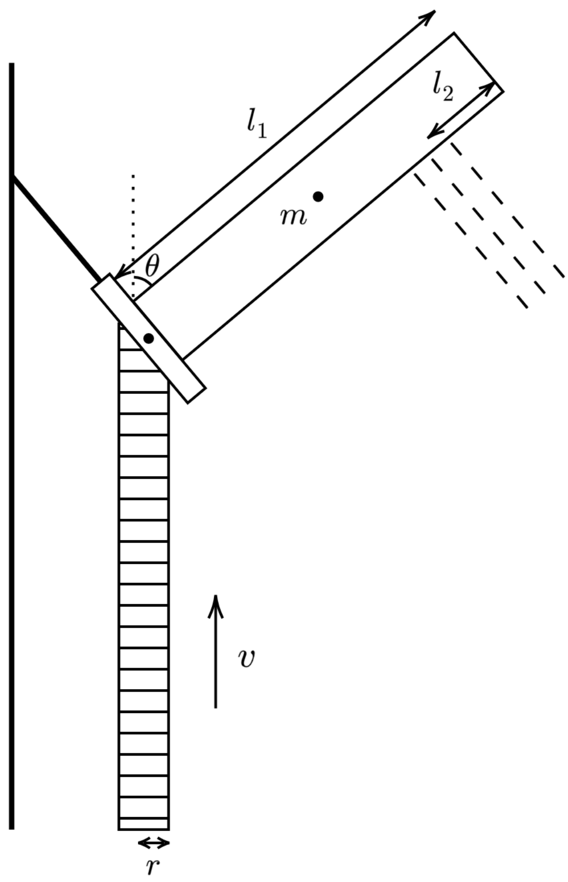
We've all been there: you're texting with your phone in the shower, when suddenly some water accidentally hits your phone's touchscreen, coincidentally tapping the video call button. And then it gets awkward...

Suppose that your phone registers a touch input when a minimum contact pressure $P_c = 25$ kPa is applied on the screen, and that the showerhead emits a water jet of uniform velocity v perpendicular to the screen. Find the minimum value of v for which you risk starting a video call in the shower. Assume that the water is brought to rest instantaneously upon contact with the screen, and neglect any accumulation of water on the screen.

Leave your answer to 2 significant figures in units of $\mathrm{m\ s^{-1}}$.

Half Hour Rush M3: Suspended Showerhead (4 points)

A uniform showerhead is connected to a flexible pipe, and is freely pivoted at this connection point (in a manner that does not disrupt the flow of water). The showerhead is a cylinder with a dry mass $m = 0.40$ kg and length $\ell_1 = 20.0$ cm. Water is supplied through the pipe which extends through the length of the handle, with a uniform inner radius of $r = 0.50$ cm:



When Robert turns on the showerhead, water begins to travel at a uniform speed $v = 5.0$ m s^{-1} in the pipe, and emerges perpendicularly from a point $l_2 = 5.0$ cm from the tip of the showerhead. Robert then notices the showerhead suspends itself at an acute angle θ from the vertical. Find θ.

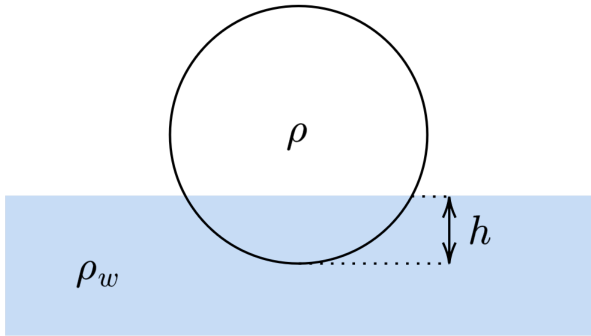
Leave your answer to 3 significant figures in units of degrees.

