4^{th} *Singapore Physics League*

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Problem 1: Simp (3 points)

Amy has a crush on Jake. Amy is hot on Jake's heels, and they are both cycling at the same constant velocity some distance apart. Jake engages the brakes and coasts to a stop at constant deceleration over a distance $d = 20$ m. At the exact moment Jake comes to a stop, Amy engages her brakes and coasts to a stop at the same constant deceleration. Coincidentally, Amy stops exactly where Jake is. How far behind Jake was Amy at the start?

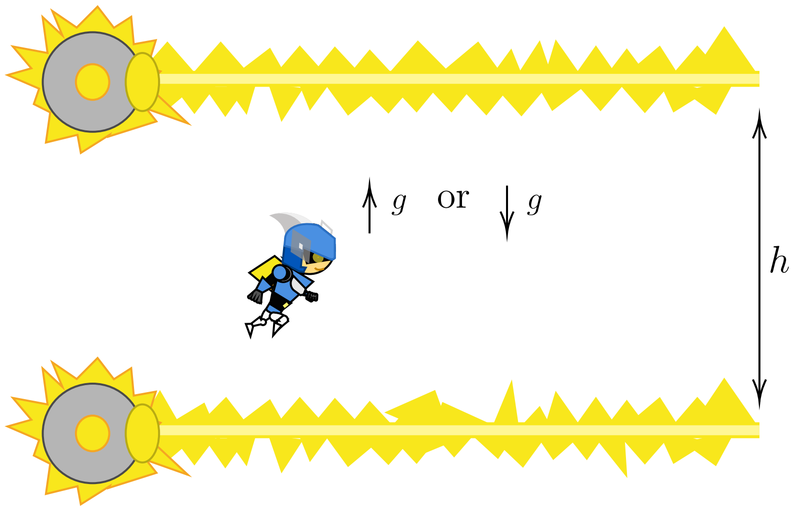
Leave your answer to 2 significant figures in units of m.

Problem A: Jetpack Joyride (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

Gravity Guy is a character in Jetpack Joyride who can reverse the direction of gravity. As Gravity Guy approaches the middle of a pair of long horizontal zappers spaced a distance $h = 0.5$ m apart vertically, he presses his gravity-switching button at a regular time interval t. What is the maximum value of t such that he can stay between the zappers indefinitely? Assume that the magnitude of gravitational acceleration remains constant at g, and that he is small enough to be considered a point mass.

Leave your answer to 2 significant figures in units of s.

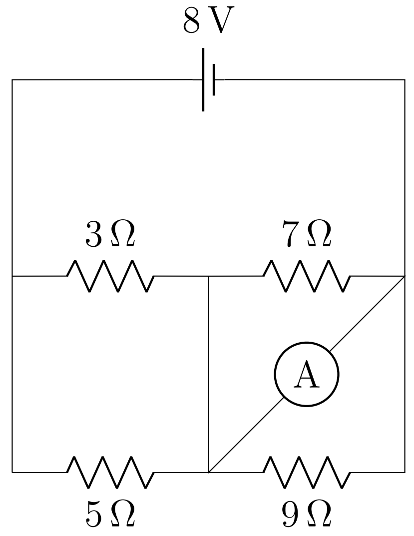


Problem B: Innocent Circuit (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

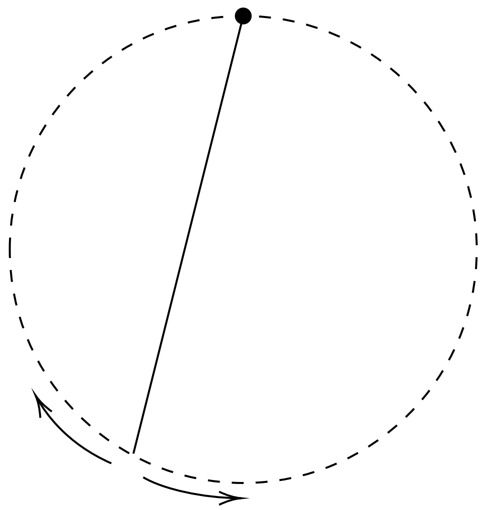
A circuit is set up with an ideal ammeter as shown below. What is the ammeter reading I?

Leave your answer to 3 significant figures in units of A.



Problem 2: Crossing a Circle (3 points)

Consider a vertical ring of radius $R = 20$ cm. A slope is constructed between two points: the top of the ring, and another point chosen anywhere on the ring. A point mass is then placed at the top of the slope and released from rest. Find the minimum possible time taken t for the mass to travel the length of the slope.



Leave your answer to 2 significant figures in units of s.

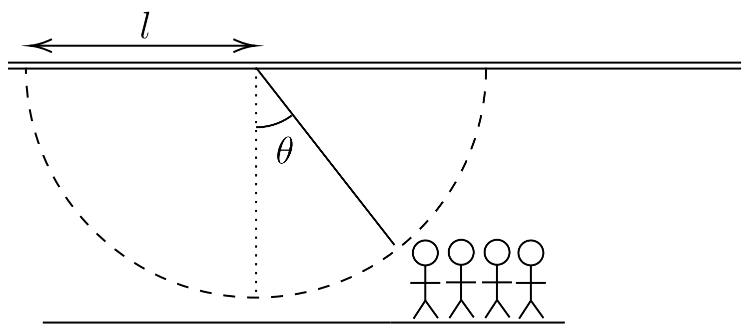
Problem C: Me And The Boys (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

Paul and his friends found a massless rope of length $l = 11$ m with one end tied to a tree branch and decided to have some fun. First, Paul holds the lower end of the rope and swings from rest and back with an amplitude $\theta_0 = 20°$, completing the 1^{st} oscillation. When Paul reaches his maximum height, his friend grabs onto him from rest, and the two friends swing for another full oscillation, completing the 2^{nd} oscillation. This process repeats itself for the infinitely many friends Paul has.

Let v_n be the maximum velocity of Paul and his friends during the n^{th} oscillation. Determine v_4. Assume Paul and his friends are all point masses with mass $m = 67$ kg, and neglect resistive forces.

Leave your answer to 2 significant figures in units of $\mathrm{m\ s^{-1}}$.



Problem D: Icy Friendship (3 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

Adam stands on a frozen lake next to Bane, who is standing on wet ground. Adam's coefficient of static friction on ice is $\mu_{\text{ice}} = 0.05$ and Bane's coefficient of static friction on the wet ground is $\mu_{\text{wet}} = 0.15$. Adam has mass m_a, while Bane has mass $m_b = 30$ kg. When Adam pushes Bane, Adam is able to remain stationary on the ice, while causing Bane to slide on the wet ground.

What is the minimum value of m_a for this situation to occur?

Hint: The maximum static friction force between surfaces $f \leq \mu N$, where N is the normal force between the surfaces.

Leave your answer to 2 significant figures in units of kg.

Problem 3: Confused Diode (3 points)

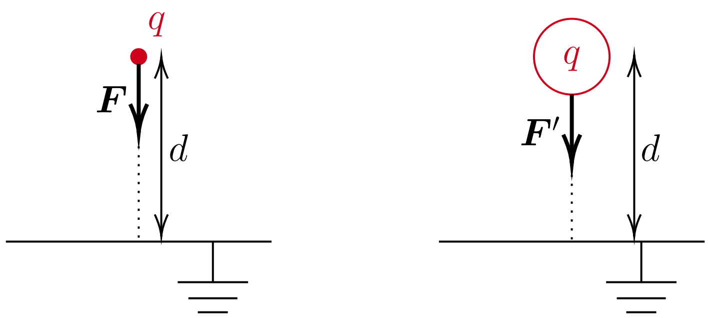
A battery is connected to an arrangement of a diode and two resistors of resistances r and R, where $R > r$, with ideal wires. Initially, the current passing through the battery is I. When the battery's polarity is reversed, the current passing through the battery becomes $I' = 4I$. Find the ratio R/r.

Leave your answer to 2 significant figures.

Problem E: Ball vs. Point (2 points)

Note: These alphabet problems are additional problems for SPhL (Junior) teams only. In contrast, numbered problems are for both SPhL (Junior) and SPhL (Senior) teams.

A point charge $+q$ is located a distance d away from an infinite grounded conducting plate. The charge experiences a force F towards the plate. Now, consider a conducting ball also with charge $+q$ whose centre is a distance d away from an identical plate. The ball experiences a force F' towards the plate. Which of the following is true?



(1) $F > F'$

(2) $F < F'$

(3) $F = F'$

Problem 4: Falling Pendulum (3 points)

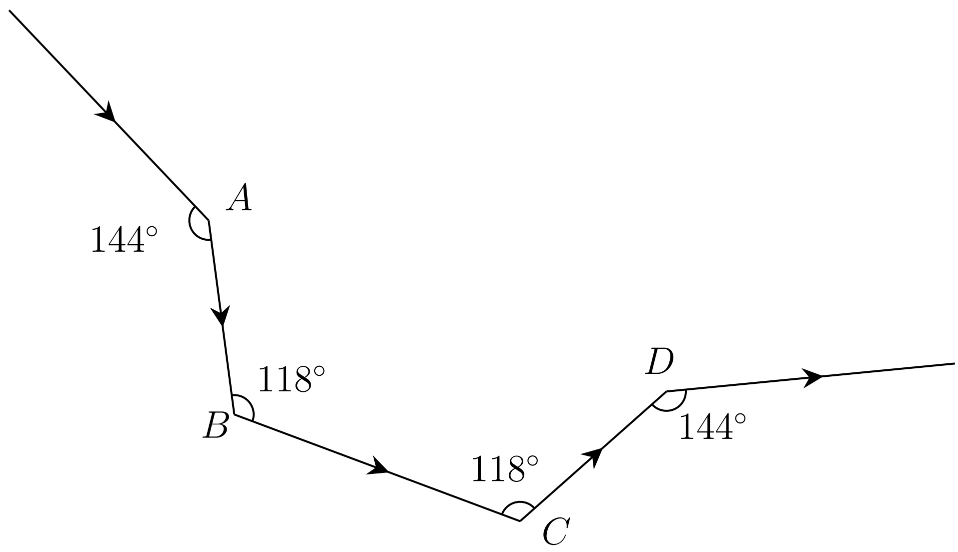
Roger is holding a toy that consists of a pendulum bob connected by a massless string of length $L = 0.40$ m to the ceiling of a sealed case. The bob oscillates with amplitude $\theta_0 = 0.251$ rad.

At time $t_0 = 0$ s, the bob is at angular position $\theta = \theta_0$. At this instant, Roger releases the case, allowing it to fall freely for $t_f = 0.15$ s. Determine the displacement s of the bob from time t_0 to t_f. Neglect resistive forces.

Leave your answer to 2 significant figures in units of m.

Problem 5: Prism Ray Tracing (4 points)

A laser beam travels from air through a transparent triangular prism with unknown orientation, forming the following path. The beam is deflected at exactly 4 points A, B, C and D, with path lengths $AB = CD$. The entire system can be taken as two-dimensional, with the beam and triangular faces of the prism all within the plane of the page. Find n, the refractive index of the prism.



Leave your answer to 3 significant figures.

Problem 6: Tilting Glass (3 points)

A light cuboidal glass with a square base of height $h = 50$ cm and side length $l = 10$ cm is placed on a flat surface. The top surface of the glass is open, allowing the glass to be filled with some water, then tilted slowly to an angle θ without any water spilling. Here, θ is defined as the angle between the base of the glass and the horizontal. Given that the volume of water in the glass can be varied, what is the maximum tilt angle θ_{max} of the glass before it topples?

You may assume the glass's mass is negligible compared to the water's mass.

Leave your answer to 2 significant figures in units of degrees.

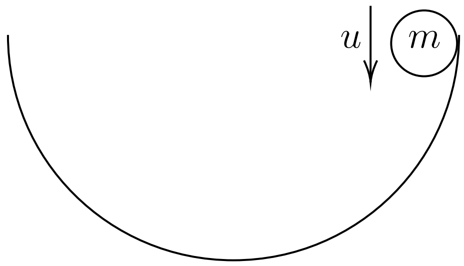
Problem 7: Hot Bulb (3 points)

Physicist S has found an incandescent light bulb which is approximately spherical with radius $R = 2.5$ cm. For reasons best known only to herself, she paints the bulb with black paint such that all light is absorbed by its now opaque glass casing, depriving it of its only purpose in life. She then connects it to a power source of voltage $V = 120$ V. A long time later, her friend passes by and unfortunately finds that the surface of the bulb is now at a temperature $T = 1300$ K. Find the current I drawn by the bulb.

Leave your answer to 3 significant figures in units of A.

Problem 8: An Average Semicircle (3 points)

Consider a smooth semicircular track in the vertical plane. A mass $m = 1.0$ kg enters one end of the track with an initial downward velocity $u = 3.0$ m s^{-1}, and takes time $t = 5.0$ s to reach the other end. Find the magnitude of the time-averaged normal force, $\left|\langle \vec{N} \rangle\right|$, exerted on the mass during its motion along the track.

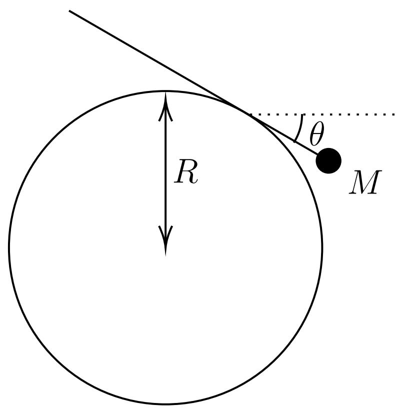


Leave your answer to 2 significant figures in units of N.

Problem 9: Slant Bar (4 points)

Roger is creating an ornament that consists of a uniform bar of mass $m = 0.20$ kg and length $L = 2.0$ m balancing horizontally on top of a horizontal cylinder of radius $R = 1.5$ m, with their centres vertically aligned. He then adds a decorative element of mass M to one end of the bar, causing the bar to rotate without slipping on the cylinder surface. At equilibrium, the bar is inclined at an angle $\theta = 30°$ to the horizontal. The cylinder remains stationary and does not rotate throughout. Determine the mass M.

Leave your answer to 2 significant figures in units of kg.



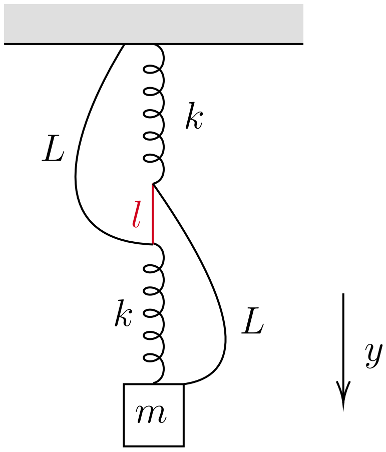
Problem 10: Single to Double (4 points)

A monochromatic laser beam is directed towards a single slit of width $a = 0.30$ mm. A screen is placed a distance $D \gg a$ away from the slit, and the intensity of the central maximum on the screen is I_0. Guangyuan covers part of the slit with an opaque film of width $b = 0.12$ mm. Now, the intensity of the central maximum is αI_0. Find α.

Leave your answer to 2 significant figures.

Problem 11: Strings and Springs (4 points)

Two springs of stiffness $k = 8\ \mathrm{N\ m^{-1}}$ are connected by a string of length $l = 0.5$ m. A mass $m = 1$ kg is hung from the ceiling using the connected springs. Additionally, two initially slack strings of length $L = 2$ m are attached as shown below. The system is initially in equilibrium.



Assuming that the springs have zero rest length, find the **signed** change in height Δy of the mass after l is cut and the system reaches equilibrium.

Leave your answer to 2 significant figures in units of m.

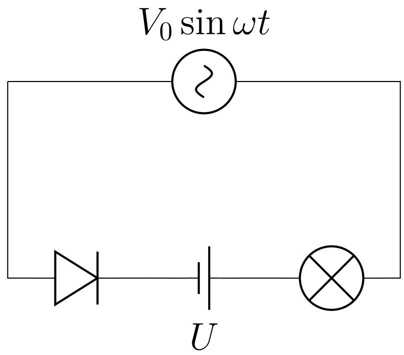
Leave a positive answer if you think the mass falls, and a negative answer if you think the mass rises.

