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Problem List

1	Kicking Pebbles	42	Love Triangle
A	Reflection	43	Strategically Accelerating Frames
B	Dress to Impress	44	Rock and Roll
2	ATLP120X	45	A Light Round of Ping Pong
3	Hexagon Force	46	Lily Pond
C	Lost in the Woods	47	67
4	Simp 1	48	Drag Me Down
D	Tilted Liquids	49	Would You Like it Hot or Cold?
E	Choo Choo	50	Ergodicity Breaking
5	Oceans Rise, Amperes Fall	51	Micromacromicrowave
6	Simulated Universe	52	Let Him Cook
7	Spintronics	53	Kansei Dorifto?!
8	Do A Barrel Roll	54	Mesmerizer
9	Audio Cue ⓘ	55	Simp 4
10	Linear or Quadratic?	M1	Faro Shuffle
11	Nuts and Volts	M2	Ribbon Spread
12	Let the Rain Pour	M3	House of Cards
13	Hanging Rope	M4	Soliton Solitaire
14	Cycle Madness B-Side	E1	Nyan Cat®
15	Through the Looking Glass	E2	It Goes in the Square Hole
16	Disappearing Water	E3	Megalovania
17	Shape of You	E4	I Am A Parallelogram
18	Aimbot	X1	Spacing Out
19	Tunnel Vision	X2	Hammer Time
20	Plate Moment ⓘ	X3	Particularly Strange
21	Simp 2	X4	Forged from Flames
22	Boing Boing		
23	Magnetic Shadows		
24	Shower Troubles		
25	Mirror Box		
26	Rubber Slingshot		
27	Sherlock Ohms		
28	Dripping Faucet		
29	Chill Out		
30	As It Was		
31	Unknown Particle in an Unknown Box ⓘ		
32	Squashing		
33	Anamorphic SPhL		
34	Flow State		
35	Resistor Grids		
36	Simp 3		
37	Hot Creep		
38	Spin-2?		
39	Hanging An L		
40	I of the Beholder		
41	Full of Hot Air		

Problem 1: Kicking Pebbles

(3 points)

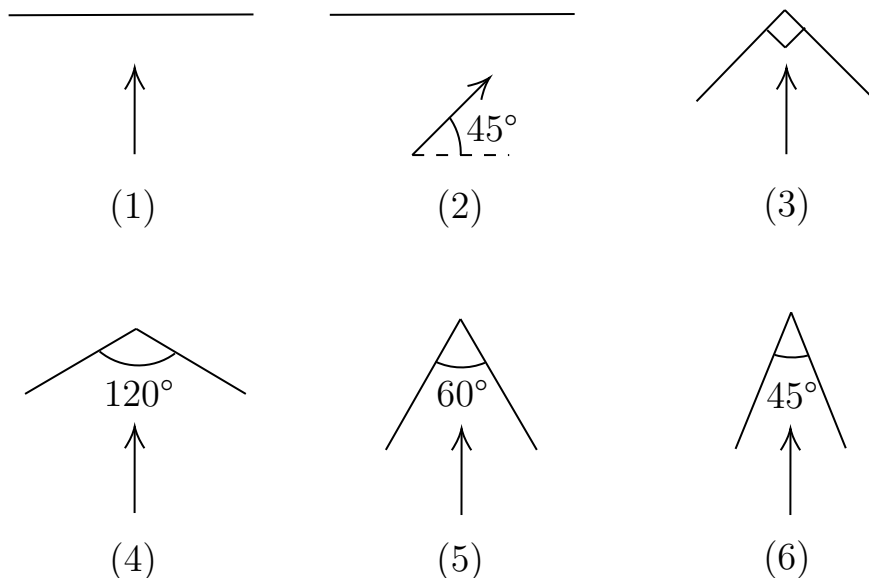
After a long day of work, Le Xi walks along a cobbled path at speed $u = 1.4 \text{ m s}^{-1}$. At any time, one of his feet is in contact with the ground, while the other moves at a constant forward velocity slightly off the ground. The moving foot collides with loose pebbles on the ground, kicking them forward at speed v relative to the ground. If the collisions between his shoe and the pebbles are perfectly elastic, determine the maximum value of v .

Leave your answer to 3 significant figures in units of m s^{-1} .

Problem A: Reflection

(3 points)

In the movie “Mulan”, she looks into a series of reflective surfaces and sings the line “why is my reflection someone I don’t know?”. This is likely because she got an eyepatch on her right eye during the war (not canonical), but her reflection has an eyepatch on its left eye. In the diagrams below, mirrors are represented by the black lines. In which of the diagrams will Mulan be looking directly at an image of herself with a right eyepatch?



Give your answer by concatenating the diagram numbers above, in ascending order. For example, if you think diagrams (1), (3), and (4) result in her looking directly at an image of herself with a right eyepatch, submit 134.

Leave your answer as an integer.

Problem B: Dress to Impress

(3 points)

Wonky is sauntering down a street, wearing his favourite hoodie. He takes steps at a frequency of $f = 130$ bpm (beats per minute), in time with Taylor Swift's hit "[Cardigan](#)". He notices that if the drawstring of his hoodie has a certain length l_0 , it will swing from side to side with a large amplitude while he walks.

Assume that the aglet of the drawstring is much more massive than the drawstring itself, causing it to behave like a simple pendulum. Determine the value of l_0 .

Leave your answer to 2 significant figures in units of cm.

Problem 2: ATLP120X

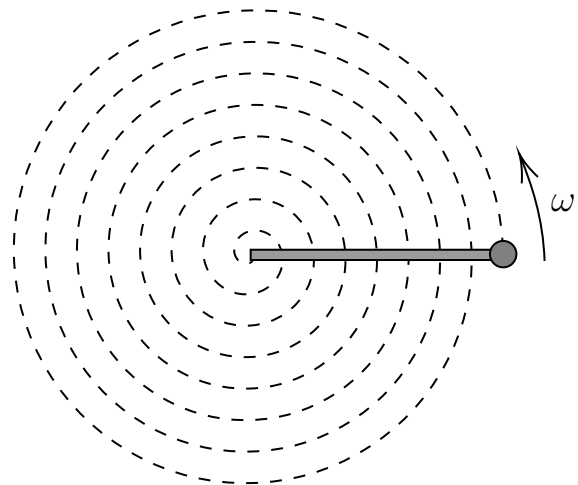
(3 points)

Vinyl records are analog recordings: music is converted into tiny grooves on the disc, which form a spiral inward. To play the record, a record player spins it at a constant angular speed of $\omega = 10 \text{ rev s}^{-1}$. A stylus is placed at the start of the spiral, and slowly moves inwards along the groove. On this particular record, the average number of grooves per radial centimeter is $n = 50.0 \text{ cm}^{-1}$.

The outermost groove is $r_s = 14.0 \text{ cm}$ from the centre of the record, and the innermost groove is $r_l = 6.00 \text{ cm}$ from the centre of the record. Determine the total playtime t of the record.



(The author likes his hobbies.)

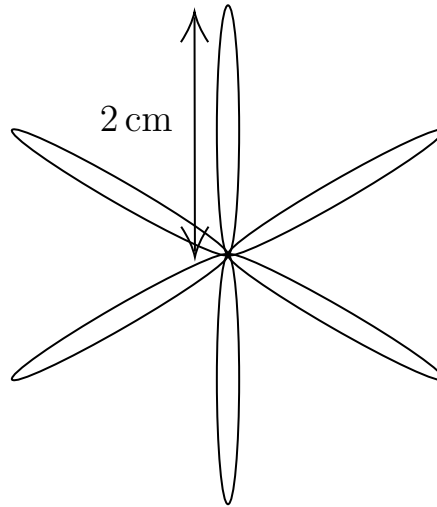


Leave your answer to 2 significant figures in units of s.

Problem 3: Hexagon Force

(3 points)

Six identical lenses with diameter $D = 2.0$ cm are arranged in a hexagon, with their edges touching each other:



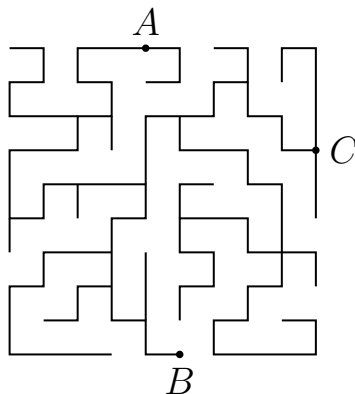
Determine the maximum focal length f of each lens such that there exists a light beam that travels in a closed loop around the hexagon.

Leave your answer to 3 significant figures in units of cm.

Problem C: Lost in the Woods

(3 points)

Tired of having to make solder in GregTech, Wenkai decides to use a soldering iron and a resistive wire with uniform resistance per unit length to make the following maze:



He then connects a 1.0 V voltage source across points A and B of the maze, with point B grounded. What is the voltage of point C with respect to the ground?

Leave your answer to 2 significant figures in units of V.

Problem 4: Simp 1

Amy has a crush on Jake. She has been chasing him for the past 5 years, unsuccessfully. Frustrated by her persistence, Jake gets the help of a highly intelligent dog to finally escape her. The characters play a game of catch on a 2-dimensional plane, with the following rules:

- A “catch” occurs when two characters occupy the same position on the plane.
- If Amy catches Jake before his dog catches her, she wins.
- If Jake’s dog catches Amy before she catches Jake, Jake and his dog win.

The top speeds of Amy, Jake and his dog are v_A , v_J and v_D respectively, and they can change their velocities instantaneously. Jake and his dog start the game close to each other and Amy starts far away from both of them. Jake and his dog must remain a finite, non-zero distance from each other (i.e. Jake cannot camp on his dog). Assume all characters play optimally.

We define v_{crit} such that if $v_A > v_{\text{crit}}$, then Amy can always win.

- (a) If $v_J = 0 \text{ m s}^{-1}$ and $v_D = 1.0 \text{ m s}^{-1}$, what is the value of v_{crit} ?