Half Hour Rush M4: Bath Fun! (5 points)

While taking a bath in water, Roger plays with his bath toy, a uniform spherical rubber ball of density $\rho = 110$ kg m^{-3}. Initially the ball is at rest on the water surface such that the bottom of the ball is a vertical height $h = 2.00$ cm below the water surface. Roger then gives the ball a small vertical displacement, causing it to oscillate with period T. Ignoring resistive forces, calculate T.

Leave your answer to 3 significant figures in units of s.

Hint: The volume V of a spherical cap with height h on a sphere with radius r is given by $V = \frac{\pi h^2}{3}(3r - h)$.



Half Hour Rush E1: Simp (3 points)

Amy has a crush on Jake. She hopes to get together with him through the charm of electrostatic attraction. From a distance of $r = 5.0$ m, she channels her superpowers and secretly transfers $N = 100$ million electrons from Jake's body to her own body. What is the magnitude of the attractive force F that she achieves?

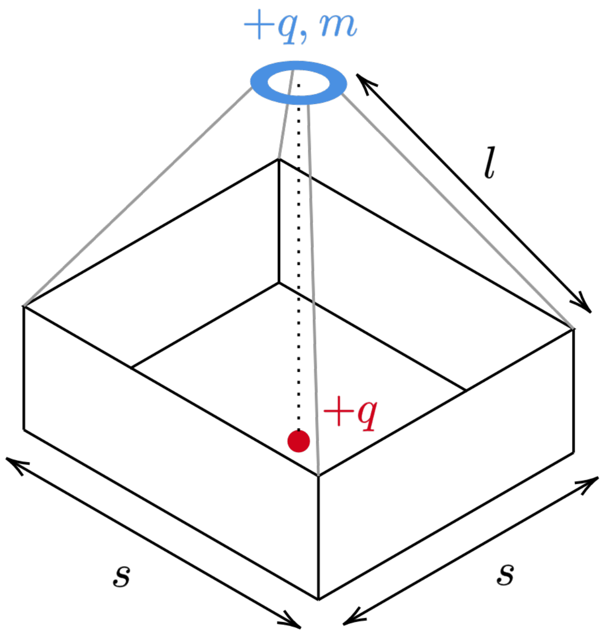
Assume that both of them are point particles that are initially neutral and do not exchange charge with the environment.

Leave your answer to 2 significant figures in units of pN.

Half Hour Rush E2: A Simple Proposal (4 points)

Josiah bought a small engagement ring of mass $m = 1.00 \times 10^{-3}$ kg, which he wanted to present to his fiancée in a box with a square base of side length $s = 0.100$ m and negligible height. On opening the box, he wanted the ring to hover a short distance above its centre. To achieve this, he hid a positive point charge $+q$ under the centre of the box and applied the same positive charge $+q$ to the ring. To constrain the ring to hover directly above the centre of the box, he tied four thin inextensible strings of length $l = 0.120$ m to the ring and secured them to the four corners of the box. Suppose the ring is small enough to be approximated by a point charge. What is the minimum charge q required to ensure the four strings remain taut while the ring hovers above the box?

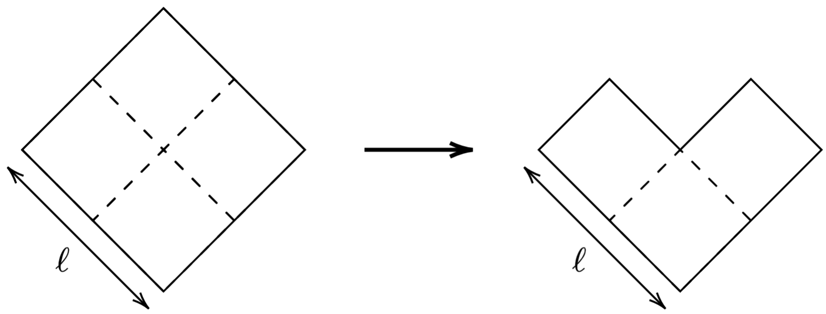
Leave your answer to 3 significant figures in units of μC.