Problem 12: AC/DC (3 points)

An AC voltage source is connected to an ideal diode, a DC voltage source, and a light bulb in series. The AC source has peak voltage $V_0 = 8$ V, while the DC voltage source has constant voltage $U = 4$ V. The bulb lights up when the potential difference across it is non-zero. Over a long time, determine the fraction of time during which the bulb is lit.

Leave your answer to 2 significant figures.

Your answer should be between 0 and 1.



Problem 13: FFFFFF

Fëanor, as High King of the Noldorin elves, wishes to create a system of units for his people to use. Since Fëanor also has a very large ego, he chooses to use 6 units that all start with the letter F:

Unit (abbreviation)	In SI units
fathom (f)	1 f $= 1.83$ m
frequency of middle F (F_4)	1 F_4 $= 349$ Hz
Franklin (Fr)	1 Fr $= 3.34 \times 10^{-10}$ C
Farad (F)	1 F $= 1$ F
Faraday's constant (F)	1 F $= 9.65 \times 10^{4}$ C mol^{-1}
Fahrenheit (°F)	1 °F $= 0.556$ K

Fëanor calls this his FFFFFF system. Fëanor also does not like prefixes (such as kilo- or milli-), and has banned their use.

(a) Fëanor wants to know the mass of a U-235 nucleus, which has mass $m = 235$ u in SI units. Find the numerical value of m in the FFFFFF system.

Leave your answer to 3 significant figures in the appropriate units. (3 points)

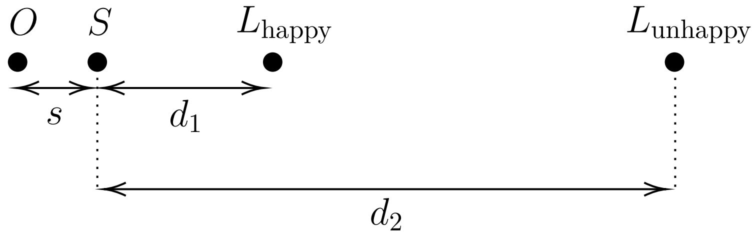
(b) Fëanor decides to create "Fëanor's constant", given by $\mathcal{F} = 2\varepsilon_0 hc/e^2$, where e is the elementary charge. Find the numerical value of $\mathcal{F}$ in the FFFFFF system.

Leave your answer to 3 significant figures in the appropriate units. (2 points)

You may refer to the Data Sheet for all of the constants cited above.

Problem 14: Controversial Concerto (4 points)

A *concerto* is performed by a soloist S in front of an orchestra O. Here, we model the soloist and the orchestra as point sources positioned a distance $s = 10$ m apart. A happy listener, seated at distance $d_1 = 20$ m from the soloist, hears the soloist to be as loud as the orchestra. On the other hand, an unhappy listener, seated at distance $d_2 = 80$ m from the soloist, hears the soloist to be softer than the orchestra. Find the ratio of sound intensities of the soloist to the orchestra I_s/I_o, as heard by the unhappy listener. Neglect any acoustic effects from the walls of the concert hall.



Leave your answer to 2 significant figures.

Problem 15: Nuclear Fusion?

In a fusion power experiment, a helium-4 nucleus undergoes fusion with a beryllium-8 nucleus. Their initial kinetic energies may be taken as negligible and their centre of mass stationary. Their fusion produces a carbon-12 nucleus and may also produce a gamma particle (a high-energy photon).

(a) Find the minimum wavelength λ_{min} of the gamma particle, if it is produced.

Leave your answer to 3 significant figures in units of fm. (2 points)

(b) Find the maximum wavelength λ_{max} of the gamma particle, if it is produced.

Leave your answer to 3 significant figures in units of fm. (2 points)

Answer 0 *if the wavelength can be arbitrarily small.*

Answer -1 *if the wavelength can be arbitrarily large or no gamma particle is produced.*

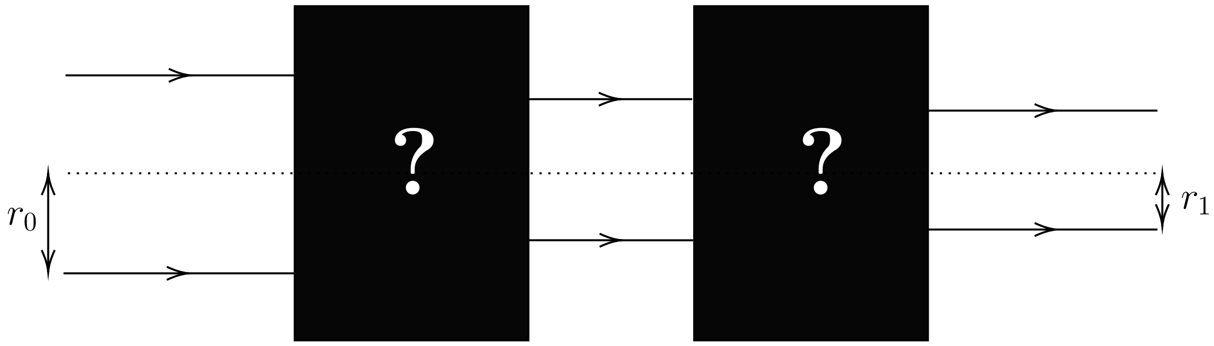
Answer -2 *to both parts if the fusion reaction is impossible or not enough information is given in the problem.*

Data:
Rest mass of a helium-4 nucleus and two electrons: $m_{\text{He}} = 4.002603$ u
Rest mass of a beryllium-8 nucleus and four electrons: $m_{\text{Be}} = 8.005305$ u

Problem 16: Always Parallel (4 points)

Bob has four thin convex lenses of focal lengths 20 cm, 10 cm, 5 cm and 2 cm. He places all the lenses in two boxes (with at least one lens in each box), ensuring that their centres are aligned along a common principal axis. He then directs a light beam of radius $r_0 = 10$ cm through the centre of the first lens, parallel to the optical axis. To his surprise, he observes that the light rays are always parallel outside the boxes.

Given that he can arrange the lenses in any order fulfilling the constraints above, what is the smallest radius of the emergent beam r_1 he can obtain?

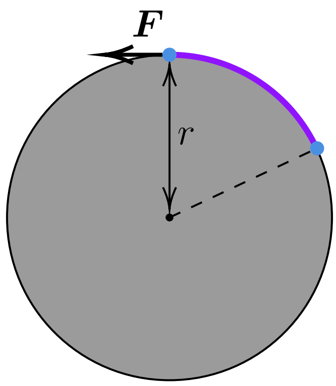


Leave your answer to 2 significant figures in units of cm.

Problem 17: Pulling Strings (4 points)

During a pre-delivery inspection of an airplane, Dave the overworked technician accidentally leaves a thin inextensible string on one of the plane's engines. When he finally notices the string sliding down the engine, Dave swiftly reaches for it and pulls on the top end of the string with a horizontal force F to hold it in place.

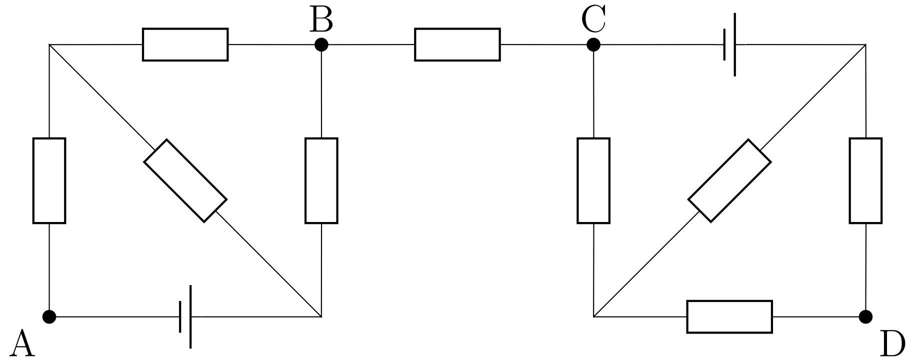
The vertical cross-section of the engine is a circle of radius $r = 1.06$ m. The top end of the string rests precisely at the top of the circle, and the string lies in the same plane as the circle. The string has length $\frac{\pi}{3}r$ and mass $m = 1.32$ kg. Find the magnitude of F so that the string remains stationary. Neglect friction.



Leave your answer to 2 significant figures in units of N.

Problem 18: Circuit Conundrum (4 points)

Shanay constructs a circuit with batteries each of voltage $V = 10$ V and resistors each of resistance $R = 1\ \Omega$:



Find the potential difference $V_{DA} = V_D - V_A$.

Leave your answer to 2 significant figures in units of V.

Problem 19: It's Getting Hot In Here (4 points)

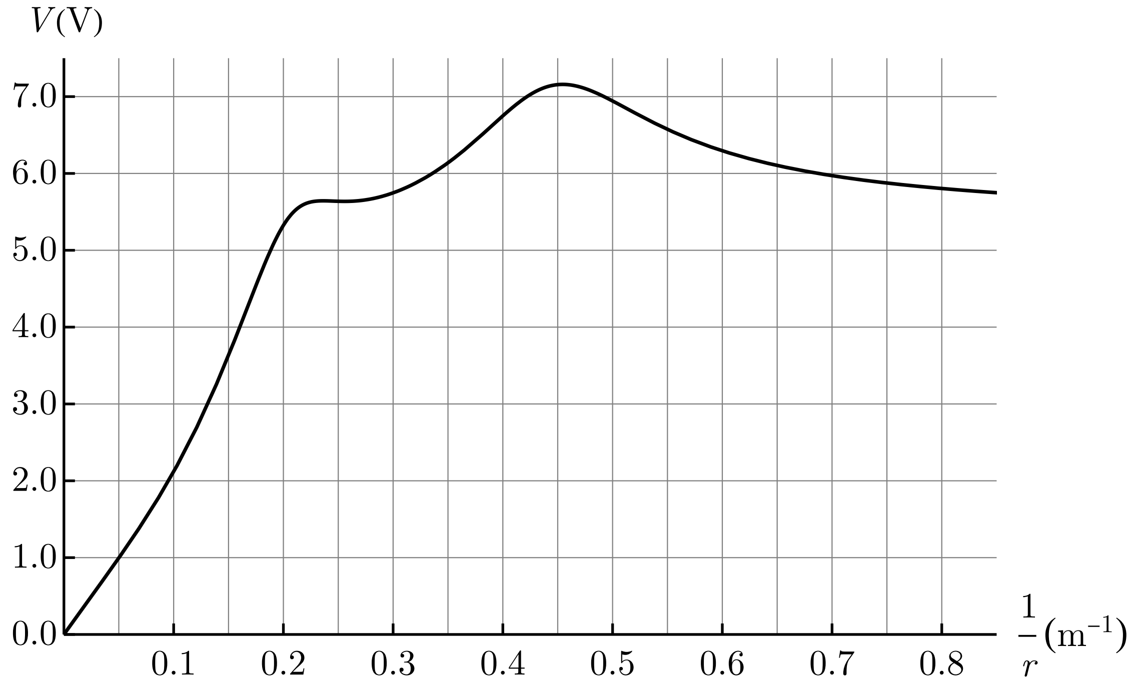
After the SPhL servers shut down during SPhL 2023, Ziwen decides to shut himself in an airtight room of constant volume $V = 30.0 \text{ m}^3$. At $t = 0$ s, the room is initially at atmospheric pressure P_0 and temperature $T_0 = 30.0\,^\circ\text{C}$. As he feels the room is too hot, he turns on the air-conditioner. The air-conditioner feeds air of temperature $T_1 = 20.0^\circ\,\text{C}$ into the room at a constant rate $r = 7.50 \text{ mol s}^{-1}$ starting at $t = 0$ s. Find the temperature T_a of the room after time $t = 300$ s.

Assume that air is a diatomic ideal gas and that the air in the room is always at thermal equilibrium. Ignore the heat from Ziwen's body and assume the walls of the room are perfectly insulating. Further assume that the air conditioner does the work to bring the air into the room, not the air itself.

Leave your answer to 3 significant figures in units of $^\circ$C.

Problem 20: Black Box Potential (4 points)

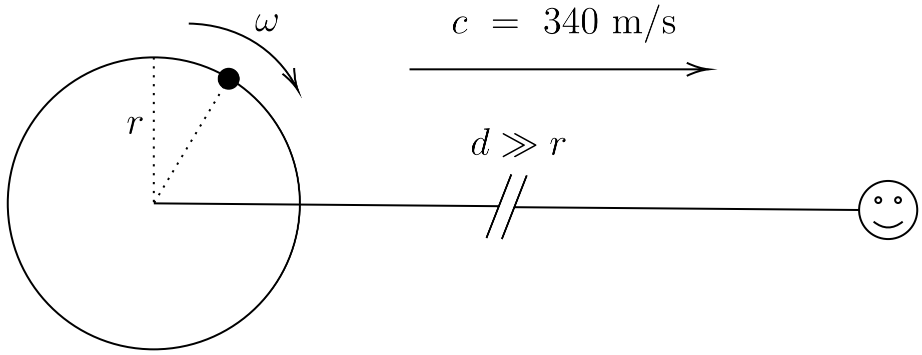
Consider a finite arrangement of charges that you know nothing about except for the following graph of the electric potential V against $1/r$, where r is the distance along an arbitrary axis starting somewhere near the charges. Determine the total charge $\sum Q$ present in the arrangement.



Leave your answer to 2 significant figures in units of nC.

Problem 21: Siren Plane (4 points)

A plane flying in circles of radius $r = 25$ m at $\omega = 6$ rad s^{-1} emits a steady sound wave of frequency $f_s = 200$ Hz. Instead of the steady frequency emitted by the plane, an observer at a far distance hears a sound which oscillates between two frequencies, a higher $f_{\max}$ and a lower $f_{\min}$.



Assume that the speed of sound in air is $c = 340$ m s^{-1}. Find $|f_{\max} - f_{\min}|$.

Leave your answer to 3 significant figures in units of Hz.

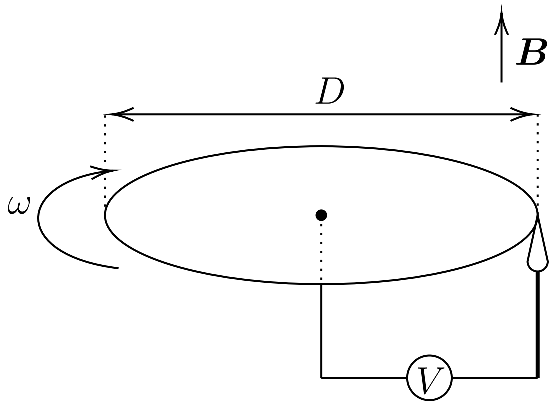
Problem 22: Conducting Plate (4 points)

A horizontal circular plate with diameter $D = 13.0$ m, constructed from an ideal conductor, is rotating clockwise (viewed from above) at an angular velocity ω about a vertical axis passing through its centre. A uniform magnetic field $B = 3.0 \times 10^{-7}$ T points vertically upwards.

A voltmeter is connected to the plate such that one probe is fixed to the centre of the plate and the other is fixed slightly beyond the edge of the plate such that it remains stationary yet it is always in contact with the edge of the rotating plate. The reading on the voltmeter is $V = 0$. Find ω, given that it is nonzero.

Leave your answer to 3 significant figures in units of rad s^{-1}.

Leave a negative answer if you think the plate must be rotating counterclockwise.