Leave your answer to 2 significant figures in units of m s^{-1} . (2 points)

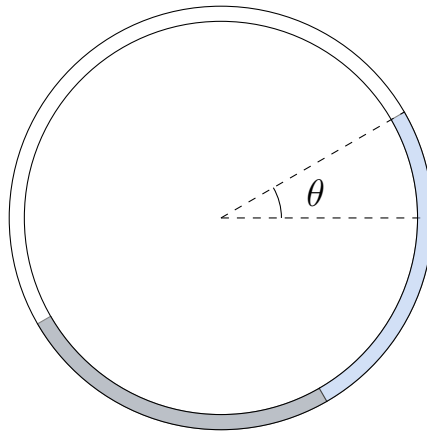
- (b) If $v_J = 0.8 \text{ m s}^{-1}$ and $v_D = 0.5 \text{ m s}^{-1}$, what is the value of v_{crit} ?

Leave your answer to 2 significant figures in units of m s^{-1} . (2 points)

Problem D: Tilted Liquids

(3 points)

A vertical ring of volume V and of negligible thickness contains 2 immiscible liquids of density ρ_1 and ρ_2 , with $\rho_2 = 3\rho_1$. The volume of each liquid in the ring is $0.25V$. The less dense liquid is floating on top of the denser liquid. At equilibrium, the liquids have the following arrangement:



Determine the value of θ .

Leave your answer to 3 significant figures in units of degrees.

Problem E: Choo Choo

(3 points)

Thomas is a uniform train of length $l = 50$ m. He wishes to complete the frictionless track below, starting at $s = 0$ m with initial velocity v along the track. What is the minimum initial velocity $v_{\min}$ required for Thomas to reach the end of the track at $s = 600$ m? Neglect air resistance.



Leave your answer to 3 significant figures in units of m s^{-1} .

Problem 5: Oceans Rise, Amperes Fall

(3 points)

A wire with resistance $R = 1.50\,\Omega$ and length $l = 2.00\,\text{m}$ is connected vertically between two plates in Earth's uniform gravitational field g . The top plate is held at $V_1 = 0\,\text{V}$ and the bottom plate at $V_1 = 10\,\text{V}$. An ammeter is attached to the wire and measures current I_1 . The entire setup is then turned upside down and, strangely, a slightly different current I_2 is measured. Determine $|I_1 - I_2|$.

Leave your answer to 2 significant figures, in units of nA.

Problem 6: Simulated Universe

(4 points)

Oon Han finds himself as the first inhabitant of a simulated universe, standing on an Earth with an identical gravitational field as ours. In this universe, time progresses in steps of duration $\Delta t = 6.40 \times 10^{-15}$ s and kinematics is described by the recurrence relations

$$\begin{aligned}\mathbf{r}(t + \Delta t) &= \mathbf{r}(t) + \mathbf{v}(t)\Delta t \\ \mathbf{v}(t + \Delta t) &= \mathbf{v}(t) + \mathbf{a}(t)\Delta t\end{aligned}$$

where $\mathbf{r}$, $\mathbf{v}$ and $\mathbf{a}$ denote position, velocity and acceleration respectively. Unfortunately, this results in total energy not being conserved, so his universe will soon meet its impending doom.

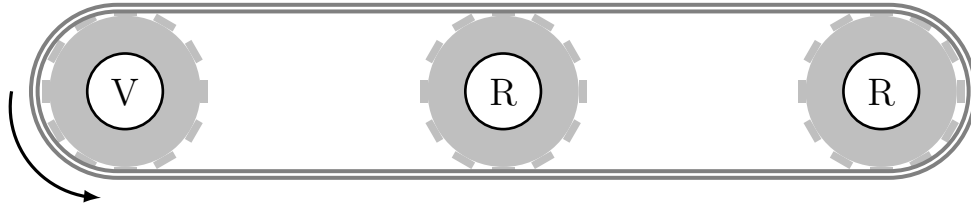
Oon Han lets an orange of mass $m = 0.140$ kg fall from rest and catches it $t_0 = 0.600$ s later. The total mechanical energy of an object is defined as the sum of its kinetic and potential energies. The change in total mechanical energy of the orange from the time it was released to right before it was caught is ΔE J. Determine $\ln |\Delta E|$.

Leave your answer to 3 significant figures.

Problem 7: Spintronics

In *Spintronics*, electrical circuit components are replaced by mechanical gears and chains. You may assume that all gears have radius $r = 5.0$ cm and negligible mass, and that chains do not slip.

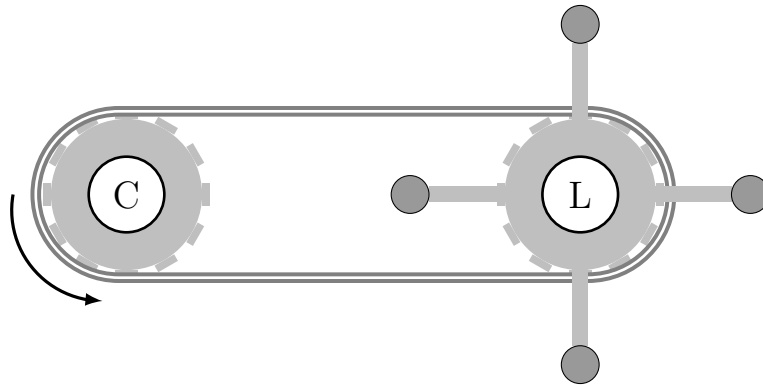
- (a) Let us consider a circuit, as shown below, with 3 gears and a chain. Gear V is a motorised gear, which exerts a constant force of $F_V = 10$ N on the chain. Each free gear R exerts a resistive force $F_R = -bv$ on the chain, where $b = 2.0 \text{ kg s}^{-1}$ is a constant and v is the linear velocity of the chain.



In the above setup, what is the final linear velocity of the chain?

Leave your answer to 2 significant figures in units of m s^{-1} . (2 points)

- (b) In the next setup, shown below, gear C is attached to a torsional spring, which obeys Hooke's law. Thus, it exerts a resistive torque $\tau_C = -k\theta r$ when rotated, where $k = 0.60 \text{ kg m s}^{-2}$ is a constant and θ is the angular displacement from its equilibrium position, in radians. Gear L has four massless rods of length $l = 5.0$ cm connected normal to the gear, with a point mass $m = 0.20$ kg at the end of each rod.



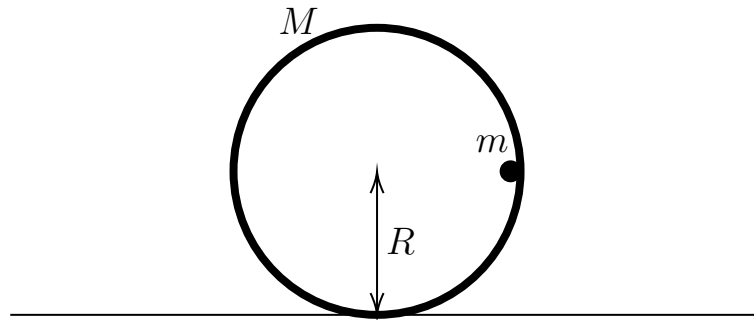
Gear C is rotated to some $\theta = \theta_0$ and released. What is the time t for gear C to first return to its equilibrium position?

Leave your answer to 2 significant figures in units of s. (3 points)

Problem 8: Do A Barrel Roll

(3 points)

A hollow frictionless cylinder of mass $M = 5.0$ kg and radius $R = 1.0$ m is placed on horizontal ground. A tiny sphere of mass $m = 1.0$ kg is placed halfway up the inner side of the cylinder, and the whole setup is released from rest. Determine the amplitude A of the sideways oscillations of the cylinder's centre.

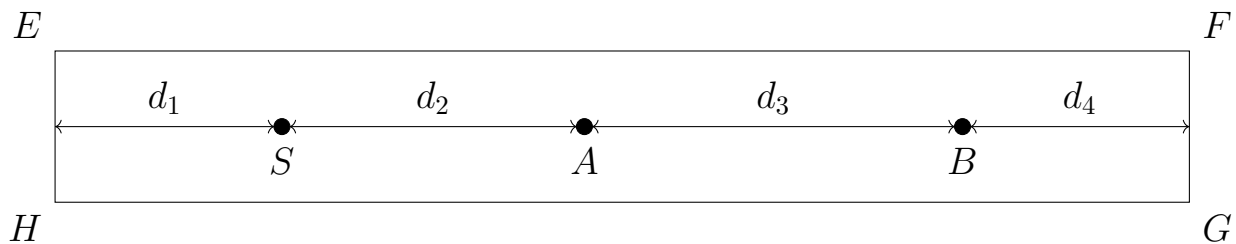


Leave your answer to 2 significant figures in units of m.

Problem 9: Audio Cue

(5 points)

Physicist S is playing a game with friends A and B using omnidirectional speakers (i.e., speakers that emit sound uniformly in all directions). Physicist S also has a microphone. All three stand along a straight line in a room of length L and negligible width. Physicist S, friend A, and friend B stand at points S , A , and B respectively. The distances d_1 to d_4 are defined as shown in the diagram below:



Their positions satisfy $d_1 < \min(d_2, d_3, d_4)$. Walls EF and GH are lined with acoustic foam, which absorbs all incident sound. Walls FG and EH reflect all incident soundwaves perfectly.

The game proceeds as follows:

1. Physicist S starts the microphone. After a delay of $t_{\text{start}} = 1.00$ s, he plays a single note of frequency $f_S = 500$ Hz from his speaker.
2. Upon first hearing S's note, Friend A immediately plays a note of frequency $f_A = 1200$ Hz.
3. Upon first hearing S's note, Friend B immediately plays a note of frequency $f_B = 1800$ Hz.
4. Physicist S stops the recording shortly after his microphone has captured each of the three notes at least three times (including the note at $t = t_{\text{start}}$).

Volume warning: The audio may be loud.

The audio file Physicist S recorded can be found [here](#). You may find it helpful to use an online tone generator to familiarise yourself with the three frequencies.

Determine the value of $d_1 + d_2 + 2d_3 + 2d_4$.

Leave your answer to 2 significant figures in units of km.

Problem 10: Linear or Quadratic?