Half Hour Rush E3: Inducing Love (4 points)

Donghang is planning to propose to his girlfriend, so he bought a square ring of side length $\ell = 2.00$ cm. To make the ring seem more special, he deformed it into the shape of a heart. As shown in the figure below, the heart-shaped ring consists of three of the four smaller squares that make up the original square ring. Given that the self-inductance of the square ring is $L_\square = 0.100$ H, find the self-inductance $L_\heartsuit$ of the heart-shaped ring.

Leave your answer to 3 significant figures in units of H.

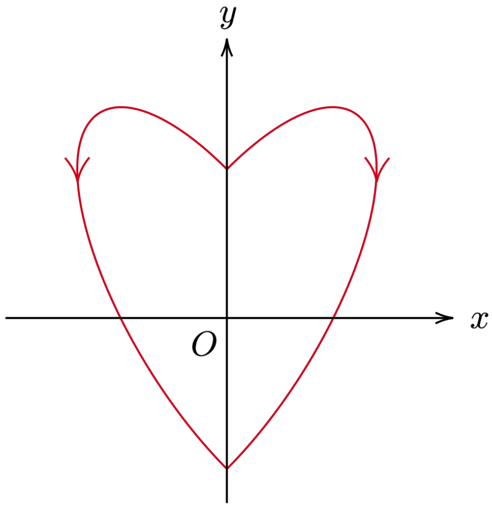


square ring heart-shaped ring

Half Hour Rush E4: An Elaborate Proposal (5 points)

Paul wants to draw a heart in an elaborate manner to impress his crush. To do this, he carefully creates a region with spatially varying magnetic field $B(\vec{r})$ in a direction perpendicular to the plane of the page. The maximum magnitude of magnetic field he is capable of producing is B_{max}. He then ejects two identical charged particles of charge $q = 0.50$ C and mass $m = 0.020$ kg with equal speeds $v = 3.0\ \mathrm{m\ s^{-1}}$. These particles travelled in mirrored paths, tracing out a heart-shaped figure with equation $x^2 + (y - |x|)^2 = 1$, where x and y are lengths in units of metres. What is the minimum value of B_{max} for this to be possible? Neglect the forces exerted by the charged particles on each other.

Leave your answer to 2 significant figures in units of T.



Half Hour Rush X1: Diabolus in Musica (3 points)

Johannes, an organ student, hears of a "cursed interval" in music, which comprises two notes whose frequencies have a ratio of $\sqrt{2}$. Messing around, he accidentally presses two notes on the organ to form this interval, summoning his music professor (who he believes to be the devil). The lower and higher pitched notes are produced with separate organ pipes of lengths L_a and L_b respectively. Find the ratio L_b/L_a.

You may assume that the sound heard is the fundamental frequency and that both pipes are open on both ends.

Leave your answer to 3 significant figures.

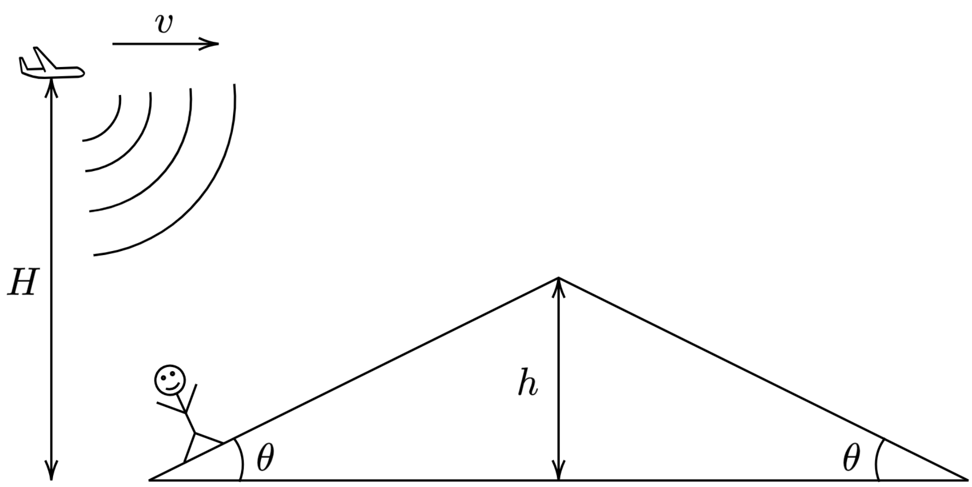
Half Hour Rush X2: Sonic Boom (3 points)