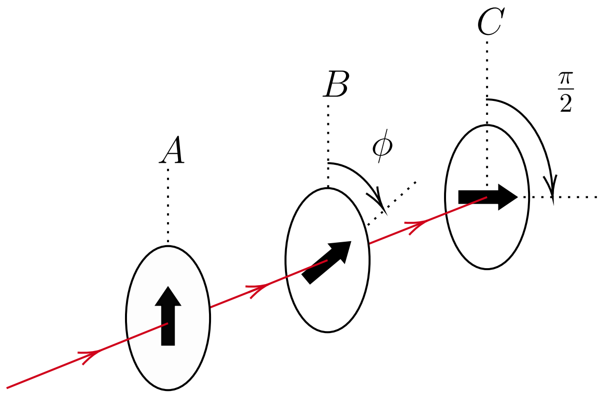
Problem 23: Interstellar (5 points)

Hoping to set up a new habitable home for the continuation of humanity, Cooper and Brand decide to establish a new colony on a distant planet. To facilitate further scientific research, they launch two identical satellites, named TARS and CASE, into space. TARS and CASE orbit around the planet in coplanar circular orbits both in the clockwise direction, with periods $T_{\text{TARS}} = 96$ days and $T_{\text{CASE}} = 144$ days respectively. At $t = 0$, the centres of mass of TARS, CASE and the planet are collinear. After how much time t_1 will the centre-to-centre separation between TARS and CASE increase at the greatest rate? Answer with the minimum possible value of t_1.

Leave your answer to 2 significant figures in units of days.

Problem 24: Wheel of Fortune (4 points)

Alice and Bob play a physics-themed wheel of fortune. The game's setup consists of 3 ideal linear polarisers A, B and C arranged in a row. Polariser A is fixed in the upright orientation, while polariser C is fixed at angle $\frac{\pi}{2}$ rad from polariser A. Polariser B is free to rotate. Unpolarised light of intensity I_0 enters polariser A, and the light intensity after the light passes through all 3 polarisers is I.



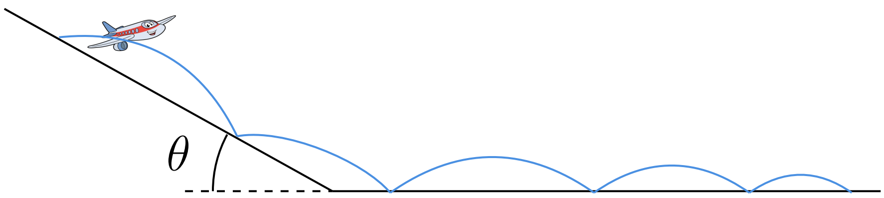
To play the game, each player spins polariser B. Once it stops spinning, the ratio $\alpha = I/I_0$ is the player's score. Alice plays first, and gets a score of $\alpha_A = 0.1$. What is the probability that Bob can get a higher score? You may assume that polariser B is equally likely to be at any angle when it stops spinning.

Leave your answer to 3 significant figures.

Problem 25: Boing Boing

A new airplane has been developed that cannot crash. Made from rubber polymers, it will just bounce. The craft was invented by Boeing, Boeing, Boeing.
- earlofdadjokes

The airplane, unfortunately, loses control and bounces exactly once on a mountain before falling onto the perfectly horizontal ground, where it bounces a few more times:



You are asked to reconstruct the path of the airplane based on the data from the flight data recorder (shown in the graph on the next page), which precisely shows you the magnitude of the airplane's velocity $|v|$ as a function of time. Unfortunately, the data is corrupted, and hence **the units for $|v|$ are arbitrary**. The units for time remain in seconds.

Assume that the mountain is a smooth slope angled at an angle θ to the ground, and that the coefficient of restitution η between the airplane and the mountain is the same as that between the airplane and the ground. $t = 0$ occurs at an arbitrary time prior to the first collision. Neglect friction.

The coefficient of restitution η between the airplane and each surface is a constant defined as the ratio of the magnitude of normal velocities of the airplane after and before the collision.

(a) Find η.

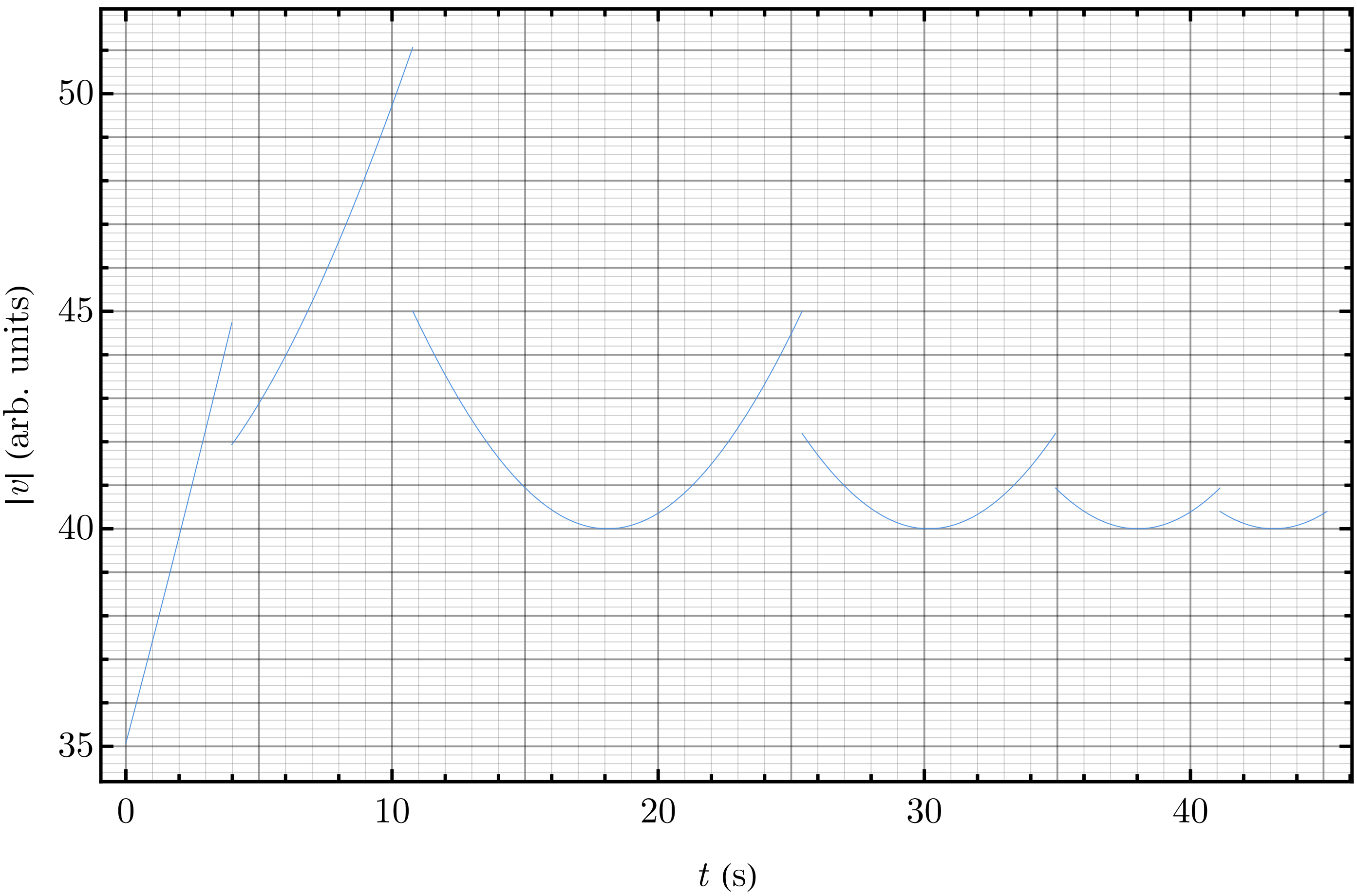
Leave your answer to 2 significant figures. (2 points)

(b) Find the conversion factor from the arbitrary units for velocity into m s^{-1}. Express your answer as k, where 1 arb. unit = k m s^{-1}.

Leave your answer to 2 significant figures. (2 points)

(c) Find θ.

Leave your answer to 2 significant figures in units of degrees. (3 points)



Note for the computations: You should try to keep a high precision in your calculations. You can zoom into the graph to find values.

Problem 26: Carbon-Free Christmas (4 points)

It's the most wonderful time of the year again, and Paul wants to generate energy $E = 156$ MJ to illuminate his Christmas lights. To reduce his carbon footprint, Paul resorts to nuclear reactions and directs a brief beam of neutrons at a large sample of plutonium-239, causing an initial number $N_0 = 5.80 \times 10^{13}$ of plutonium-239 nuclei to undergo the fission reaction ${}^{239}_{94}\mathrm{Pu} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{134}_{54}\mathrm{Xe} + {}^{103}_{40}\mathrm{Zr} + 3 \cdot {}^{1}_{0}\mathrm{n}$.

Paul notices that the neutrons produced go on to trigger further generations of the same fission reaction, each with $k = 1.02$ times as many fissions as the preceding generation, with an average time $T = 24.2$ μs elapsed between two consecutive generations. After how much time t since the first fission generation will Paul amass sufficient energy for his Christmas lights? You may assume that no other reactions take place in the sample, and that all the energy released is converted into electricity.

Leave your answer to 2 significant figures in units of ms.

<u>Data:</u>
Rest mass of a neutron, ${}^{1}_{0}\mathrm{n}$: $m_\mathrm{n} = 1.008665$ u
Rest mass of a zirconium-103 nucleus, ${}^{103}_{40}\mathrm{Zr}$: $m_\mathrm{Zr} = 102.926600$ u
Rest mass of a xenon-134 nucleus, ${}^{134}_{54}\mathrm{Xe}$: $m_\mathrm{Xe} = 133.905394$ u
Rest mass of a plutonium-239 nucleus, ${}^{239}_{94}\mathrm{Pu}$: $m_\mathrm{Pu} = 239.052157$ u

Problem 27: In All Directions

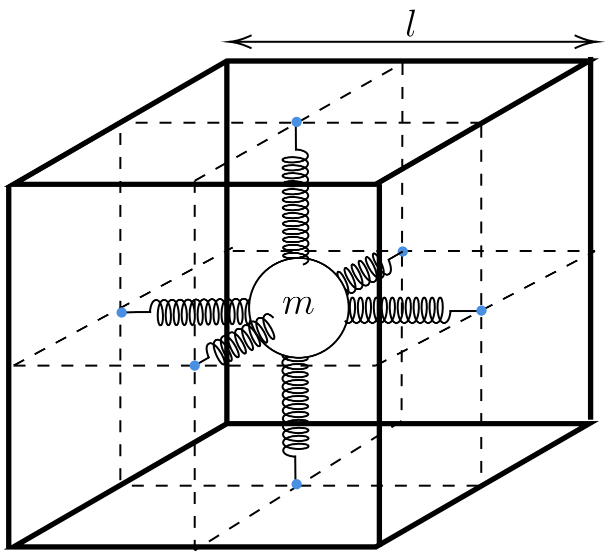
6 identical springs, with spring constant $k = 8$ N m^{-1}, are each attached to the centre of the faces of a cube with side length $l = 50$ cm. The other ends of each spring are all connected to a point mass $m = 3$ kg at the centre of the cube. Ignore the effects of gravity for the entire question.

(a) Given that the springs have zero rest length, find the period of small oscillations of the mass.

Leave your answer to 3 significant figures in units of s. (2 points)

(b) Given that the springs have a rest length of $\frac{l}{2}$, find the period of small oscillations of the mass.

Leave your answer to 3 significant figures in units of s. (3 points)



Problem 28: Confused Inductor

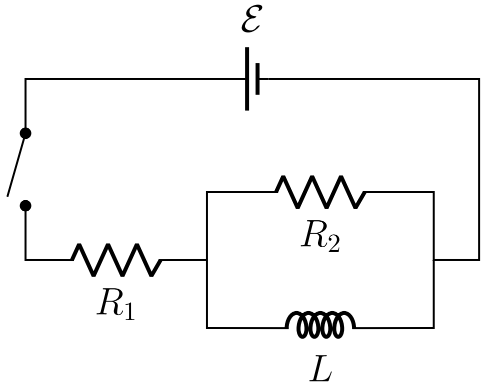
A battery of voltage $\mathcal{E} = 12$ V is connected to an arrangement of two resistors with resistances $R_1 = 20\ \Omega$ and $R_2 = 10\ \Omega$, and an ideal inductor as shown below. Initially, the switch has been open for a long time.

(a) The switch is now closed. Find the voltage $|V_L|$ across the inductor at the instant after the switch is closed.

Leave your answer to 2 significant figures in units of V. (2 points)

(b) After a long time, the switch is now opened again. Find the voltage $|V_L'|$ across the inductor at the instant after the switch is opened.

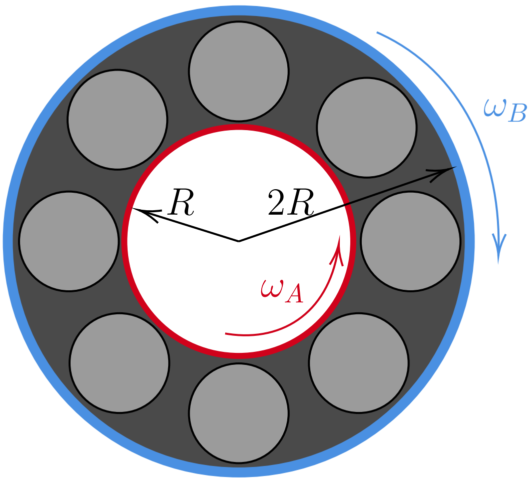
Leave your answer to 2 significant figures in units of V. (3 points)



Problem 29: Ball Bearing (5 points)

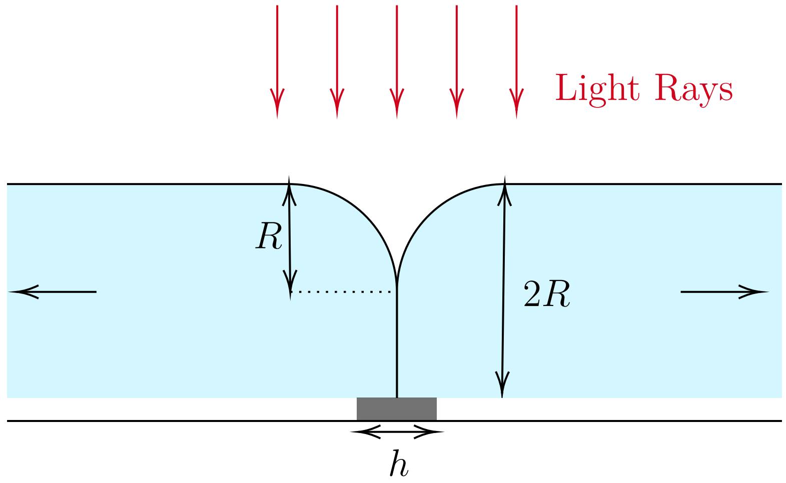
A ball bearing has inner radius R and outer radius $2R$, with 8 balls of equal diameter stuck between the two cylindrical surfaces. Suppose the inner cylinder A is rotating counter-clockwise at angular velocity $\omega_A = 35$ rad s^{-1}, and the outer cylinder is rotating clockwise at angular velocity $\omega_B = 10$ rad s^{-1}, both measured in the lab frame. There is no slipping between the ball bearing and the two surfaces. Let t_0, t_A and t_B denote the time taken for the balls to complete one revolution about the centre of the ball bearing in the lab frame, inner cylinder frame (A) and outer cylinder frame (B) respectively. Let the ratio $t_0 : t_A : t_B$ be $O : A : B$, where $\gcd(O, A, B) = 1$. Find $\overline{OAB}$.

Leave your answer as an integer. For example, if the ratio is $2 : 4 : 8 \equiv 1 : 2 : 4$, *enter* 124 *as your answer.*



Problem 30: Casting Shade (4 points)

Two identical glass blocks with circular corners of radius of curvature $R = 10$ cm cut out are placed touching each other, as shown in the diagram. The blocks have height $2R$ and a long width. Parallel vertical light rays are shone onto the blocks, leaving a dark region of width h centred where the glass blocks meet. Given that the refractive index of the glass is $n = 1.5$, determine h.



Leave your answer to 3 significant figures in units of cm.