(4 points)

E-Jay, a basketball enthusiast, is interested in determining the nature of drag experienced by basketballs so he can improve the accuracy of his shots. He hypothesises that the drag force acting on a basketball is either always linear or always quadratic, i.e:

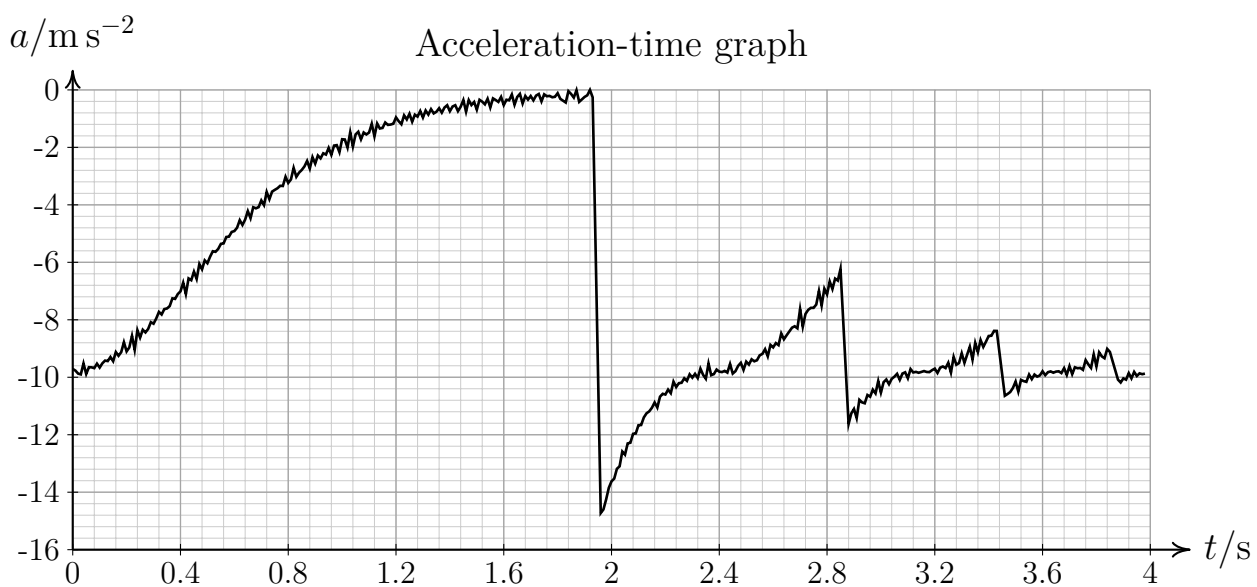
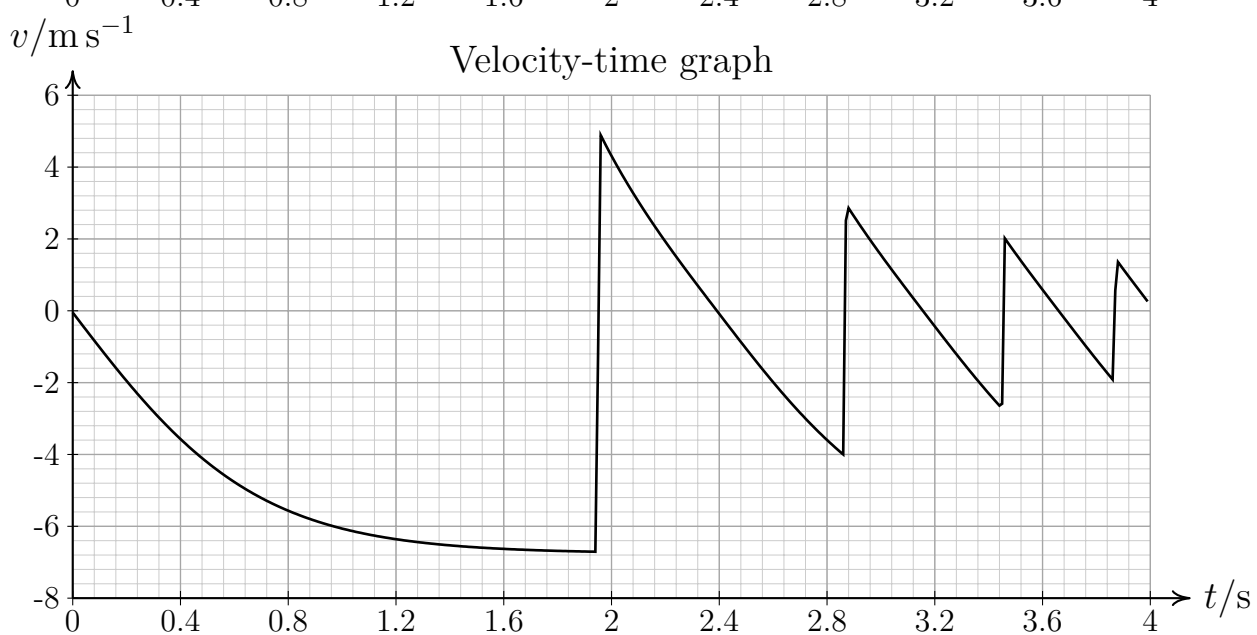
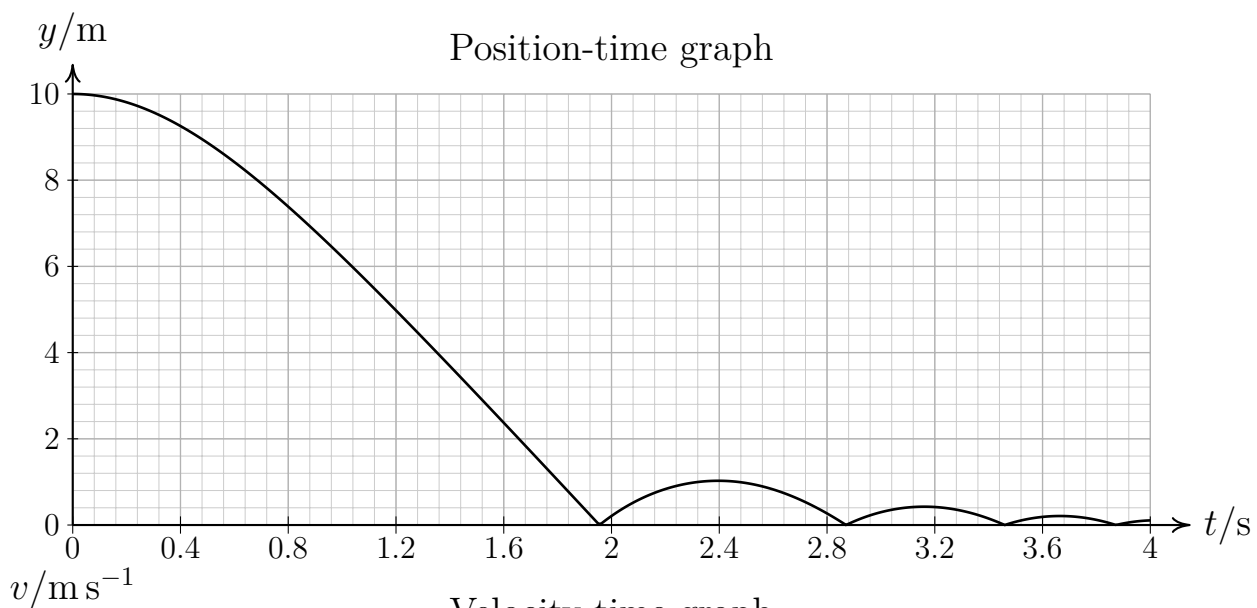
$$\underset{\text{(linear)}}{F_{\text{D}} = bv} \quad \text{or} \quad \underset{\text{(quadratic)}}{F_{\text{D}} = bv^2}$$

where F_{D} is the magnitude of the drag force, v is the speed and b is a constant.

To determine which form it is, E-Jay conducts an experiment. He drops a basketball of mass $m = 0.600$ kg from rest at some height above the ground and tracks its motion. He obtains the position, velocity and acceleration data for the basketball and plots graphs of their values against time, which are shown on the following page. By considering whether the drag force is linear or quadratic, determine b .

Leave your answer to 2 significant figures in units of either kg s^{-1} or kg m^{-1} .

If you think there is not enough information to determine the form of drag, input -1 .

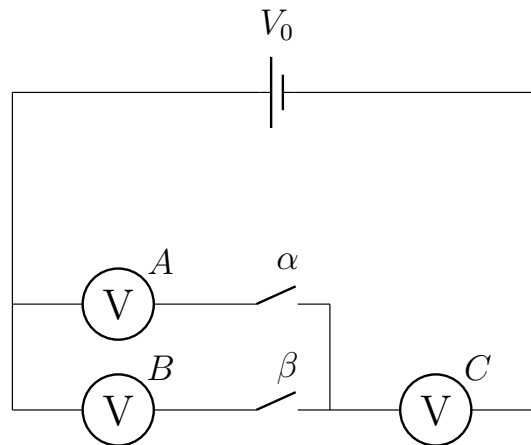


Problem 11: Nuts and Volts

(3 points)

Scrat has three voltmeters, two switches and a battery of voltage $V_0 = 2.00$ V. He labels the three voltmeters A, B, C and the two switches α, β . Unfortunately, in a scramble to retrieve his acorn, he accidentally breaks the displays of voltmeters A and B , so he can only see the voltage displayed by C .

Scrat connects the components in a circuit as shown below, where both switches are initially open. He closes β and reads that the voltage measured by C is $V_1 = 1.28$ V. Keeping β closed, he closes α and reads that the voltage measured by C is $V_2 = 1.40$ V.



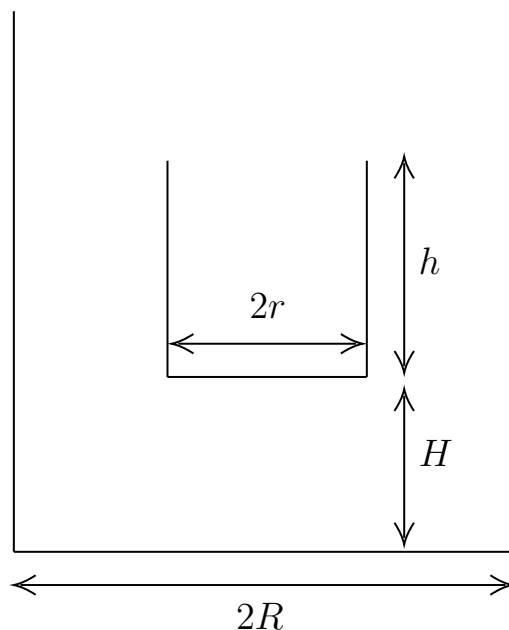
Just as Scrat opens β and tries to catch a glimpse of the new voltage V_3 displayed by C , the ice cracks below his feet and he falls into the abyss beneath the surface. Determine V_3 .