A military jet is flying over a hill of vertical height $h = 500$ m and angle of inclination $\theta = 30°$, from west to east. Chris is standing on a point along the west side of the hill to get a picture of the jet. Approaching from a distance away, the jet flies overhead at height $H = 1000$ m with a horizontal velocity of $v = 680$ m s^{-1}. As this is greater than the speed of sound in air, a shockwave is generated — a sonic boom.

Let the time at which the jet is directly on top of the western base of the hill be $t = 0$. Let the time at which Chris is hit by the sonic boom be $t = T$. Find the maximum value of T.

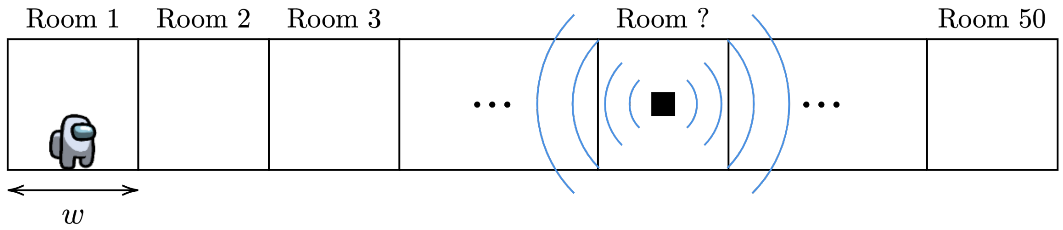
Take the speed of sound in air to be $c = 340$ m s^{-1}.

Leave your answer to 2 significant figures in units of s.



Half Hour Rush X3: An Impostor Among Us (4 points)

Gray is a crewmate aboard a spaceship consisting of a long row of 50 adjacent rooms, each of width $w = 10.0$ m, labelled sequentially from Room 1 to Room 50.



One day, Gray picked up a suspicious encrypted signal and tried to narrow down its source using an intensity meter. He took a measurement at the centre of Room 1 and another at the centre of Room 2, and deduced that the source is at the centre of Room 39. However, Room 39 turned out to be empty.

This is because Gray's intensity meter is suffering from a zero error that leads it to output measurements that are $\Delta = 2.000 \times 10^{-5}$ W m^{-2} lower than the actual wave intensities detected. If Gray's intensity meter showed $I_1 = 1.000 \times 10^{-4}$ W m^{-2} at the centre of Room 1, deduce the actual room number of the signal's source.

Gray had assumed that the waves came from a point source and that the reflection or absorption of these waves by the intervening walls was negligible. Suppose Gray's assumptions and calculations were valid, barring the incorrect measurements.

Leave your answer as an integer from 1 to 50.

Half Hour Rush X4: Doppler's Confusion (5 points)

In one of his late night musings, Doppler came up with a sophisticated way to measure the speed of sound. The following day, he went to the lab to conduct two experiments. In the first one, a sound detector moves at a constant speed v along a circular path of radius $R = 1.00$ m. A speaker is then placed a distance $d = 0.380$ m away from the centre of the circular path. In the second experiment, the detector and the speaker swap positions, as shown in the figure below. The frequency data collected by the detector in both experiments is shown in the graph below. Unfortunately, he forgot to note down which curve corresponds to each experiment. Find c, the speed of sound.

Leave your answer to 3 significant figures in units of m s^{-1}.