Problem 31: Not a Relativity Problem

Physicist S is on a rocket moving away from Earth at velocity $v = 0.8c$. Mission Control on Earth waits for one day to pass on Earth after she departs before sending a message to her. As it is their first message to her, the message header is "Mission Control 0". Afterwards:

- When Physicist S receives the message with header "Mission Control X" for some natural number X, she immediately sends a message back to Mission Control with header "Physicist S X".
- When Mission Control receives the message with header "Physicist S X" for some natural number X, they immediately send a message back to Physicist S with header "Mission Control $X + 1$".

Assume that all messages travel at the speed of light c.

(a) After 1000 years have passed on Earth since the departure of Physicist S, the agency funding the mission and operating Mission Control closed down. What is the number in the message header of the final message from Mission Control received by Physicist S?

Leave your answer as an integer. (3 points)

(b) According to special relativity, it should be impossible for two observers who are both non-accelerating to determine which of them is stationary. Hence or otherwise, how many days does Physicist S think has passed since her departure before she receives the first message with header "Mission Control 0"?

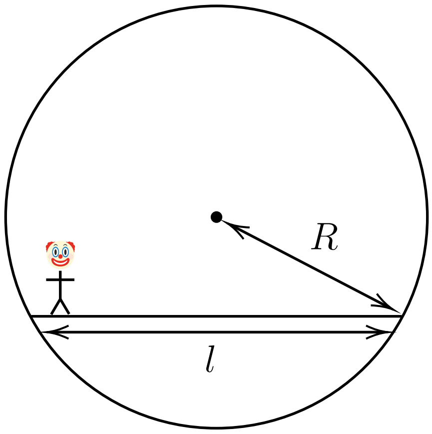
Leave your answer to 3 significant figures in units of days. (2 points)

Problem 32: Tightrope Walk (5 points)

A rod of length $l = 0.7$ m is placed within a vertical, fixed circular hoop of radius $R = 0.5$ m. The rod is initially horizontal, with both ends touching the hoop. The ends of the rod are free to slide along the hoop with no friction, but both ends must always contact the hoop.

A circus clown of mass $m = 70$ kg, wishes to walk across this rod from one end to the other, starting from rest. To make his act more impressive, the clown wants to ensure that the rod remains *completely stationary* throughout his walk. How long will the clown take to complete his act?

Leave your answer to 2 significant figures in units of s.

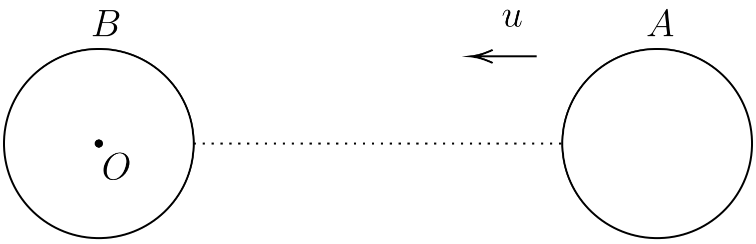


Problem 33: Fluffball Interactions (5 points)

Yueyang loves watching adorable cat videos where furry felines saunter towards each other and snuggle up in the most endearing ways. Below are two frames of a video[1] Yueyang thoroughly enjoys.

In an attempt to model this interaction, Yueyang treats each cat as a uniformly charged insulating sphere of radius $r = 10$ cm, mass $m = 4.0$ kg, and charge $q = +0.50\ \mu$C. Now, consider the case as shown in the diagram below, where cat A, with an initial velocity $u = 0.10$ m s^{-1} to the left, approaches cat B from infinitely far away along the straight line joining the centres of the two cats. Cat B is initially stationary with its centre at the origin O.

Yueyang expects the centre-to-centre separation between the two cats to be minimal at a point in time during the interaction. At this instant, what is the displacement s of the centre of cat B from the origin O? Take the leftward direction to be positive and neglect friction.

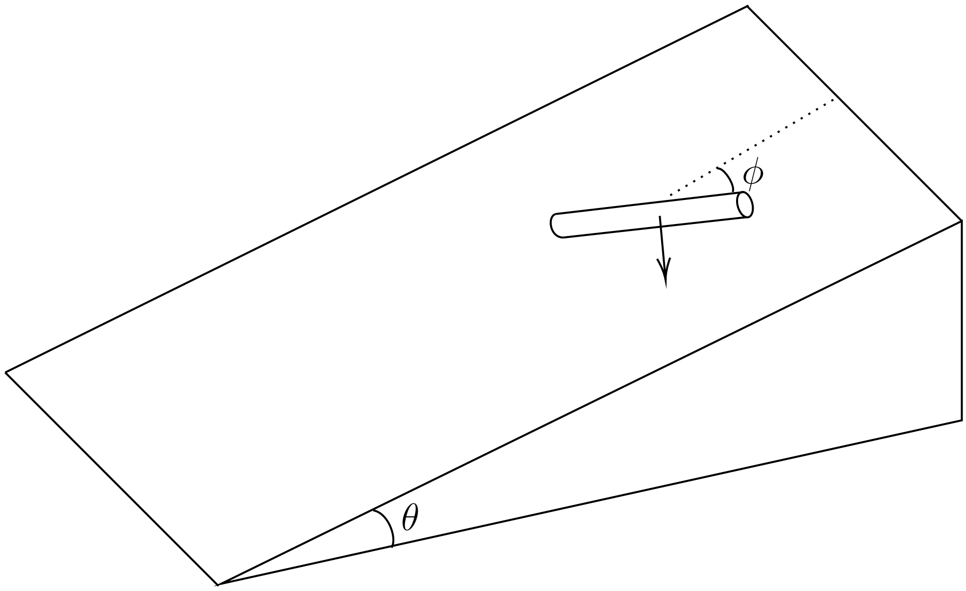


Leave your answer to 2 significant figures in units of m.

[1]These frames were taken from a YouTube video by studio GNYANG.

Problem 34: Double Slanted (5 points)

A uniform solid cylinder is placed on a slope fixed to the ground and inclined at an angle $\theta = 15°$ from the horizontal. The axis of the cylinder makes an angle $\phi = 30°$ with the direction of inclination. Given that the cylinder rolls without slipping, find the minimum coefficient of static friction μ between the cylinder and the slope.



Leave your answer to 3 significant figures.

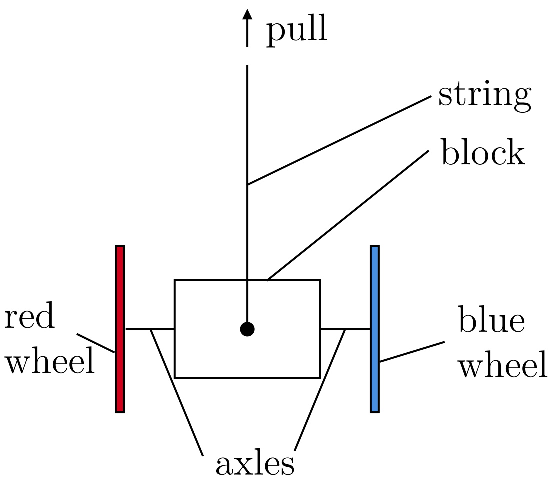
Problem 35: Tracing an Ellipse (3 points)

Niko has a toy consisting of a wooden block attached to two thin circular wheels and a string. The wheels (labelled in red and blue) have the same radius and are free to rotate about their axles which lie on a common axis. The axles of the two wheels are not connected, allowing the wheels to spin independently of each other. The two wheels are fixed a distance $l = 1.0$ m apart.

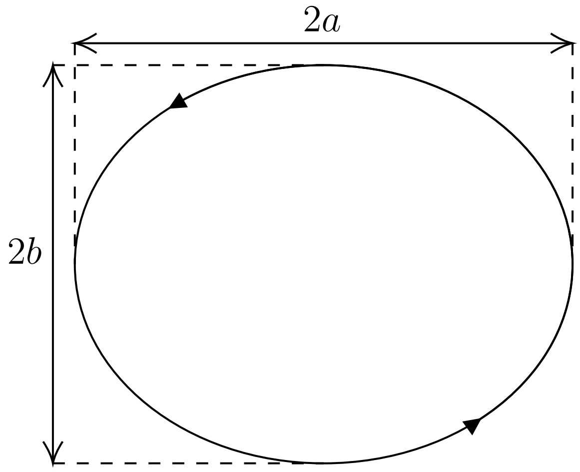
Niko then pulls the string, such that the point of the red wheel contacting the ground traces out exactly one anti-clockwise loop around an ellipse. The ellipse has semi-major axis $a = 50$ m and semi-minor axis $b = 40$ m. At all times, the axles of the toy are perpendicular to the direction of motion.

Let the distance travelled by the centres of the red and blue wheels be d_r and d_b respectively. Find the difference between these two distances $\Delta d = d_b - d_r$.

Leave your answer to 3 significant figures in units of m.



a) The toy (top view)



b) The path of the red wheel

Problem 36: Hot Balls (5 points)

Bob has a pair of identical solid balls. Bob gives Alice one of his balls. Initially, Alice's ball is at temperature $T_a = 200$ K and Bob's ball is at temperature $T_b = 300$ K. They want their balls to be at the same temperature but they can only transfer heat between balls via an ideal heat pump. What will the maximal final temperature T_f of the two balls be after equilibrium is reached?

Leave your answer to 3 significant figures in units of K.

Problem 37: Water Bending (5 points)

Consider the fictitious *SPhL* ocean somewhere on Earth, with a flat ocean floor a distance $D = 4000$ m below sea level. A cylindrical meteor with radius $R = 2000$ m and height $h = 100$ m lands onto the bottom of the ocean floor with its axis orientated vertically. As a result, the water level above the cylinder rises. Find δ, the increase in sea level directly above the centre of the meteor as compared to regions far away, at equilibrium. The density of the meteor is $\rho_m = 5321$ kg m^{-3}.

Leave your answer to 2 significant figures in units of mm.

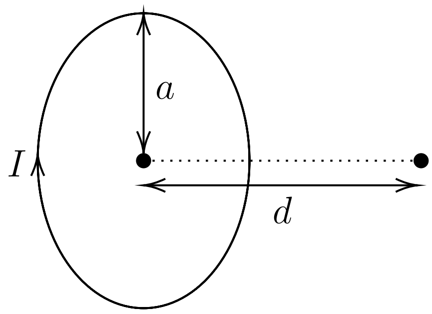
Leave a negative answer if you think the sea level above the meteor falls.

Problem 38: Magnetic Charge

Consider, in analogue to an electric charge, a hypothetical *magnetic charge*, which creates a uniform magnetic field pointing radially outwards from itself. The magnitude of the magnetic field B is given by $B = \frac{k_m q_m}{r^2}$, where $k_m = \frac{\mu_0}{4\pi}$, $q_m = 100$ A m is the magnetic charge, and r is the distance from the charge.

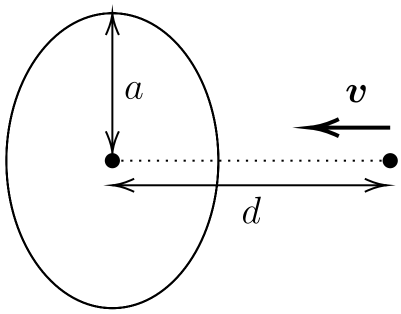
(a) A magnetic charge is placed a distance $d = 0.100$ m away from the centre of a circular wire loop of radius $a = 0.100$ m, along its axis of symmetry. A current $I = 10.0$ A flows through the loop. Find the magnitude of the force F on the magnetic charge due to the loop.

Leave your answer to 3 significant figures in units of N. (3 points)



(b) Consider the same setup as in part (a), but let there be no current I initially flowing through the wire loop. The magnetic charge now moves towards the wire loop with speed $v = 1000$ m s^{-1}, along its axis of symmetry. Find the magnitude of the instantaneous force F on the charge due to the loop. The electrical resistance of the wire loop is $R = 1.00 \times 10^{-3}$ Ω.

Leave your answer to 3 significant figures in units of N. (3 points)



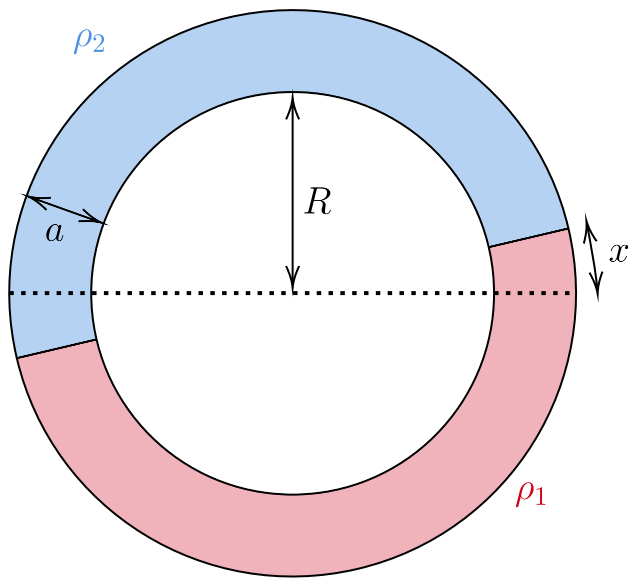
Problem 39: Instability (5 points)

A vertical circular O-tube in gravity has its lower half filled with a liquid of density $\rho_1 = 1000$ kg m^{-3}, and upper half filled with a liquid of density $\rho_2 = 2000$ kg m^{-3}. The radius of the circle $R = 1.00$ m is much larger than the radius of the tube a.

Initially, the system is at equilibrium. After a small initial perturbation of the fluid surface x_0, the fluid-fluid interface may begin to move. At small time-scales, this perturbation grows exponentially such that the perturbation x is of the form $x = x_0 e^{\gamma t}$ where γ is known as the growth rate. Find the value of γ.

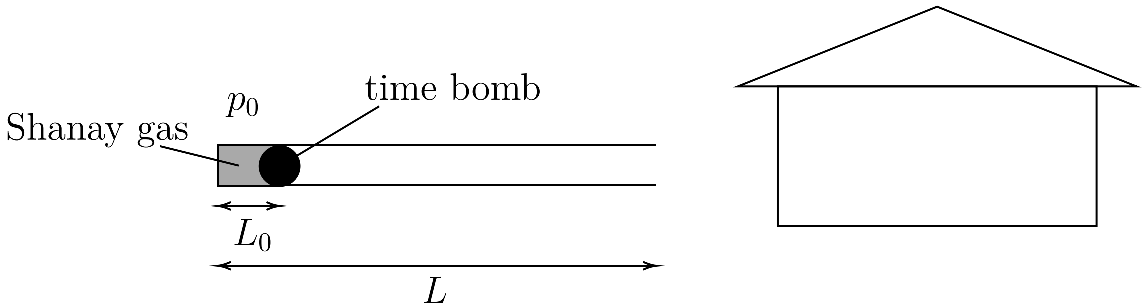
Assume $x \ll a$ throughout the motion, and neglect any effects of viscosity, friction or surface tension.

Leave your answer to 3 significant figures in units of s^{-1}.



Problem 40: Return to Sender (5 points)

Shanay constructs a cannon and points it at your house, intending to launch a time bomb of mass $m = 6$ kg. The cannon shoots by initially pressurizing some Shanay gas of adiabatic index $\gamma = 2$ to pressure $p_0 = 1000\,p_{\text{atm}}$ and then allowing it to expand rapidly. Take $p_{\text{atm}} = 1.01 \times 10^5$ Pa as the pressure of the air to the right of the bomb, and throughout the atmosphere. The bomb initially sits at a distance $L_0 = 0.01$ m away from the left end of the cannon, and is spherical with radius $r \ll L_0$.



You know that if the cannon is of a minimum length L, the bomb will eventually return towards Shanay (exploding him). Thus, you quickly extend the cannon while Shanay is away. Find the value of L.

You may assume that the bomb and the walls of the cannon are perfectly insulating and the bomb fits snugly in the barrel while moving with no friction along the length of the cannon. You may further assume that Shanay gas behaves like an ideal gas.

Leave your answer to 3 significant figures in units of m.