Leave your answer to 3 significant figures in units of V.

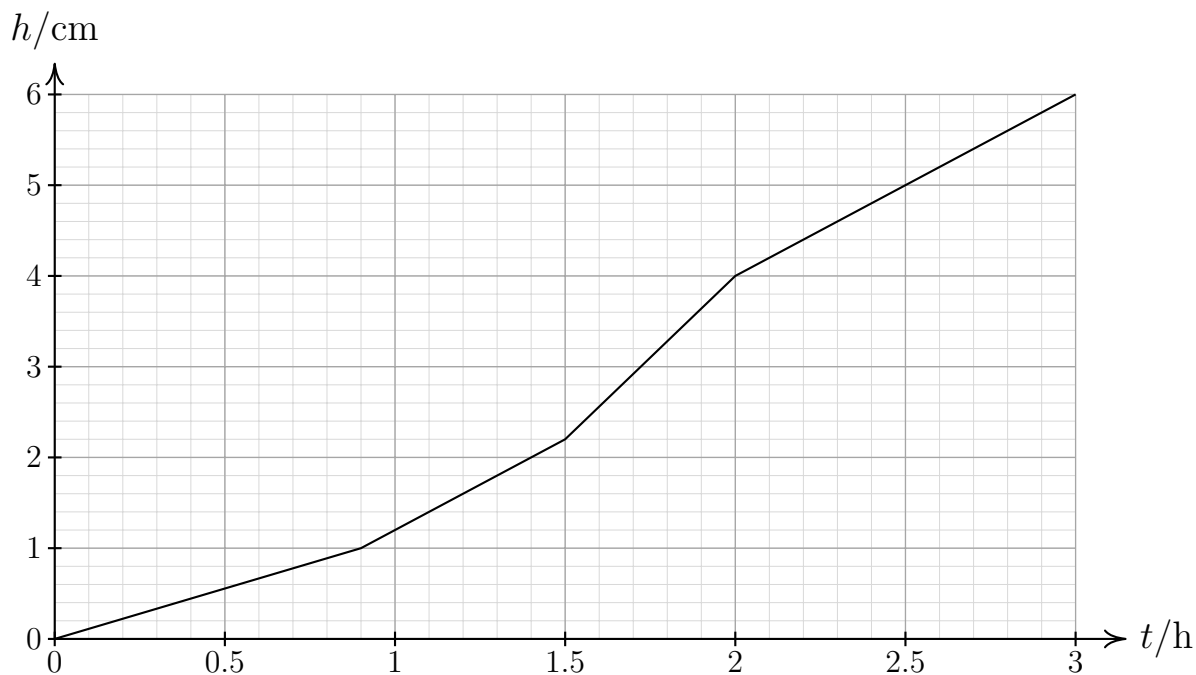
Hint: The voltmeters are non-ideal.

Problem 12: Let the Rain Pour

In a spark of engineering mania, Sid decides to build a (somewhat useless) device to collect rainwater. The device consists of two cylinders with closed bottoms, where the smaller cylinder is suspended inside the larger cylinder. The following diagram shows a cross-section of the device.



During a particularly long rainstorm, the water level of the outer cylinder is measured as a function of time to obtain the following graph. Assume that the rate of rainfall is constant.



- (a) Determine the rate of rainfall v .

Leave your answer to 2 significant figures in units of cm h^{-1} .

(2 points)

It turns out that from the data alone, we cannot uniquely determine the values of H and h – there are 2 possible sets of values which can produce this graph.

- (b) Determine the average of the 2 possible values of H .

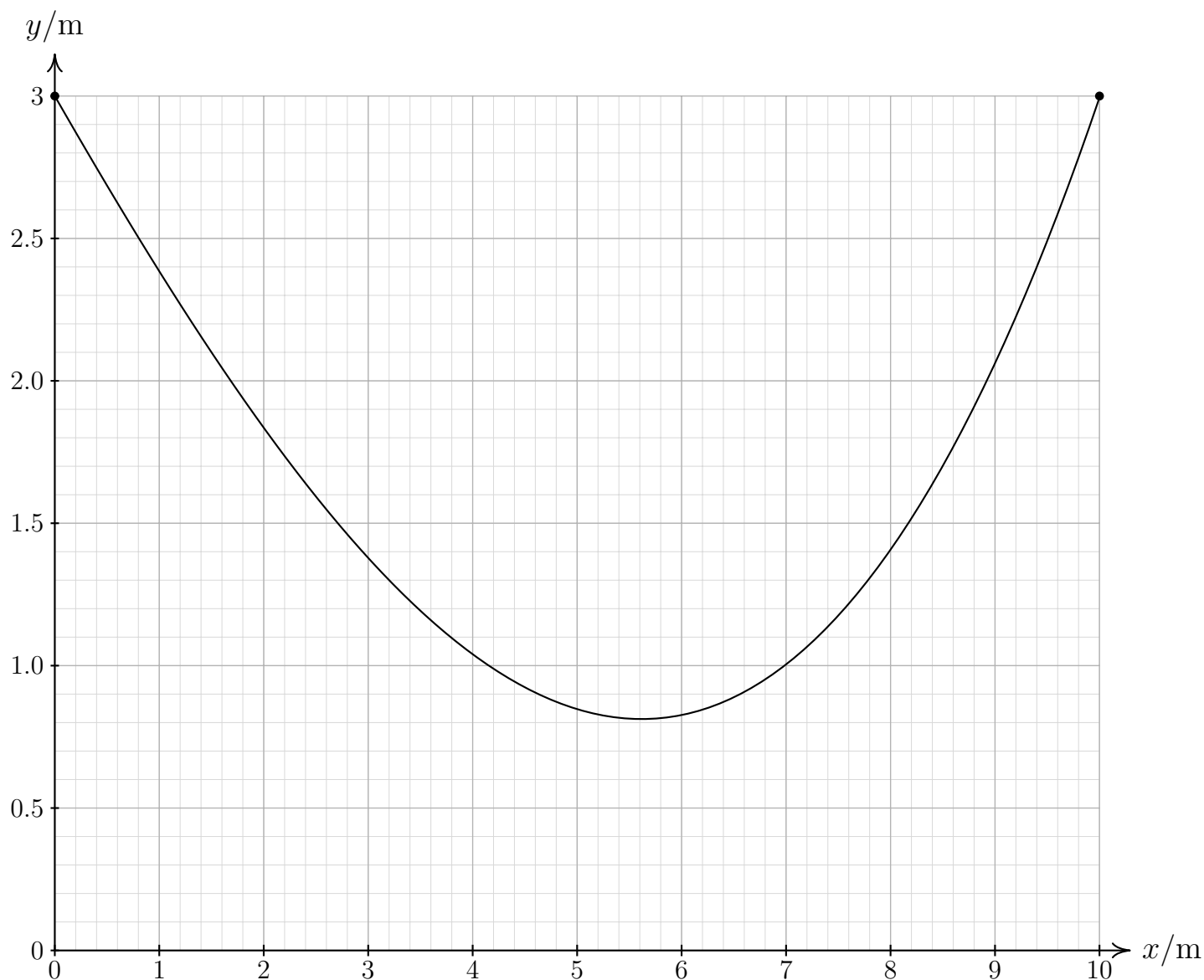
Leave your answer to 2 significant figures in units of cm.

(2 points)

Problem 13: Hanging Rope

(3 points)

An inextensible, non-uniform rope is hung between two supports at $x = 0$ and $x = 10$ m. Its linear density, measured from one end of the rope, is given by $\lambda(l) = al^b$, where a and b are unknown constants. A plot of the rope profile at equilibrium is shown below.



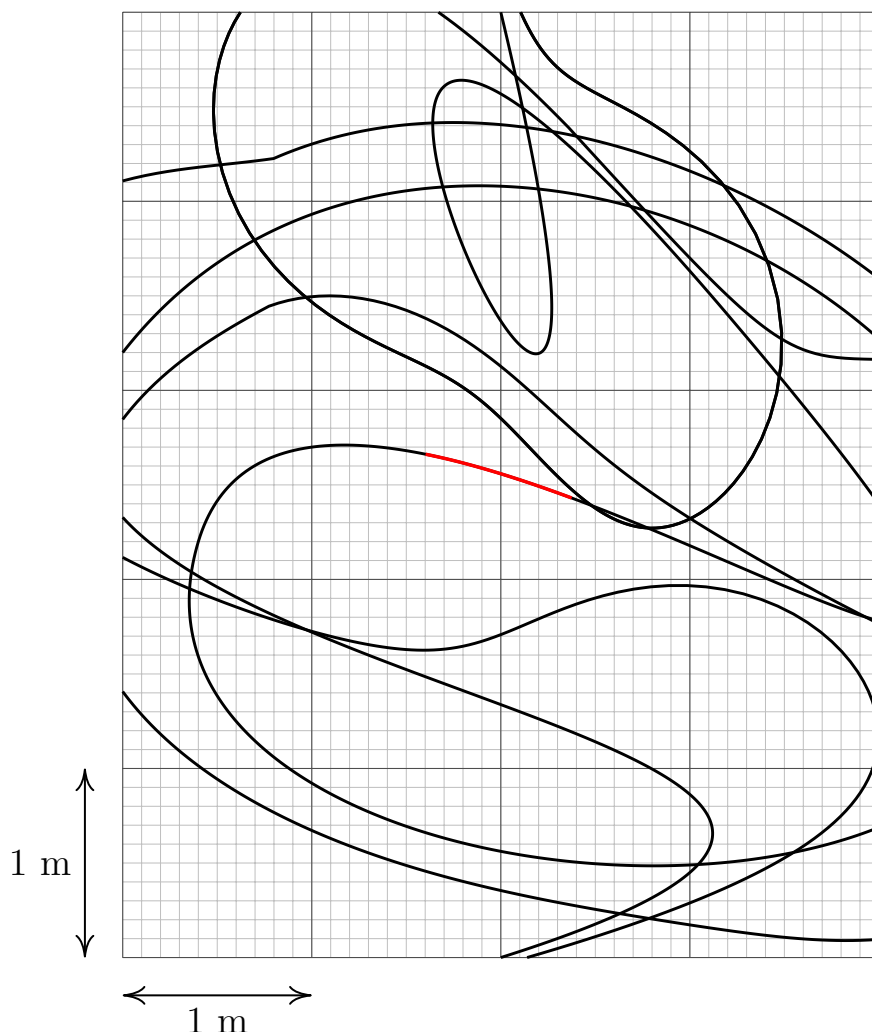
Find the x -coordinate of the center of mass of the rope.

Leave your answer to 2 significant figures in units of m.

Problem 14: Cycle Madness B-Side

(5 points)

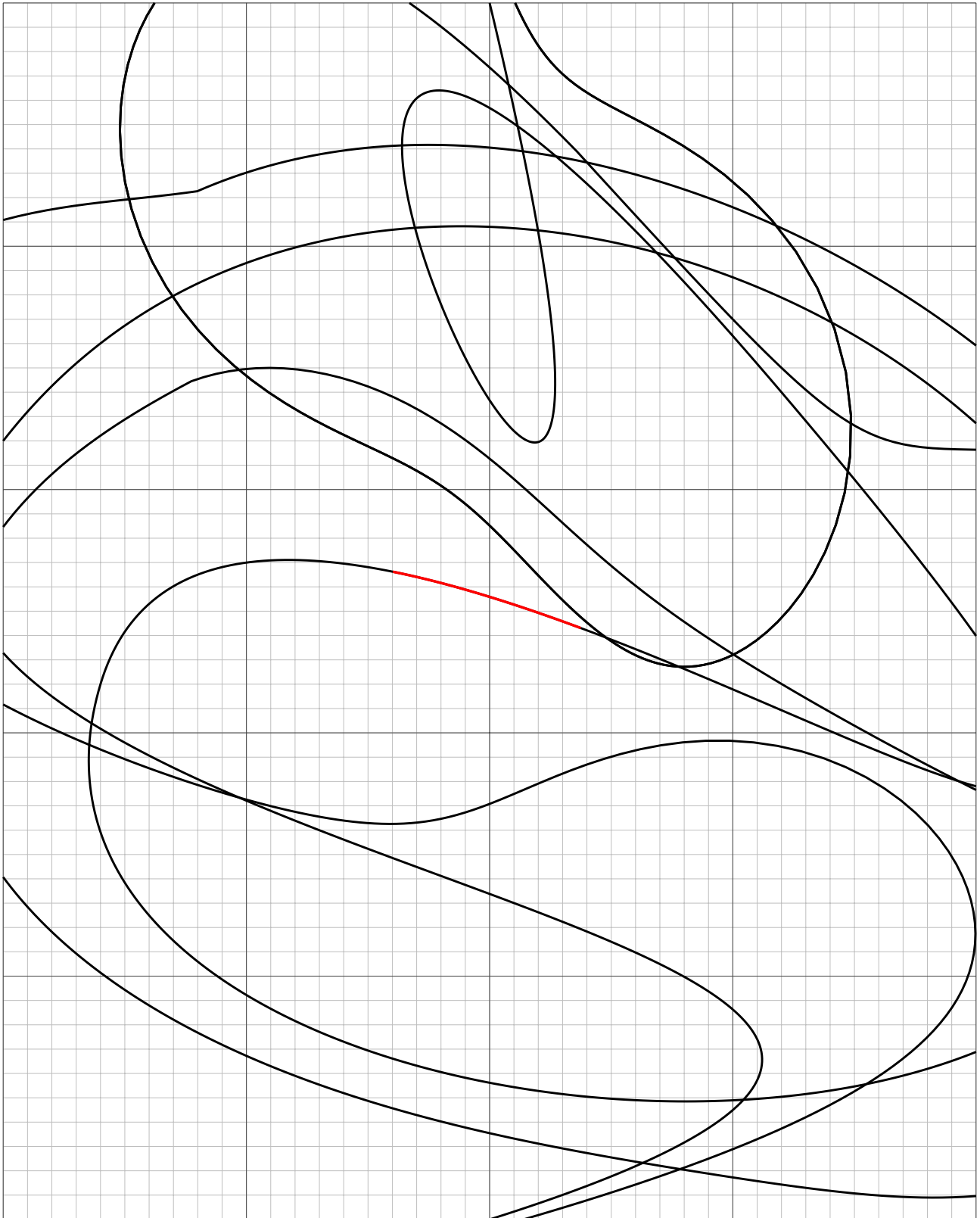
Madeline is cycling when she accidentally cycles into a puddle of water. As a result, both her wheels are wet and hence leave some tracks on the ground. The figure below shows part of the tracks that were formed (you may refer to the next page for a larger version of the same figure). Each major grid square represents a $1\text{ m} \times 1\text{ m}$ square.



Refer to next page for larger version.

The distance between the centres of the two wheels of Madeline's bicycle is l . Assume that Madeline's wheels do not slip. Let $\alpha = +1$ if the wheel that traced out the red segment is moving left to right, and $\alpha = -1$ if it is moving right to left. Given that $l < 2\text{ m}$ and that Madeline is always cycling forwards, determine $\alpha \times l$.

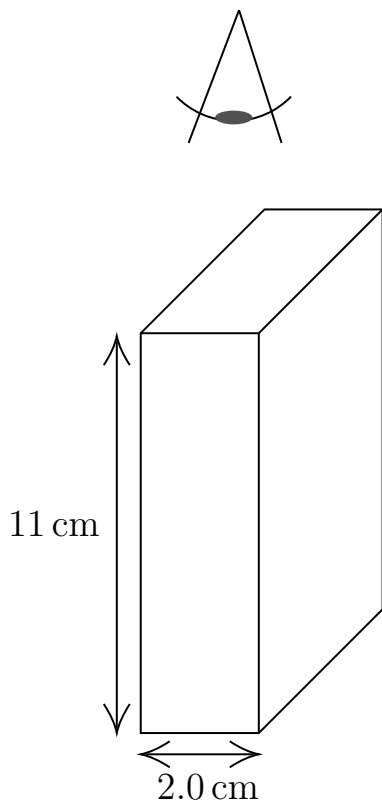
Leave your answer to 2 significant figures in units of m.



Problem 15: Through the Looking Glass

(3 points)

Ryab was polishing glass panels (as is his pastime), when he noticed something interesting. Looking into a panel from the top, its base appears to be laterally duplicated several times. A photo of what he saw is shown below.



The glass panels have height $h = 11$ cm and width $w = 2.0$ cm. Estimate the refractive index of the glass, n .

Leave your answer to 2 significant figures.

Problem 16: Disappearing Water

(3 points)

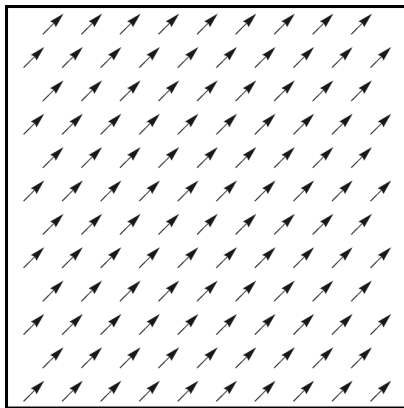
To fulfil his combat medic obligations, Le Xi needs to learn how to administer IV therapy. He heats water to exactly 100 °C and fills a syringe with 30 mL of it, taking care to avoid trapping any air. He then seals the nozzle and pulls the plunger from the 30 mL to the 31 mL mark. After a long time, Le Xi notices that a very small amount of the water has gone missing!

Assume the syringe is perfectly thermally insulating. The density of water at 100 °C is 0.958 g/mL. Determine $|\Delta V_w|$, the change in the volume of liquid water.

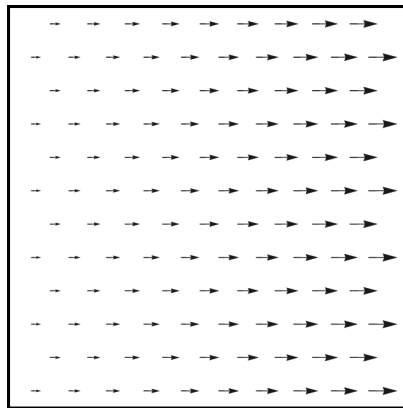
Leave your answer to 2 significant figures in units of μL .

Problem 17: Shape of You

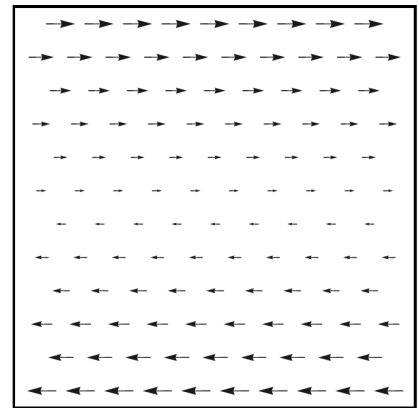
A series of vector fields are shown below. Diagrams (1-3) are vector plots while diagrams (4-6) are streamline plots.



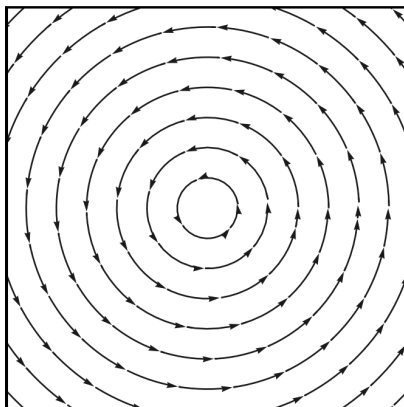
(1)



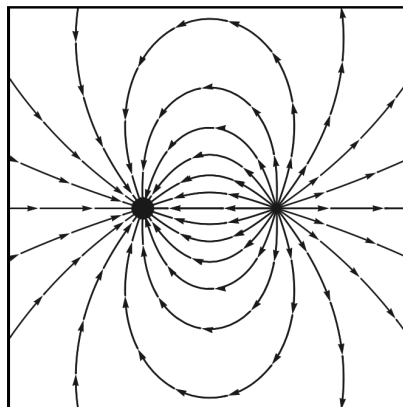
(2)



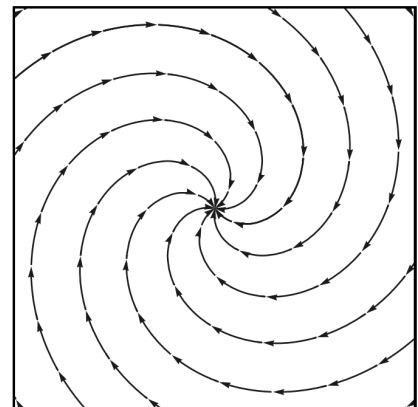
(3)



(4)



(5)



(6)

For each of the following parts, give your answer as a concatenation of the numbers of the diagrams above, in ascending order. For example, if you think the answer is diagrams (1), (2), and (3), then submit 123.