Problem 41: Interstellar (Continued) (5 points)

At the beginning of the film *Interstellar*, the spacecraft Endurance travels from Earth to Saturn with the help of Mars. Mars has mass $M = 6.00 \times 10^{23}$ kg and orbits at velocity $V_i = 24$ km s^{-1} around the Sun. Endurance has mass $m = 2 \times 10^6$ kg and its velocity is given by $v_i = 2V_i$. Endurance approaches Mars at an angle $\theta = 60°$ with respect to the direction of Mars' velocity. The trajectory of Endurance is chosen such that its final velocity after leaving Mars is perpendicular to its initial velocity, and is as large in magnitude as possible. Find the change in the kinetic energy ΔK of Mars through this process.

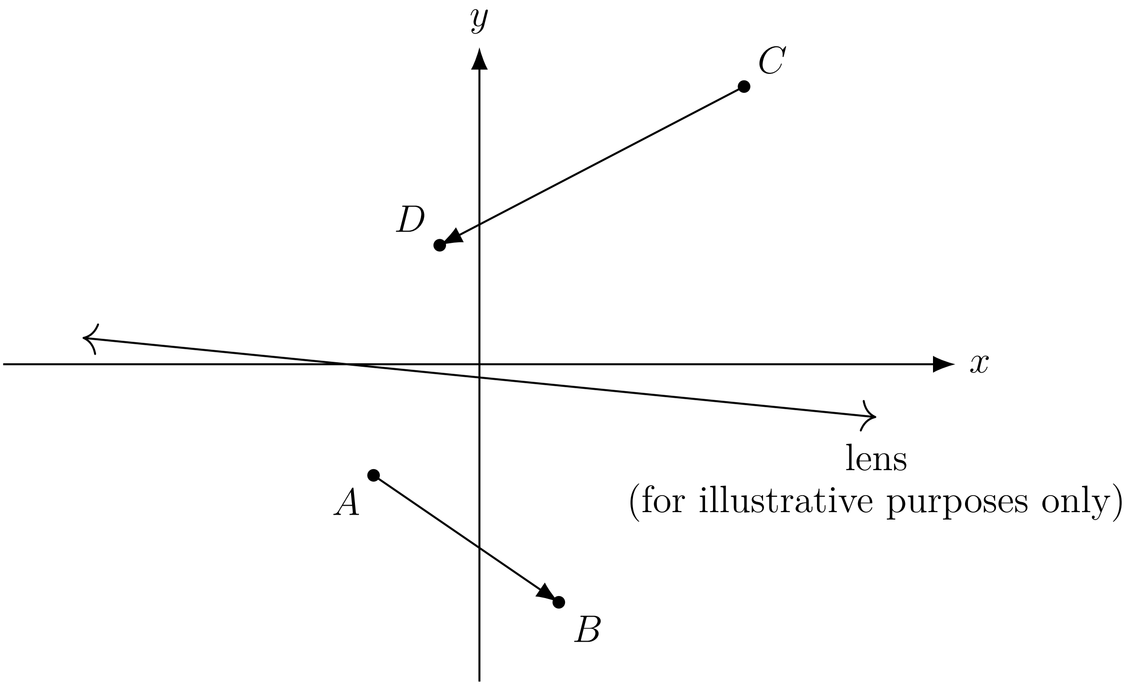
You may model both Endurance and Mars as point masses. The magnitude and direction of all velocities are given in the frame of the Sun. You may also assume that the orbit of Mars is always circular.

Leave your answer to 3 significant figures in units of TJ.

Leave a negative answer if you think the kinetic energy decreases.

Problem 42: Object and Image (5 points)

A convergent ideal thin lens is placed such that the plane of the lens is perpendicular to the plane of the paper, and the optical axis lies within the plane of the paper. Define points $A(-4.0, -4.2)$, $B(3.0, -9.0)$, $C(10.0, 10.5)$ and $D(-1.5, 4.5)$ within the plane of the paper. If the vector $\overrightarrow{AB}$ is used as an object for the lens, then the real image $\overrightarrow{CD}$ is formed. Find the focal length of the lens.

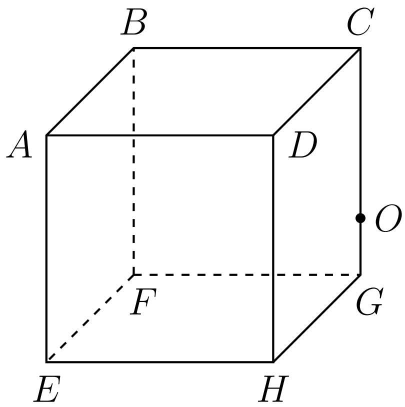


All coordinates are given in centimetres.

Leave your answer to 2 significant figures in units of cm.

Problem 43: Electric Cube (7 points)

Paul constructs a cube structure $ABCDEFGH$ using 12 uniform, identical wires each with resistance $R = 1.0\ \Omega$, as shown in the figure below. While connecting it to an external circuit via points A and G, Paul notices that the contact supposedly at G unexpectedly slips towards C along side CG. He denotes O as the new contact point along side CG. Given that O can be at any point along CG, what is the maximum effective resistance R_{AO} between points A and O?



Leave your answer to 2 significant figures in units of Ω.

Problem 44: Heat Capacity

A perfectly conducting block with heat capacity $C = 455$ J K^{-1} is connected to a heat source by a cylindrical rod of negligible heat capacity, length $L = 91.0$ cm, radius $r = 3.25$ cm and conductivity $k = 398$ W m^{-1} K^{-1}. The heat source has temperature $T_s(t) = T_0 + \Delta T \sin(\omega t)$ at time t, where $T_0 = 299$ K, $\Delta T = 52.0$ K and $\omega = 0.0161$ s^{-1}. We may assume that the setup is given sufficiently long to reach steady-state before time $t = 0$ and that there is negligible heat loss to the surroundings.

(a) Find the maximum temperature T_{max} of the block after time $t = 0$.

Leave your answer to 3 significant figures in units of K. (4 points)

(b) At what time t_0 does the block first reach this temperature after time $t = 0$?

Leave your answer to 3 significant figures in units of s. (2 points)

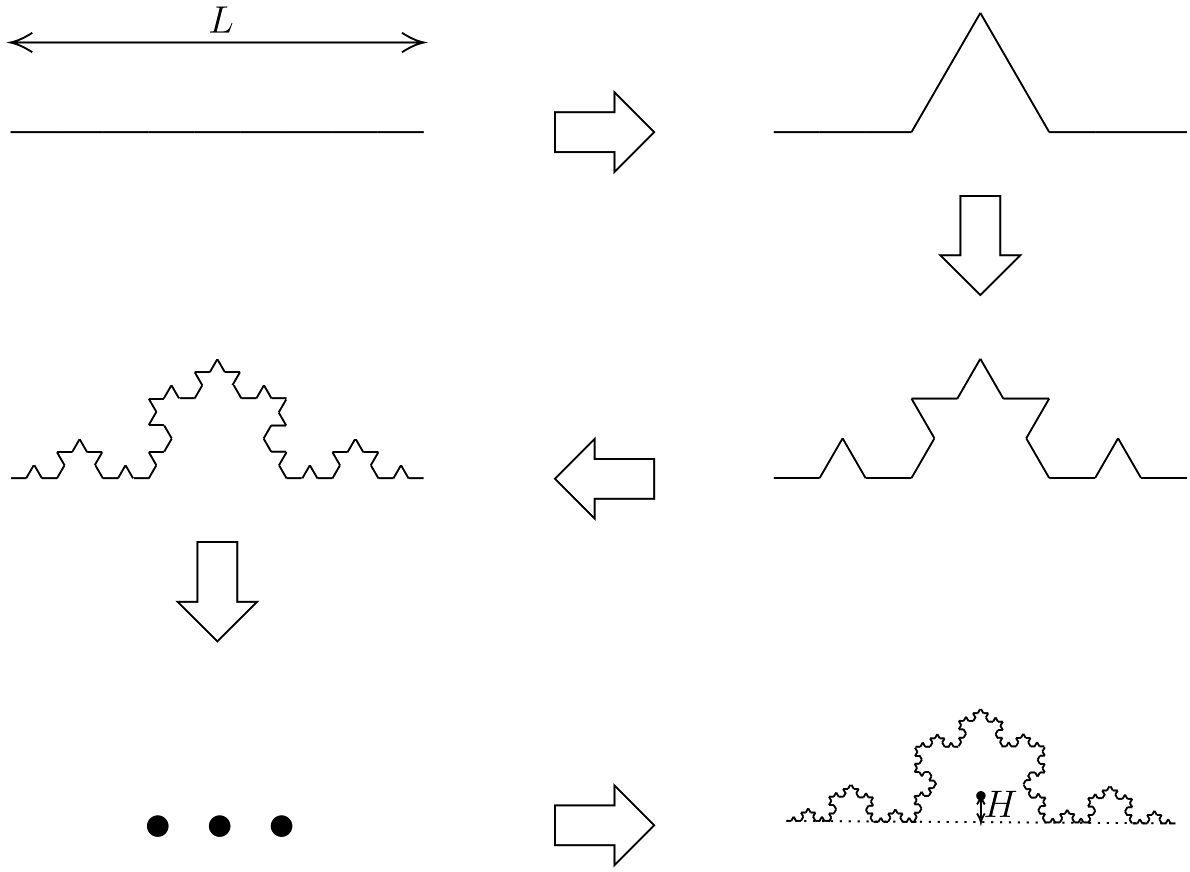
Problem 45: Snowflake Fragment (5 points)

A fractal is produced as follows:

1. Start with a uniform thin straight wire of length $L = 1.00$ m.
2. Cut out the middle third of the wire segment, and stretch it to twice its initial length. Its mass remains constant, distributed uniformly along its length (i.e. this new segment has half the mass per unit length of the original segment).
3. Bend the wire into two such that the two branches form an angle of 60°, and reattach it to the other twos segments, facing outwards.
4. Repeat Steps 2 and 3 for each straight segment of the wire recursively, down to arbitrarily small scales.

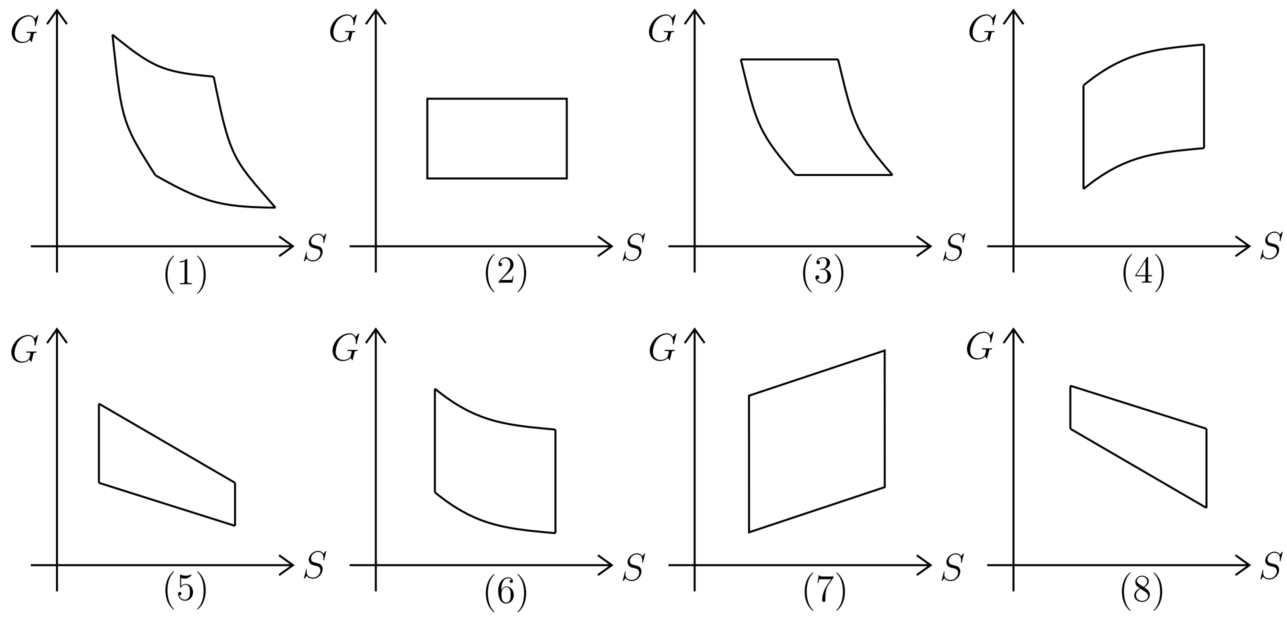
The resulting fractal-shaped wire has centre of mass at a distance H from the original centre-of-mass position of the straight wire. Find the value of H.

Leave your answer to 3 significant figures in units of cm.



Problem 46: Carnot's Strange Cycle

(a) The Gibbs free energy of a gas (with internal energy U, pressure p, volume V, temperature T and entropy S) is defined as $G = U + pV - TS$. For a Carnot engine using an ideal gas working between 2 large heat reservoirs, which G-S diagram is the most accurate representation of the cycle, and in which direction? You may assume that G and S are defined such that they are always positive.



Leave your answer as a non-zero integer from -8 to 8. If you think the cycle is clockwise, leave you answer as the option number. If you think the cycle is anti-clockwise, add a negative sign in front of your answer. (3 points)

(b) Suppose the Carnot engine uses $n = 0.800$ mol of ideal monoatomic gas, between 2 large heat reservoirs at temperature $T_h = 400$ K and $T_c = 300$ K. The volume of the gas doubles during the isothermal expansion at temperature T_h. Furthermore, the maximum entropy of the system is three times the minimum entropy. What is the area A enclosed by the G-S graph?

Leave your answer to 2 significant figures in units of $\mathrm{J^2\ K^{-1}}$. (4 points)

Problem 47: Revenge of the Crazy Electron (5 points)

An electron is projected with initial velocity $v_0 = 150\ \text{m s}^{-1}$ at an initial distance $y_0 = 0.36$ m from an infinitely long wire carrying current $I = 0.0014$ A. The direction of initial velocity is perpendicular and directed away from the wire. Determine the maximum distance y_{max} of the electron from the wire.

Leave your answer to 2 significant figures in units of m.

Problem 48: Hot Ring

Annatar has a ring of radius $R = 0.700$ m, situated in the x-y plane with its centre at the origin (the z-direction points upwards). The ring has a total surface area of $A = 0.0300$ m^2 and a small cross-sectional area $S \ll A$. The ring is a perfect black body with temperature $T_0 = 10000$ K. A small thin disc with radius $r = 1.00$ mm $\ll R$ is located at a height $H = 1.75$ m directly above the centre of the ring (so the centre of the disc is at $(0, 0, H)$). The plane of the disc is parallel to the x-y plane.

(a) Calculate the intensity of thermal radiation I at the centre of the disc.

Leave your answer to 2 significant figures in units of kW m^{-2}. (2 points)

(b) Calculate the equilibrium temperature T of the disc. Assume heat is lost from the disc only through radiation. The disc can be treated as a perfect heat conductor, and has emissivity $\varepsilon = 0.170$.

Leave your answer to 2 significant figures in units of K. (3 points)

(c) The disc is able to hover in position, due to the radiation pressure from the thermal radiation of the ring. Calculate the mass m of the disc.

Leave your answer to 2 significant figures in units of ng. (3 points)

Problem 49: Bobbing Ball (5 points)

A uniform solid spherical ball of radius r floats in water. The ball is displaced vertically downwards by a distance $a \ll r$ and released from rest, after which it oscillates with period T. After a long time $t_1 \gg T$, its amplitude of oscillations drops below a fixed value $\delta \ll a$. If a ball of the same material and increased radius $2r$ is displaced by the same distance a, the time taken for its amplitude to drop below δ becomes t_2. Determine the ratio t_2/t_1.

Assume that the damping force on the ball is given by Stokes' Law as $F = -6\pi r\eta v$, where η is the viscosity of the water and v is the velocity of the ball. Assume also that the ball's oscillations are underdamped.

Leave your answer to 2 significant figures.

Problem 50: Friendly Fire

The Death Star continuously fires a deadly, perfectly collimated laser with power $P = 9.0 \times 10^{14}$ W at an initially stationary rebel X-wing with mass $m = 100$ kg and positioned a distance l away. Much to Darth Vader's anger, the X-wing has a perfectly reflecting surface, and the moment the first photon fired from the laser returns to him, his laser is destroyed. The X-wing and Death Star are both initially stationary in the lab frame, and the mass of the Death Star is large enough such that it remains effectively stationary throughout.

(a) If $l = 2.0 \times 10^3$ m, find the final velocity of the X-wing in the lab frame.

Leave your answer to 2 significant figures in units of m s^{-1}. (2 points)

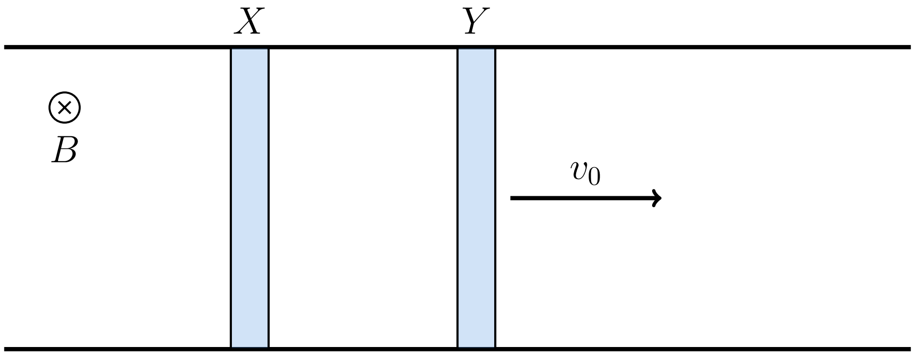
(b) If $l = 2.0 \times 10^{12}$ m, the final velocity of the X-wing in the lab frame is given by $v = \beta c$, where c is the speed of light. Find β.

Leave your answer to 2 significant figures. (5 points)

Problem 51: Catch-up Capacitor (6 points)

Two conducting rods, X and Y, are placed perpendicular on top of smooth, parallel rails. Each rod can be considered a series combination of a capacitor and resistor. The capacitance of X across its length is $C_x = 100\,\mu\text{F}$ and the capacitance of Y across its length is $C_y = 2C_x$. Each of them has resistance $R = 10\ \Omega$ and mass $m = 0.10$ kg. A strong magnetic field $B = 10$ T is directed into the page. The rails are a distance $d = 0.50$ m apart and have zero resistance.

Y moves at an initial speed $v_0 = 10\ \text{ms}^{-1}$ to the right. This induces X to move to the right following Y. Sadly, X never catches up with Y. Once steady state is reached, at what rate does the distance between them increase?

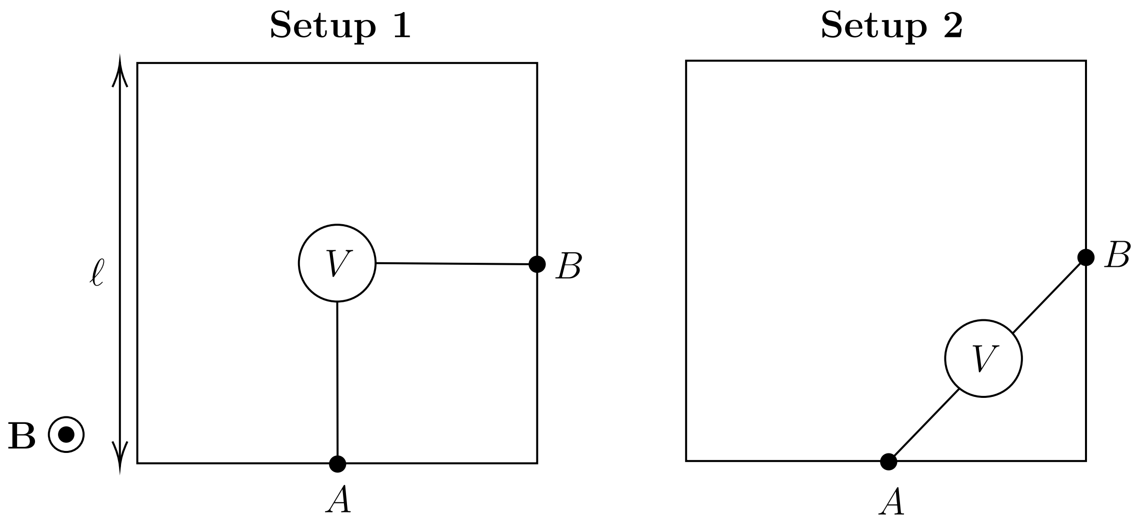


Leave your answer to 2 significant figures in units of m s^{-1}.

Problem 52: Sussy Voltmeter (5 points)

Bobby places a uniform, square-shaped wire with side length $\ell = 1.0$ m and total resistance $R = 10\ \Omega$ in a plane perpendicular to a magnetic field. The magnetic field points out of the page and increases in magnitude at a constant rate $\alpha = 10\ \mathrm{T\ s^{-1}}$.

He wants to measure the potential difference between points A and B as shown, where A and B are both the midpoints of the sides of the square wire. He connects the voltmeter in two different setups, and records two different readings V_1 and V_2, despite placing the voltmeter probes at the exact same two points on the wire. The voltmeter has an internal resistance $r = 1000\ \Omega$. Determine $|V_1 - V_2|$.



Leave your answer to 3 significant figures in units of V.

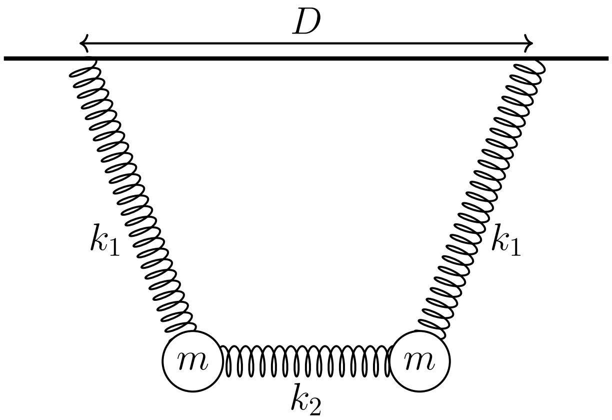
Problem 53: Escaped Proton (6 points)

An infinitely long charged wire with linear charge density $\lambda = 5 \times 10^{-9}$ C m^{-1} and radius $R = 0.1$ m is placed inside a uniform magnetic field $B = 0.1$ T directed parallel to the wire. A proton leaves the surface of the wire at an initial speed v_i in an arbitrary direction. What is the **minimum** magnitude of v_i required such that the **maximum** distance from the surface of the wire that the proton can reach is also R?

Leave your answer to 3 significant figures in units of m s^{-1}.

Problem 54: Spring Swing

A "swing" is made of 3 springs and 2 masses. Two identical springs of spring constant $k_1 = 100\ \mathrm{N\ m^{-1}}$ are attached to a ceiling at points a distance $D = 0.50$ m apart. At their other ends, they are each attached to identical point masses $m = 20$ kg, which are connected by a spring of spring constant $k_2 = 200\ \mathrm{N\ m^{-1}}$. Take all springs to have zero natural length.



(a) Find the distance d between the two masses at equilibrium.

Leave your answer to 2 significant figures in units of m. (2 points)

(b) Find the sum of all distinct resonant frequencies, $\sum \omega_i$, of the system.

Leave your answer to 3 significant figures in units of rad $\mathrm{s^{-1}}$. (5 points)

Problem 55: Relativistic Submarines (5 points)

There are two submarines underwater, Submarine A and Submarine B, operated by Operator A and Operator B respectively, moving along the same axis towards each other. Relative to the water, Submarine A is moving at a speed of v_A while Submarine B is moving at a speed of $v_B = 5v_A$.

Operator A shines a monochromatic red light, which Operator A measures to have a wavelength of $\lambda_i = 666$ nm in air, at Submarine B. The light bounces off the reflective window of Submarine B and returns to Submarine A, where Operator A sees it as dark blue light and measures it to have a wavelength of $\lambda_f = 420$ nm in air. Some of the light passes through the reflective window of Submarine B and is seen by Operator B, who measures the light to have a wavelength of λ_m in air. Find λ_m.

Take the refractive index of water to be exactly $n = \frac{4}{3}$. Take the refractive index of air to be 1.

Leave your answer to 3 significant figures in units of nm.

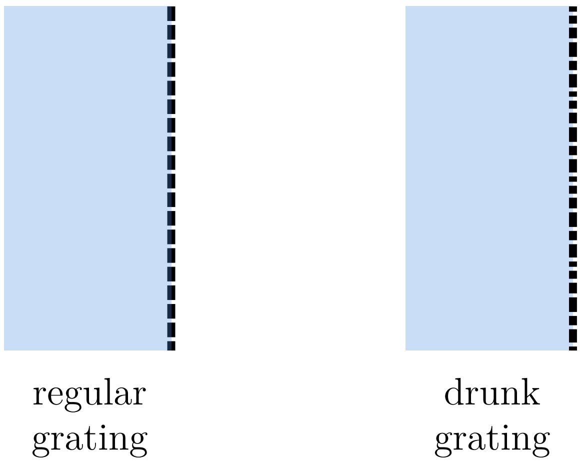
Problem 56: Drunk Grating (5 points)

Jennifer uses a 3D printer to create diffraction gratings. Each diffraction grating has $N = 1 \times 10^6$ slits, with adjacent slits separated by distance $d = 1200$ nm. She tests the grating by passing a laser of wavelength $\lambda = 600$ nm through the grating onto a screen placed a distance $D \gg d$ away. She records the intensity I_0 of light at an angular position $\theta_0 = 0°$.

One day, while drinking on the job, she accidentally sets the slit separation between each adjacent slit to be a random value between 0 and d with uniform probability, while keeping the total number of slits constant at N. Now, she conducts the test again with the same laser and screen, but records the intensity I_1 of light at an angular position $\theta_1 = 30°$ instead. Determine the expected value of the ratio $\frac{I_0}{I_1}$.

Assume that the width of each slit is constant and much smaller than slit separations in both gratings. Assume further that the beam illuminates the whole grating.

Leave your answer to 2 significant figures.



Problem 57: Toilet Bowl

Why do some toilet bowl lids close so slowly?

Let us consider a model of a hinged lid falling towards the ground. The lid is of a square shape with side length $\ell = 40$ cm and has uniform mass $m = 0.50$ kg. Assume that there are vertical walls blocking the red cross-section at the sides (as shown in the figure), such that air between the lid and ground can only escape from the blue cross-section at the edge furthest from the hinge.

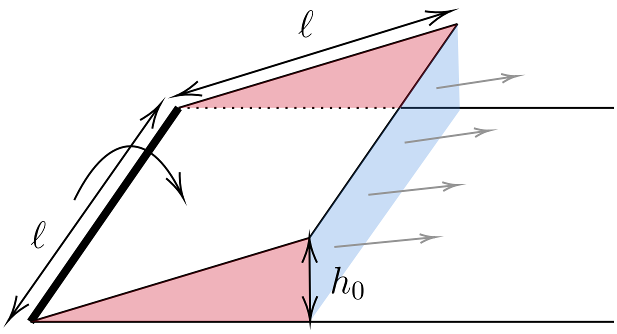
The air has density $\rho = 1.3$ kg m^{-3} and the airflow is laminar and incompressible. The slowing of air near surfaces can be ignored, and the air quickly becomes stationary once it escapes and diffuses into the surroundings.

(a) When the angular velocity of the lid is ω, the velocity of the air flow v at a radial distance x from the hinge is given by $v = \alpha\omega x$, where α is a dimensionless constant. Determine α when the height of the lid $h = 1.0$ cm (measured from the highest side to the ground).

Leave your answer to 2 significant figures. (3 points)

(b) The lid is released from rest at a small initial height $h_0 = 3.0$ cm. Determine the angular velocity ω_1 of the lid when its height above ground reaches $h_1 = 1.0$ cm.

Leave your answer to 3 significant figures in units of rad s^{-1}. (3 points)



Problem 58: Pulling an Inductor

In the following circuit, a very long solenoidal inductor with $n_0 = 5000\ \mathrm{m}^{-1}$ turns per unit length is connected to a capacitor in series. The length ℓ and radius r of the inductor satisfy $\ell \gg r$, so only the magnetic field inside the inductor needs to be considered. The initial maximum current in the circuit is $I_0 = 1.00$ A, and the initial period of oscillations of the current is T_0.

At time $t = 0$, there is no current flowing through the wires. At this instant, the inductor is pulled on and stretched out, such that the turns per unit length decreases to $n_1 = 2000\ \mathrm{m}^{-1}$. This process is done in time t_p. Find the new maximum electric current I_1 in the circuit if:

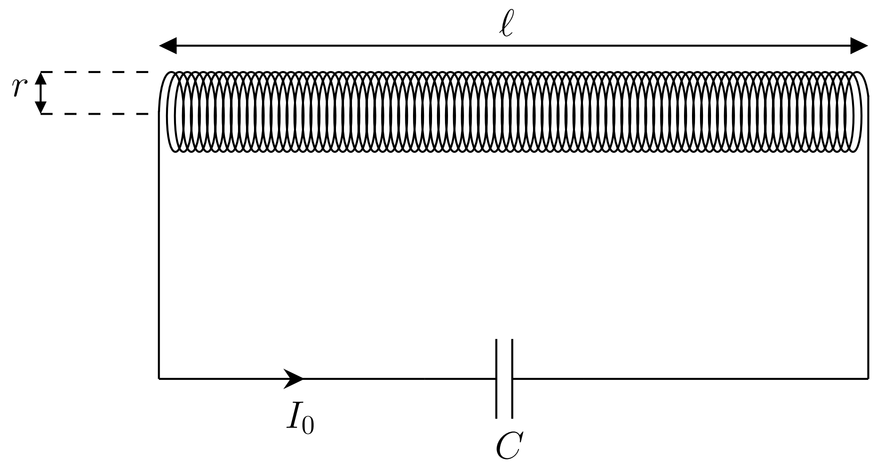
(a) $t_p \ll T_0$.

Leave your answer to 3 significant figures in units of A. (3 points)

(b) $t_p \gg T_0$.

Leave your answer to 3 significant figures in units of A. (4 points)

You may assume the turns remain equally spaced apart throughout this process.

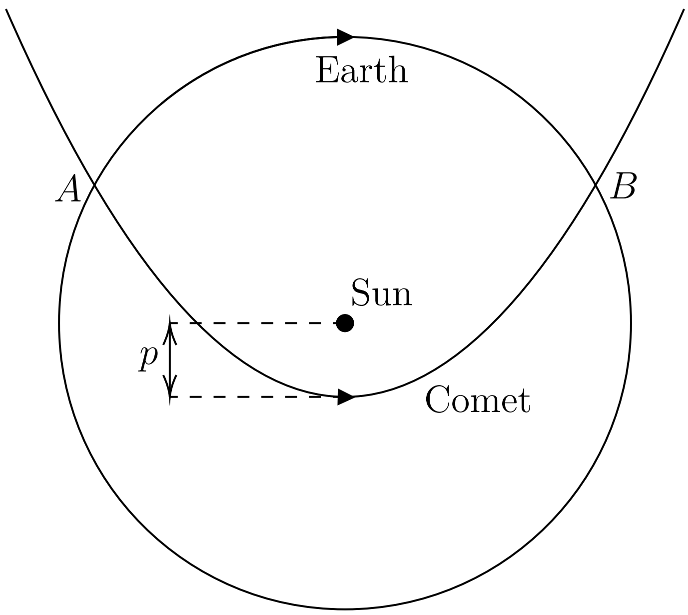


Problem 59: Viewing a Comet (6 points)

A comet travels in a parabolic path around the Sun. Its closest distance to the Sun is p. The comet's position first coincides with the Earth's position at point A, and again at point B. You may assume that:

- Although the Earth and comet are close to each other at points A and B, they do not exert any gravitational influence on each other. Only the gravitational force from the Sun acts on the comet.
- The Earth's orbit is circular with radius R.
- The Earth travels along the minor arc AB to pass by the comet again.

Find the value of $\frac{p}{R}$.