- (a) Which of these represents a possible part of the electric field that can be produced by a static configuration of electric charges?

Leave your answer as an integer.

(2 points)

- (b) Which of these represents a possible part of the velocity field in a sealed 2D tank of water?

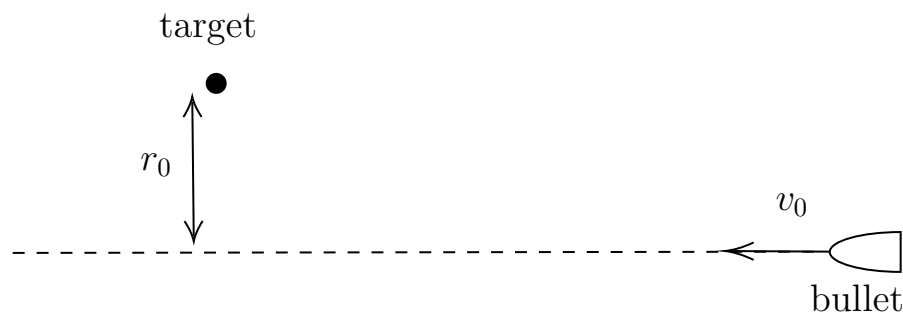
Leave your answer as an integer.

(2 points)

Problem 18: Aimbot

In the game *Valorant*, a first-person shooter, player A's character, the "target", keeps getting hit by bullets fired by the opponent. Fed up, he decides to investigate the situation and observes two scenarios where the bullet strayed from a straight line to hit the target. He hypothesizes that this is because of electrostatic forces. The target, teammate, and bullet can be treated as points, and both cases occur in a vacuum with no gravity.

- (a) He figures that the reason why the bullet keeps hitting the target is that they attract each other. Assume that the fixed target is charged with $q = 0.50 \text{ mC}$ and the bullet is charged with $-q$. The bullet of mass $m = 40 \text{ g}$ is fired from a gun far away from the target, at an initial horizontal velocity v_0 , with $r_0 = 1.6 \text{ m}$.

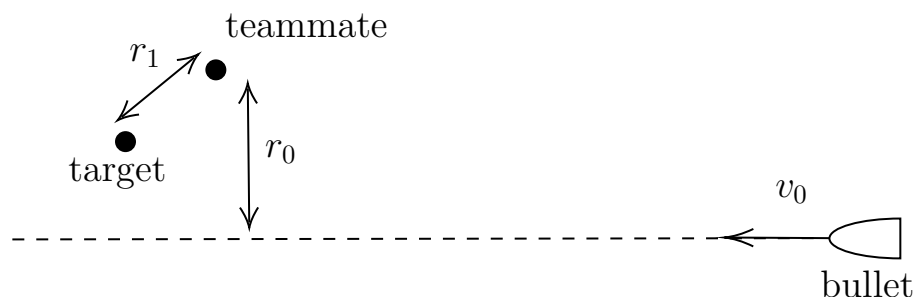


Determine the value of v_0 that allows the bullet to hit the target.

Leave your answer to 3 significant figures in units of m s^{-1} . (2 points)

Input -1 if no value of v_0 exists.

- (b) Next, player A decides to blame his stationary teammate for being charged with $+q$ instead, while the fixed target is a neutral insulator. The bullet of mass $m = 40 \text{ g}$ is fired from a gun far away from the target, at an initial horizontal velocity $v_0 = 500 \text{ m s}^{-1}$, with $r_0 = 1.6 \text{ m}$.



Let r_1 be the distance between the target and the stationary teammate. Given that the bullet hits the target at its maximum velocity, determine the value of r_1 .

Leave your answer to 3 significant figures in units of m . (3 points)

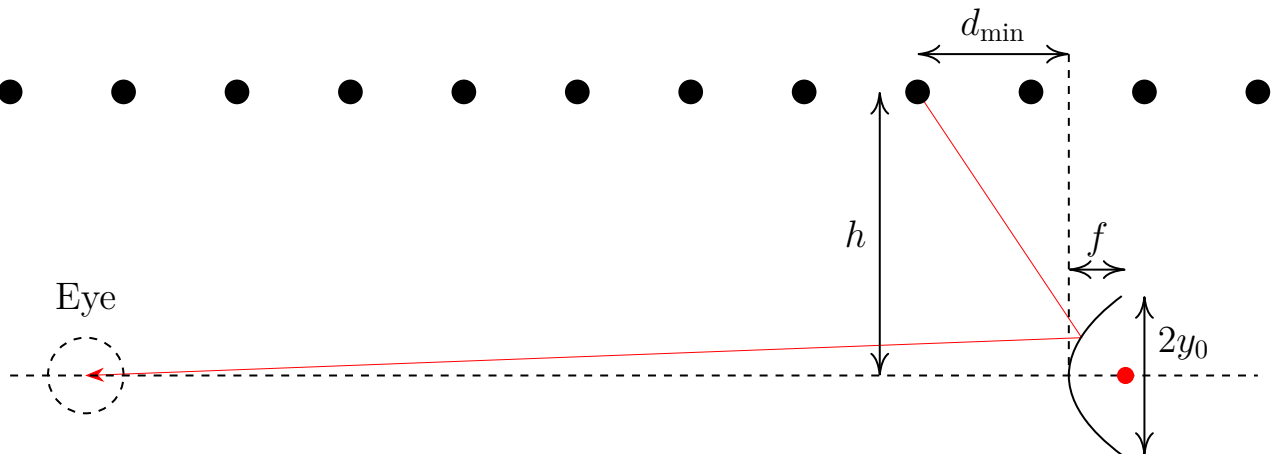
Input -1 if no value of v_0 exists.

Problem 19: Tunnel Vision

(4 points)

Jamie is driving far behind Gindon on the KPE and notices reflections of the many overhead lights on his rear window.

We can model the rear window as a parabolic mirror whose axis of symmetry is horizontal, as shown in the figure below. Its focal length is $f = 2.5$ m and its height is $y_0 = 0.30$ m. Jamie's eye lies on the axis of symmetry. The tunnel lights are directly above the cars at a height $h = 5.0$ m above the midpoint of the window. Assume each tunnel light is a point source and that they are spaced very closely together.



Find the value of $d_{\min}$, the horizontal distance between the midpoint of the rear window and the nearest tunnel light behind Gindon whose reflection can be seen by Jamie. Assume the window is small enough that only lights located behind it are reflected.

Hint: A vertical parabola of focal length f has equation

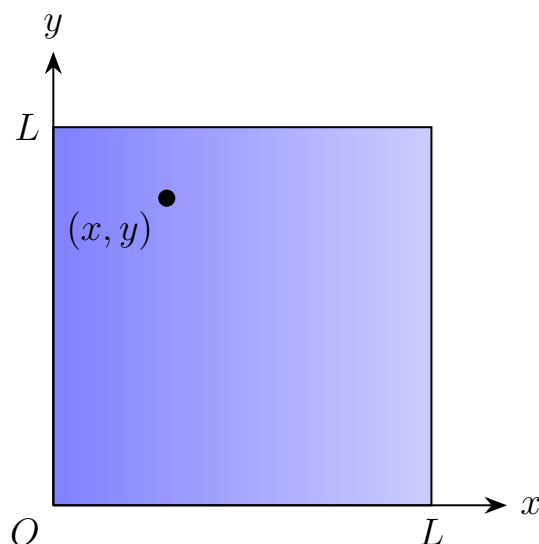
$$y = \frac{x^2}{4f}$$

Leave your answer to 2 significant figures in units of m.

Problem 20: Plate Moment

(5 points)

Keith has a thin **non-uniform** square plate with side length $L = 1.000$ m. By using a special machine, Keith is able to measure the moment of inertia of the plate along any axis passing through the plate at the coordinates (x, y) and perpendicular to the plate.

**Format**

You are to input the coordinates of your query x and y to **3 decimal places** in units of m with $0 \leq x, y \leq L$. The simulation will output I , in units of kg m^2 to **5 significant figures**.

Hypothetical Example		
x (m)	y (m)	Output: I (kg m^2)
0.3000	0.8123	1.6969

Find the mass of the plate, m .

Leave your answer to 2 significant figures in units of kg.

You are given 3 simulation queries per problem attempt.

Problem 21: Simp 2

(4 points)

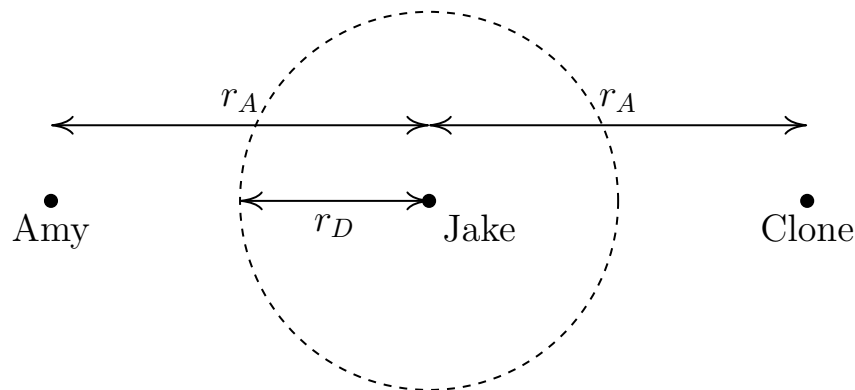
Amy, Jake, and his dog decide to play the same game of catch on a 2-dimensional plane. However, Amy has brought a clone of herself to make things a little harder for Jake and his dog (and for you!). They play with the same rules as before:

- A “catch” occurs when two characters occupy the same position on the plane.
- If Amy or her clone catches Jake before his dog catches her, she wins.
- If Jake’s dog catches both Amy and her clone before either catches Jake, Jake and his dog win.

Having barely escaped Amy in the previous game, Jake is now exhausted and cannot move. He also does not have the energy to entertain his hyperactive hyper-intelligent dog. As such, they add the following rules:

- Jake must remain stationary.
- Jake’s dog can only catch either Amy at a distance greater than r_D from Jake.