Leave your answer to 2 significant figures.

Problem 60: Bouncing in a Valley

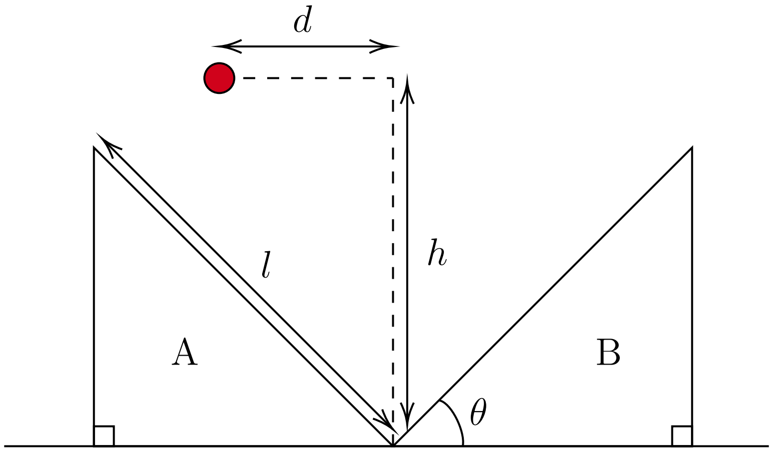
Slope A and Slope B, two smooth right triangular prisms, are fixed to the ground and joined at their bottom corners, forming a V-shaped valley. Each slope has slope length l and is inclined at $\theta = 45°$ above the horizontal. A ball is dropped above Slope A, at a height h above the ground, and a horizontal distance d from the centre of the valley. Model the ball as a point mass and assume that all collisions are perfectly elastic and frictionless.

(a) Given that $l = 2.00$ m, $h = 1.25$ m, $d = 0.35$ m, find the time taken t_1 for the ball to first return to its starting point. If the ball never returns to its starting point, enter -999 as your answer.

Leave your answer to 3 significant figures in units of s. (4 points)

(b) Given instead that $h = 2.50$ m while l and d take the same values as in part (a), the ball will eventually escape over the edge of the valley. Find the duration t_2 between the ball's initial release and its final contact with the prisms.

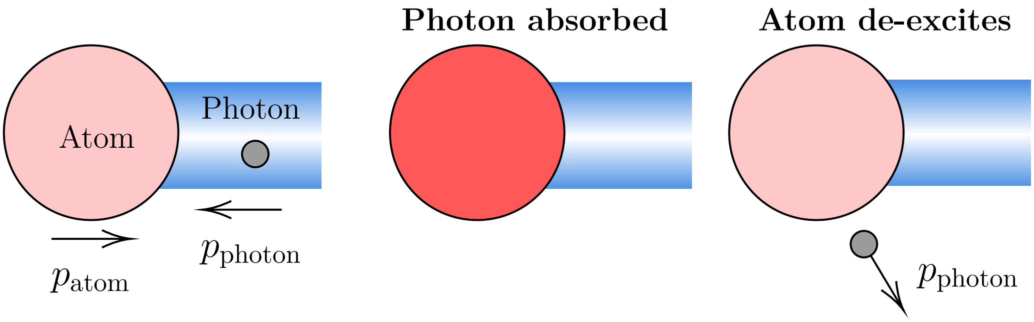
Leave your answer to 3 significant figures in units of s. (4 points)



Problem 61: Why So Cool? (5 points)

It was recently discovered that one can cool an atom (i.e. slow it down) by directing a laser at it.

Consider the simplest case of 1D motion. A photon is absorbed when its frequency **in the atom's frame** coincides with the atom's "resonant frequency". The excited atom will then quickly de-excite by emitting a photon spontaneously but in a random direction. The net result is that the atom's speed decreases.

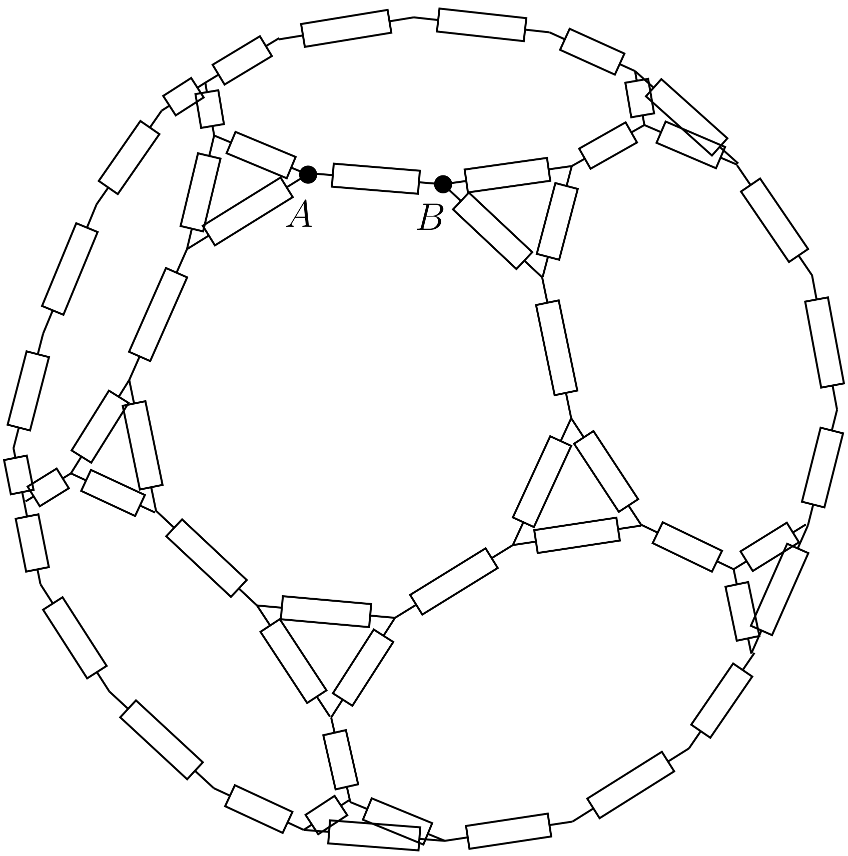


Consider a Ca^{2+} ion of mass 6.6×10^{-26} kg and initial velocity $v_0 = 0.10c$ at $t = 0$ in the lab frame, where c denotes the speed of light. The ion can absorb photons with a frequency range of 5.3×10^{14} Hz $< f <$ 5.5×10^{14} Hz (in the ion's frame). The laser is directed against the atom's initial velocity, and has a frequency $f_0 = 4.9 \times 10^{14}$ Hz with power $P_0 = 1.0 \times 10^{-8}$ W as observed in the lab frame. Assume that only **half** the photons from the laser reach the atom and get absorbed. Find the time taken t for the atom to stop absorbing photons.

Leave your answer to 2 significant figures in units of ms.

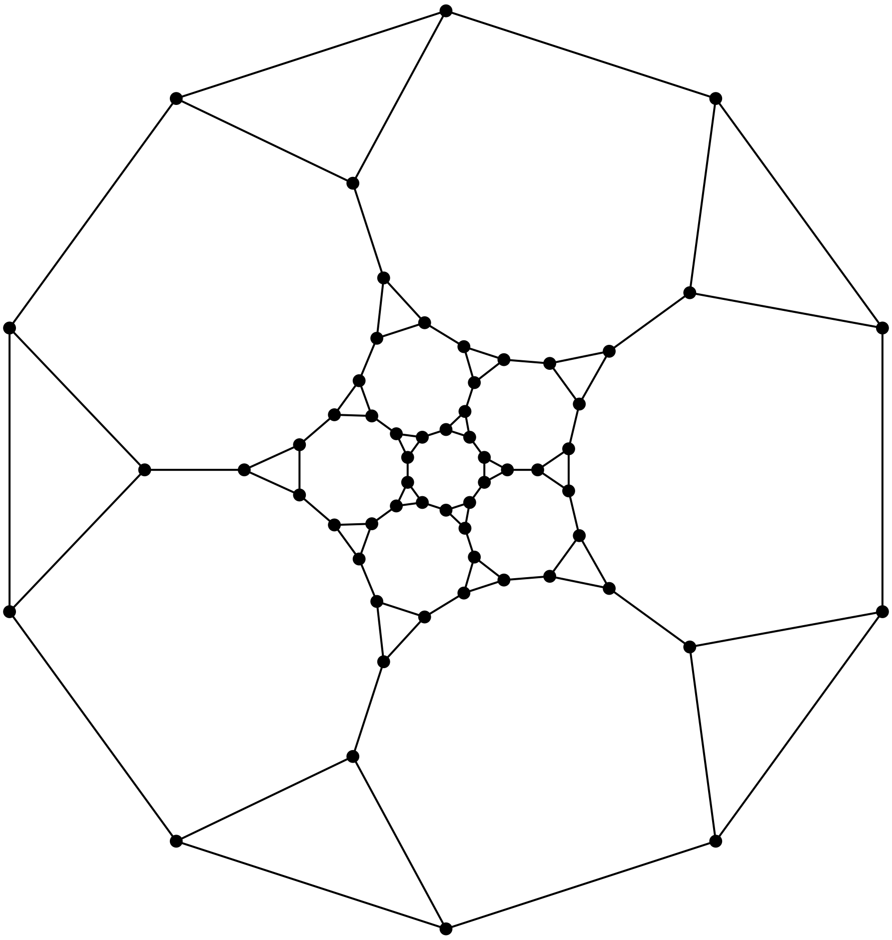
Problem 62: Truncated Dodecahedron (6 points)

Identical resistors of resistance $R = 1.0\ \Omega$ are connected by wires of zero resistance into the shape of a truncated dodecahedron, which is a polyhedron formed by cutting off the corners of a regular dodecahedron. The resistors thus form 12 decagonal faces and 20 triangular faces. Determine the resistance R_{AB} between points A and B.

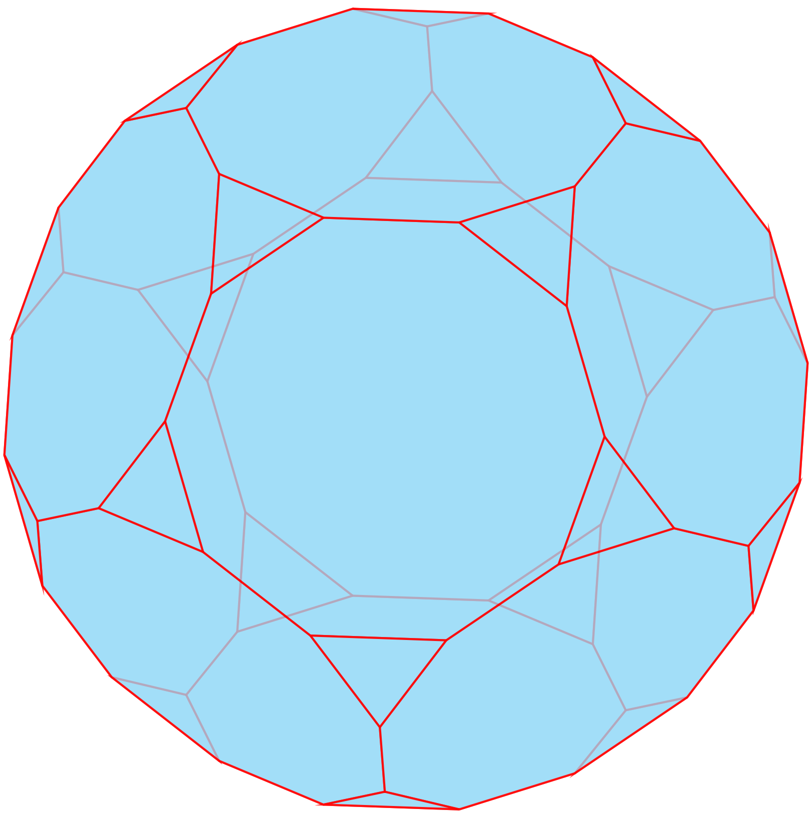


Note that in the circuit diagram, only the resistors in the front are shown to avoid clutter. To see more diagrams of the truncated dodecahedron, see the next page.

Leave your answer to 2 significant figures in units of Ω.



A truncated dodecahedron, as a planar graph

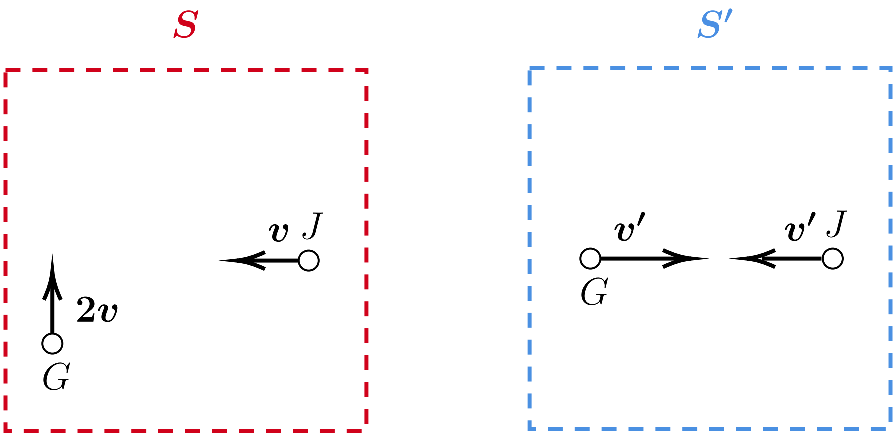


A truncated dodecahedron

Problem 63: Delulu is the Solulu (5 points)

Guangyuan is a Twice stan. At a Twice concert, Jihyo is moving at velocity $v = \frac{2}{5}c$ in an inertial frame S. Unfortunately for Guangyuan, he can only move at velocity $2v$ in a direction perpendicular to her, as viewed in frame S. In his delusion, he argues that they are actually moving directly towards each other, as there exists an inertial frame S' in which his velocity $v' = \beta' c$ has the same magnitude and opposite direction to Jihyo's (though its exact direction is not necessarily as shown in the diagram). Determine β'.

Leave your answer to 3 significant figures.



Half Hour Rush M1: Weightlifting (3 points)

In a simplified model of weightlifting, an athlete applies an initial upwards impulse to the weight, then allows it to move freely under the influence of only gravity. The maximum height of the weight can then be taken as the point where it reaches zero velocity.

The athlete can lift the weight in two ways:

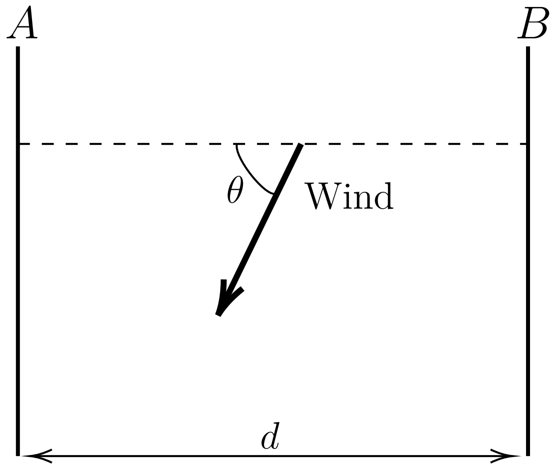
1. **Clean and Jerk**: the weight is lifted in two stages, first to a momentary rest at a height $l = 0.90$ m, then lifted once again to height $2l$.
2. **Snatch**: the weight is lifted to height $2l$ immediately.

If a weightlifter lifts a maximum mass $m_c = 200$ kg for the clean and jerk, determine the maximum mass m_s he can lift for the snatch. Assume that the maximum impulse that the weightlifter can impart remains the same.

Leave your answer to 2 significant figures in units of kg.

Half Hour Rush M2: Smooth Sailing (3 points)

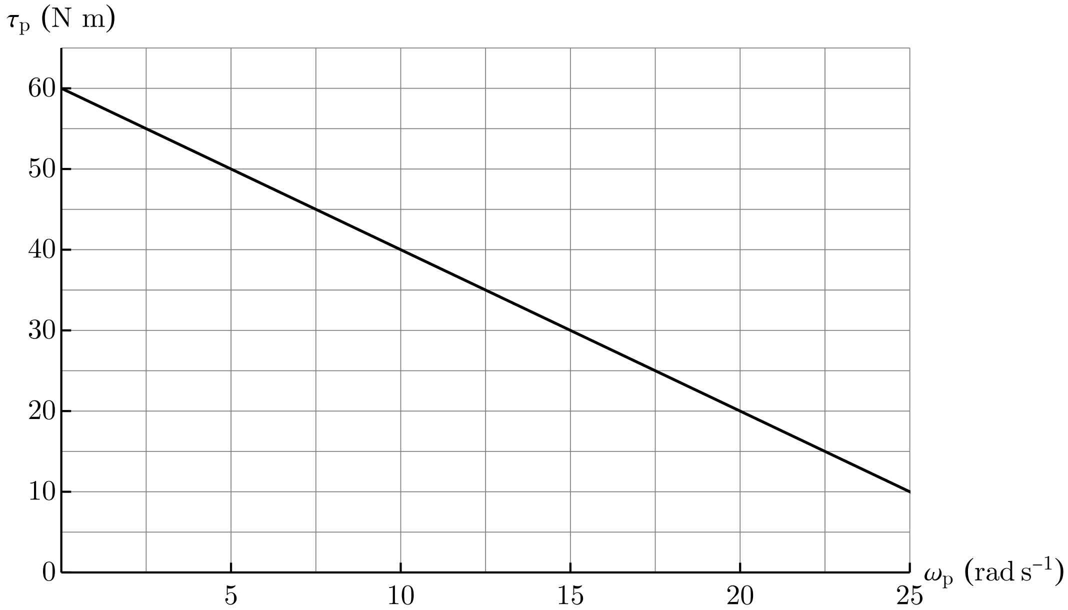
Roger is an Olympic sailor. He needs to travel from Shore A to Shore B, separated by a perpendicular distance $d = 2000$ m. However, the wind blows against him at an angle $\theta = 70^\circ$ to the axis perpendicular to the shores. His sail reflects the wind perfectly and can be oriented in any direction. He orientates it in a constant direction such that he reaches the other shore in the shortest possible time. Determine the distance l that he travels from Shore A to Shore B.



Leave your answer to 2 significant figures in units of m.

Half Hour Rush M3: Tour de France (4 points)

Paul is an Olympic cyclist. The maximum torque τ_p that he can supply to the pedals on his bicycle is dependent on the angular velocity ω_p at which he pedals. The graph below shows the relation between τ_p and ω_p.



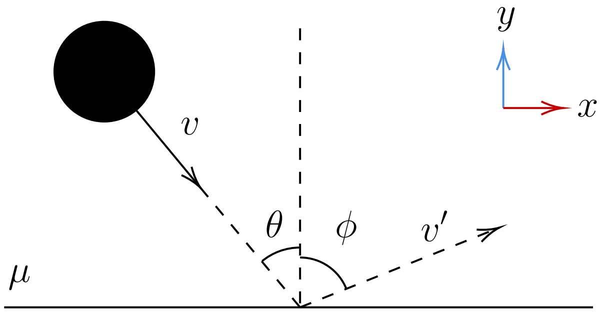
Paul starts from rest pedalling with a 5 : 1 gear. At any time while cycling, he can choose to switch to a 10 : 1 gear. The gear ratio $a : b$ represents the ratio of the number of rotations of the wheels, a, to the number of rotations of the pedals, b.

Given that the radius of his wheels is $r = 0.20$ m, at what velocity v of his bicycle should he switch gears to reach his top speed within the shortest time?

Leave your answer to 2 significant figures in units of m s^{-1}.

Half Hour Rush M4: Tennis Ball (5 points)

Paul, an Olympic tennis player, throws a spherical tennis ball at a wall with coefficient of kinetic friction $\mu = 0.5$. The ball travels in a straight line with initial speed $v = 6.9$ m s^{-1}, at an angle $\theta = 50°$ with respect to the normal.



(Top-down View)

The coefficient of restitution e of the ball-wall collision is a constant, defined as the ratio of the magnitude of normal velocities of the ball after and before the collision. Find e such that the speed of the ball after the collision v' is minimised.

Assume the ball is always slipping against the wall. Ignore gravity and air resistance and assume all energy loss comes from the reaction forces (i.e. friction and normal forces).

Leave your answer to 2 significant figures.

Half Hour Rush E1: Karma (3 points)

Gerrard and Ziwen are both point masses standing in the same plane P. Gerrard tries to hit Ziwen with a small ball of mass $m = 0.50$ kg by shooting it towards him directly. Unbeknownst to him, the ball has charge $q = 3.0$ C and there is a uniform magnetic field $B = 0.20$ T directed perpendicular to P. Determine the time taken t for Gerrard to be hit by his own ball.

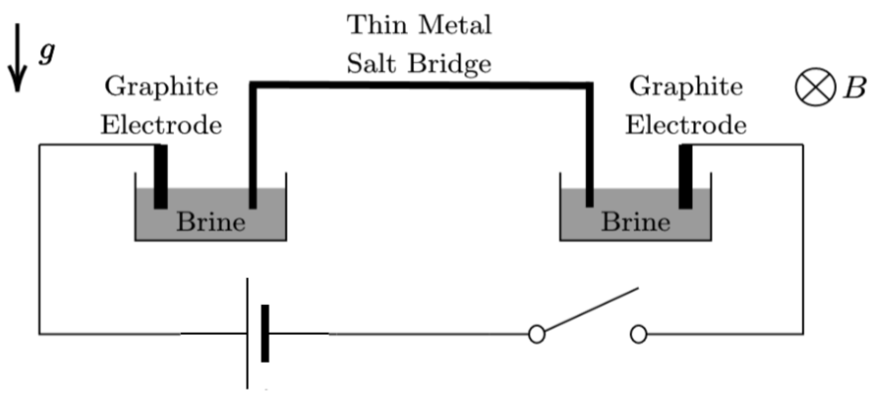
Leave your answer to 2 significant figures in units of s.

Half Hour Rush E2: You Need To Calm Down (4 points)

Claurine is about to manufacture copious amounts of chlorine gas at home via the chemical setup shown below. To make Claurine calm down, her sister Claudine exposes the horizontal section of the thin conducting salt bridge of length $l = 0.20$ m to a uniform magnetic field $B = 0.10$ T directed into the plane containing the salt bridge. The salt bridge of total mass $m = 0.030$ kg is initially at rest but free to move vertically.

Claurine then closes switch S and is shocked to observe the entire salt bridge jump up instantly and rise to a maximum height $h = 2.0$ m above its original position. Find the magnitude of the total charge q that flows in the salt bridge throughout the process.

Assume that the salt bridge loses contact with the brine completely before reaching its maximum height.



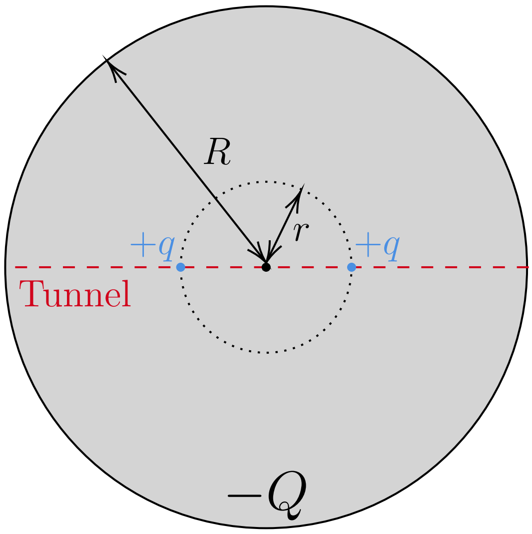
Leave your answer to 2 significant figures in units of C.

Half Hour Rush E3: We Are Never Ever Getting Back Together (4 points)

An infinitesimally narrow tunnel is drilled through an insulating solid sphere with radius R and uniform charge $-Q$. Romeo and Juliet are two point charges each with charge $+q$, both located inside this tunnel an initial distance $r = 2.00$ m away from the centre of the sphere. Upon release from rest, Romeo and Juliet are separated by a maximum distance d during their subsequent motion. Calculate d.

Take $\frac{Q}{q} = \frac{1}{2}$ and $\frac{R}{r} = 2$.

Leave your answer to 3 significant figures in units of m.



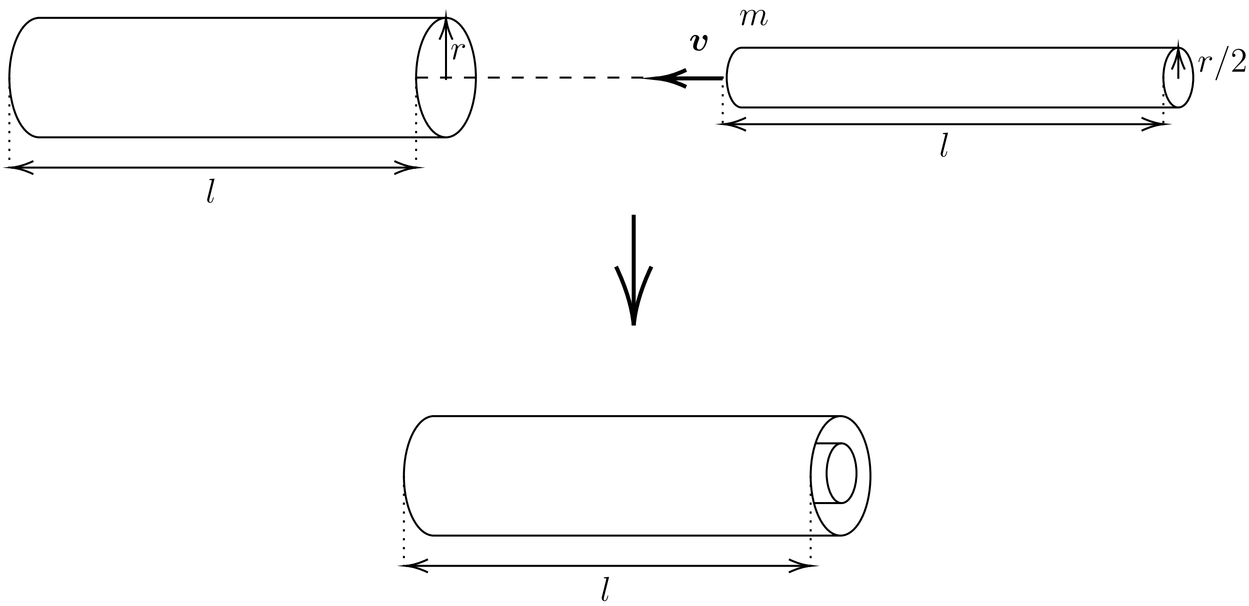
Half Hour Rush E4: Blank Space (5 points)

A fixed superconducting cylindrical shell A of radius $r = 0.05$ m and length $l = 100$ m initially has uniform current flowing in its azimuthal direction with total magnitude $I = 500$ A. A second superconducting cylindrical shell B of mass $m = 2.5 \times 10^{-6}$ kg, radius $\frac{r}{2}$ and length l initially has no current flowing through it. It is positioned infinitely far away from shell A, and is allowed to move.

Shell B is launched towards shell A with initial velocity v. The axes of symmetry of the two cylinders are aligned throughout the subsequent motion of shell B. Determine the velocity v such that shell B will fill the length of the blank space within shell A exactly, after a long time.

Hint: The magnetic flux through a superconductor is conserved.

Leave your answer to 2 significant figures in units of m s^{-1}.



Half Hour Rush X1: Stuck Ball (3 points)

Tom seals an empty bottle lying horizontally with a solid spherical ball. The opening of the bottle and the ball have the same cross-sectional area $A = 4 \times 10^{-4}$ m^2. Jerry then punctures a small hole in the bottle, pumps all the air out of the bottle and reseals it, creating a vacuum. Suppose that static friction between the ball and the bottle is at its maximum value, and is precisely sufficient to keep the ball in place. Determine the minimum force F required for Tom to remove the ball.

You may refer to the Data Sheet for the value of the atmospheric pressure p_0.

Leave your answer to 2 significant figures in units of N.

Half Hour Rush X2: My Ball Is Shockingly Bright (4 points)

Theo has a solid ball of radius $R = 0.690$ m made of the metal rhodium, with charge $Q_0 = +1.11$ nC on it. Theo then shines monochromatic ultraviolet radiation of wavelength $\lambda = 37.0$ nm on the ball. Rhodium has a work function of $\Phi = 4.98$ eV. After a very long time, the charge on the ball stays relatively constant. If the number of photoelectrons that have escaped over the entire duration is N, calculate $\log_{10} N$.

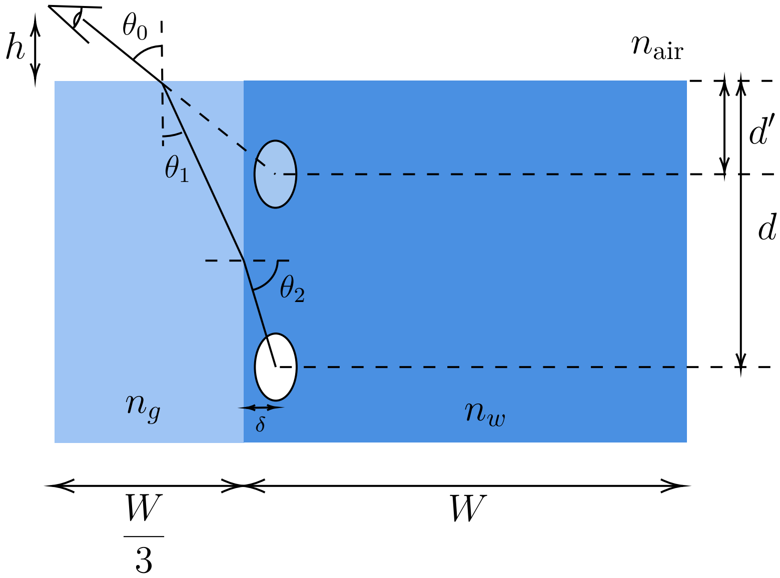
Leave your answer to 2 significant figures.

Half Hour Rush X3: Ball's Deep (4 points)

Jed, who has height $h = 1.33$ m, stands at the edge of a deep pool of width $W = 5$ m and glass walls of thickness $W/3$.

Jed accidentally drops his favourite ball into the pool at a small distance $\delta \ll W$ from the edge. Moving to the far edge of the glass wall to get a better look through the glass, he observes that the ball stops sinking at an apparent depth d'. Find d'.

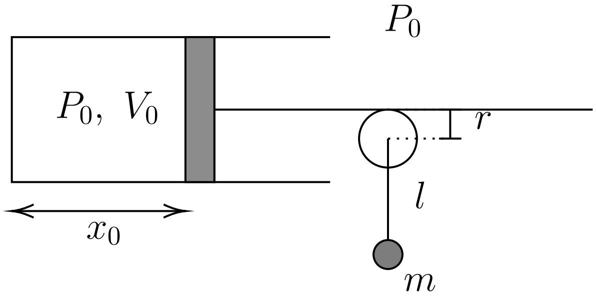
Refractive indices:
Air: $n_{\text{air}} = 1$
Water: $n_w = 1.33$
Glass: $n_g = 1.50$



Leave your answer to 2 significant figures in units of m.

Half Hour Rush X4: Pendulous Balls (5 points)

A rigid massless rod of length $l = 2$ m is fixed to the centre of a massless wheel of radius $r = 0.1$ m on one end, and a mass $m = 10$ kg on the other end, forming a pendulum. The wheel's rotation drives the shaft of a movable piston without slipping. Sealed in the container is an ideal gas initially at atmospheric pressure $P_0 = 1.01 \times 10^5$ Pa and initial volume $V_0 = 1$ m^3. Initially, this system is in equilibrium at length $x_0 = 1$ m.



The walls of the container are thermally conductive, causing the gas to stay at constant temperature. Yueyang taps the ball of the pendulum *lightly*. Find the angular frequency ω of the subsequent small oscillations.

Leave your answer to 2 significant figures in units of rad s^{-1}.