Jake and his dog begin at the origin, while Amy and her clone begin on opposite sides of Jake and his dog, at a distance r_A . The Amys have a top speed v_A , and the dog has a top speed v_D . All of them can change their velocities instantaneously.



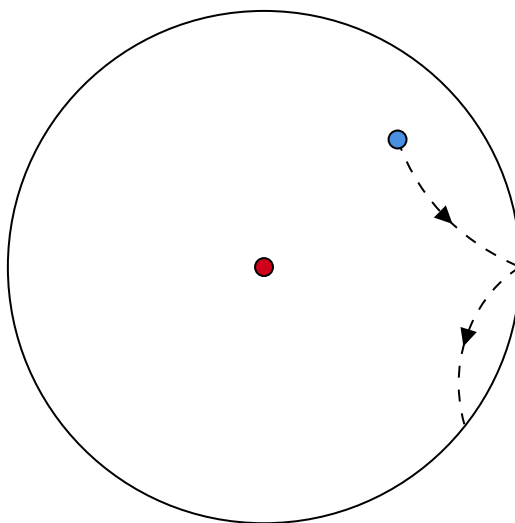
They start the game with $r_A/r_D = 2$. Assuming all characters play optimally, Amy wins if $v_A/v_D > \chi$. Determine χ .

Leave your answer to 3 significant figures.

Problem 22: Boing Boing

(3 points)

A proton is placed at an unknown position within a circular container. An fixed positive point charge of unknown magnitude is located at the centre of the circle.



At $t = 0$, the proton is projected with an unknown velocity $\mathbf{v}_0$. It subsequently moves within the container, following the dashed arrow shown in the figure and colliding inelastically with the walls, where each collision has coefficient of restitution e . Aaron, standing outside the container, is able to hear each impact and records the times of the first 15 collisions as shown in the table below:

No.	1	2	3	4	5	6	7	8
$t/\mu\text{s}$	2.0450	7.3476	12.679	17.724	22.254	26.162	29.433	32.117
No.	9	10	11	12	13	14	15	16
$t/\mu\text{s}$	34.289	36.031	37.421	38.525	39.401	40.094	40.642	41.075

From the data, estimate e . You may neglect gravity.

Leave your answer to 2 significant figures.

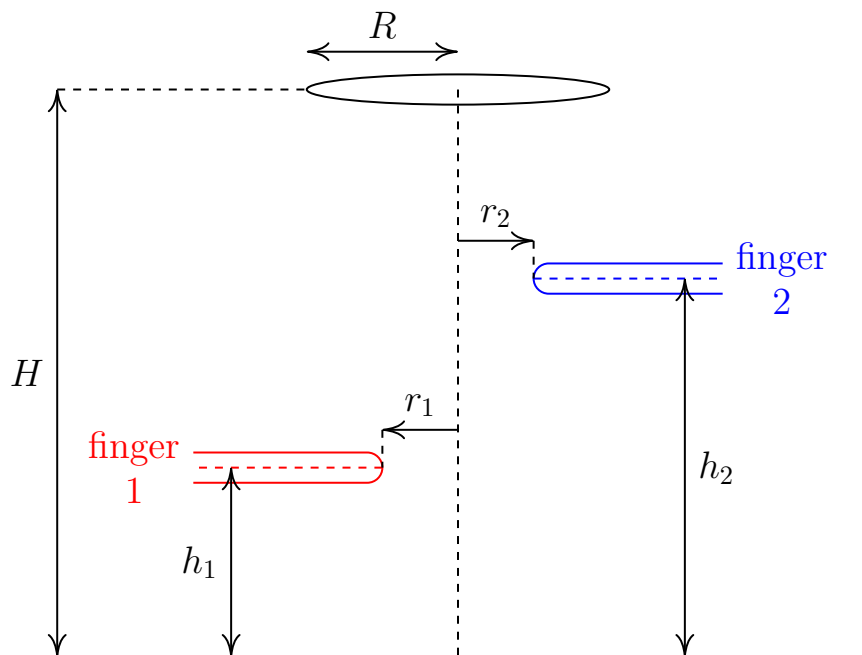
Problem 23: Magnetic Shadows

(4 points)

When two objects form shadows due to a non-point light source, the shadows may seem to “attract” each other as they get closer. Note that when we use the term “attract”, we refer specifically to the completely unilluminated parts of the shadows expanding. The image below shows an example.



Imagine a circular light source of radius $R = 4.0$ cm, positioned $H = 15$ cm above the ground, shining directly downwards.



A person puts two fingers below the light source, pointing towards each other. From the top view, both fingers are aligned to the same diameter of the light source. Finger 1 is $h_1 = 5.0$ cm above the ground, and finger 2 is $h_2 = 10$ cm above the ground. The horizontal distance between the tip of the lower finger and the centre of the light source is $r_1 = 2.0$ cm.

Keeping h_1 and h_2 constant, he slowly moves finger 2 towards the centre. What is the minimum horizontal distance r_2 between the centre and the tip of finger 2 before the

two shadows start to “attract” each other?

Leave your answer to 2 significant figures in units of cm.

Problem 24: Shower Troubles

(4 points)

A shower water heater consists of a large well-mixed tank of volume V and inner surface area A , as well as an outlet. The tank walls are maintained at temperature T_h by a heating system, while heat is transferred to the water with a convective heat transfer coefficient h . When the shower is turned on, the heated water exits the showerhead at a mass flow rate q , and is replaced by cold water at room temperature $T_0 = 25^\circ\text{C}$.

The shower has 2 unmarked control knobs: one controls T_h , while the other controls q . Each knob has 2 fixed settings labelled *low* and *high*, but the numerical values are unknown. The shower is allowed to reach steady state under 3 different knob settings, and the outlet temperatures are recorded below:

Temperature setting	Flow rate setting	Outlet temperature/ $^\circ\text{C}$
low	low	$T_{11} = 36.7$
low	high	$T_{12} = 33.3$
high	low	$T_{21} = 43.9$
high	high	$T_{22} = ???$

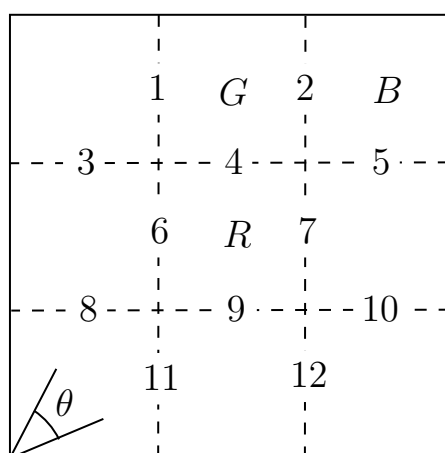
Unfortunately, the shower broke before the final measurement could be taken. Using the available data, determine the unknown temperature, T_{22} .

Leave your answer to 3 significant figures in units of $^\circ\text{C}$.

Problem 25: Mirror Box

A square room is divided into a 3×3 grid. A red ball (R), blue cylinder (B) and green cube (G) are placed as shown in the diagram below, at different heights. The walls, floor, and ceiling of the room are mirrors, and the ceiling of the middle cell is a light source.

A camera is placed at one corner of the room, looking horizontally at the opposite corner. The horizontal field of view of a camera is defined as the angle between the leftmost and rightmost rays of light that are captured by the camera, and is labelled by θ in the diagram below. The camera used has a field of view which is a multiple of 10, when measured in units of degrees.



Two photos are taken by the camera, shown on the following page:

- Photo 1: A photo with the objects in the room as described above.
- Photo 2: A photo with the objects in the same position, but with mirrors on 2 of the grid segments (numbered in the diagram above).

(a) What is the horizontal field of view of the camera?

Leave your answer to 1 significant figure in units of degrees. (2 points)

(b) Which 2 grid segments have mirrors placed on them in Photo 2? Concatenate the numbers of the segments in your final answer, in ascending order. For example, if you think the mirrors are at grid segments 4 and 12, give your answer as 412.

Leave your answer as an integer. (3 points)

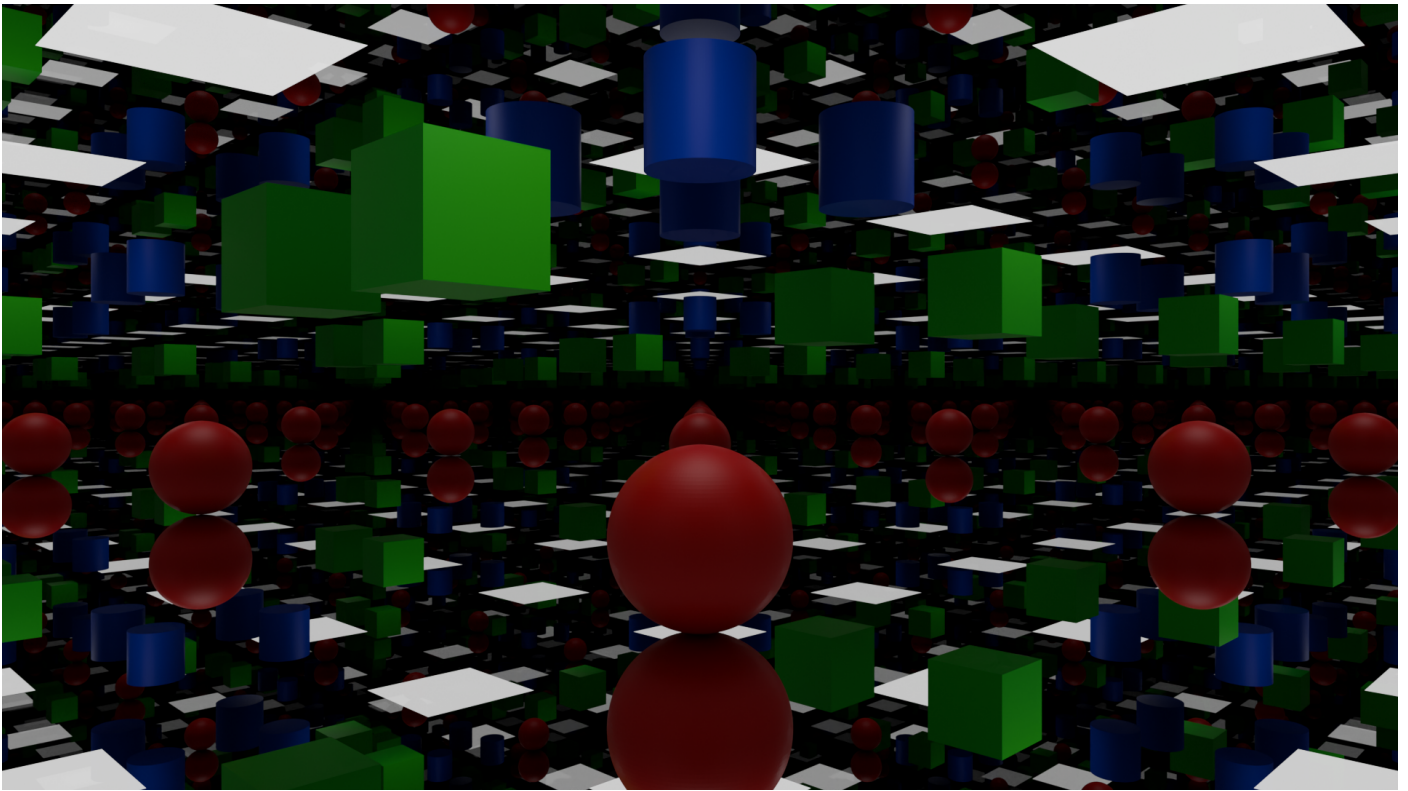


Photo 1: No added mirrors

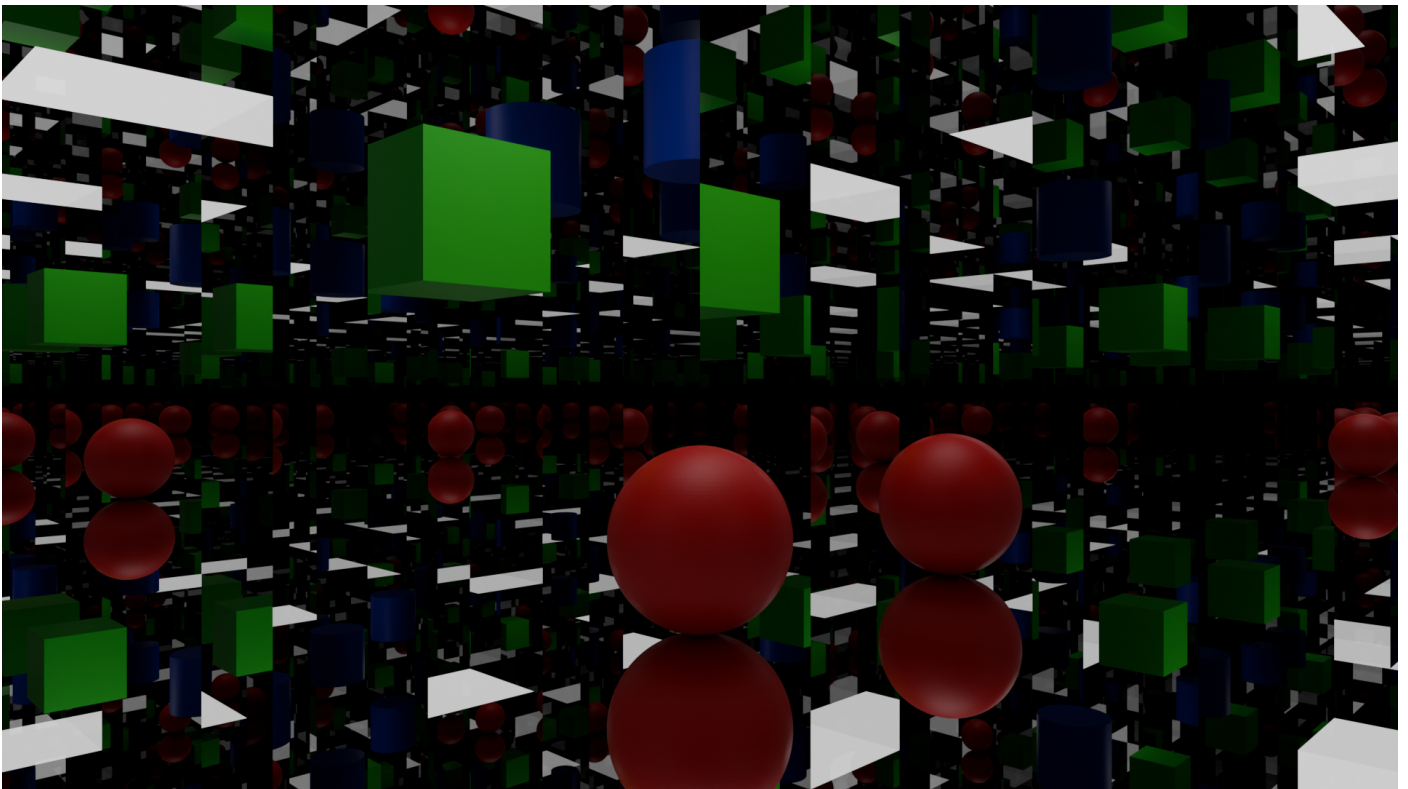
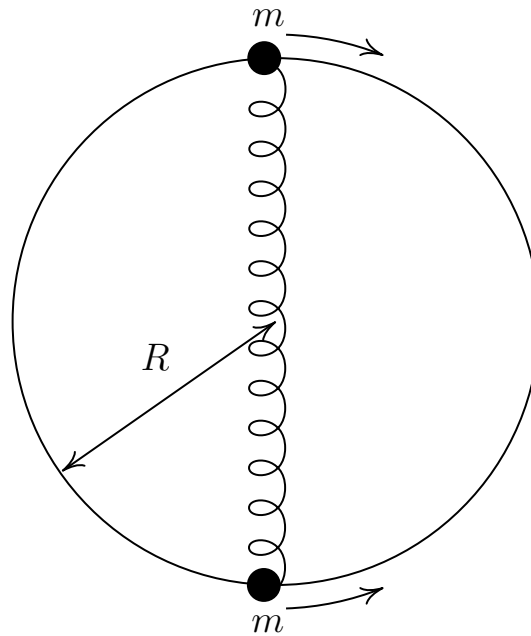


Photo 2: With added mirrors

Problem 26: Rubber Slingshot

(3 points)

Raphael has a frictionless circular disk of radius $R = 15.0$ cm and a rubber band that he wants to launch forwards. To do so, he holds the disk horizontally, wraps the rubber band around the diameter of the disk and gives it a small impulse in the x -direction, as shown below.



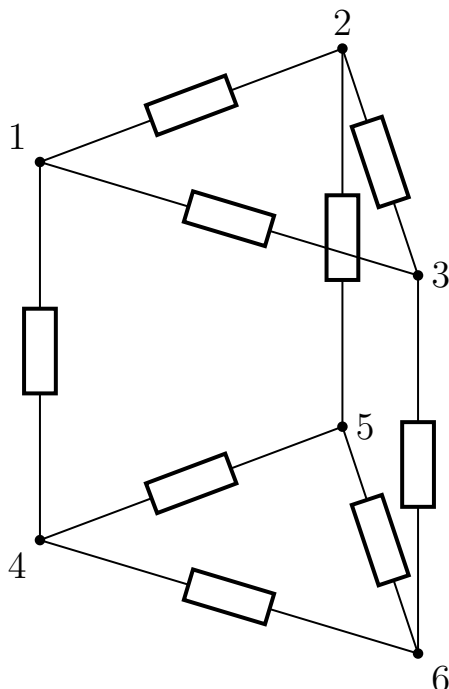
The rubber band may be modelled as two point masses of mass $m = 13.0$ g connected by a spring of spring constant $k = 60.0$ N m⁻¹ and zero rest length. You may ignore gravity. Determine the maximum horizontal velocity v the rubber band achieves during its subsequent motion.

Leave your answer to 2 significant figures in units of m s⁻¹.

Problem 27: Sherlock Ohms

(6 points)

There are 9 resistors arranged in the form of a prism, with vertices labelled 1 to 6. Eight resistors have the same resistance R , while one has a different resistance R_{odd} . You are given an ohmmeter that can measure the resistance between 2 vertices.



Determine R_{odd} .

Leave your answer to 2 significant figures in units of Ω .

Format

You are to input a **2 digit number** corresponding to the labels of the 2 vertices $\overline{ab}$ that the ohmmeter will be attached to. Input the vertices in ascending order with $a < b$. The simulation will output R , the resistance measured, in units of Ω to **4 significant figures**.

Hypothetical Example	
Vertices	Output: R (Ω)
12	1.234

You are given 5 simulation queries for your first attempt, and 2 more per subsequent attempt.

Problem 28: Dripping Faucet

(3 points)

Being the careless person he is, Joan has left the tap in his kitchen dripping. The tap can be modelled as a cylindrical tube with inner radius $a = 0.50$ cm. Water drips periodically from the tap, with $T = 3.0$ s between drops. The surface tension of water is $\sigma = 0.071$ N m⁻¹. Assuming the contact angle between the water and inner surface of the tap is $\theta = 0^\circ$, find the average velocity u (over a cross-section) of the water within his tap.

Leave your answer to 2 significant figures in units of mm s⁻¹.

Problem 29: Chill Out

(4 points)

A box of helium gas is initially at pressure $P_0 = 1.01 \times 10^5$ Pa and volume $V_0 = 0.590$ m³. The gas is isothermally compressed to a pressure of $2P_0$ and then adiabatically expands back to pressure P_0 . This sequence is repeated infinitely many times. Assume the helium always behaves as an ideal monatomic gas and all processes are reversible. Determine the net heat Q lost by the gas throughout the entire process.

Leave your answer to 3 significant figures in units of kJ.

Problem 30: As It Was

(4 points)

In the music video for “[As It Was](#)”, there is a scene where Harry Styles runs on a horizontal rotating disk with a central axle. The axle is fixed in place and the disk can rotate freely about it. The disk has radius $R = 1.8$ m and mass $M = 100$ kg; Harry Styles has a mass $m = 70$ kg. Harry Styles and the disk start at rest.

If Harry Styles has a maximum running speed of $v_{\text{max}} = 10$ km h⁻¹ relative to the surface of the disk, what is the maximum angular velocity ω that he can impart to the disk? Ignore air resistance, and assume that Harry Styles does not jump off the disk.

Leave your answer to 3 significant figures in units of rad s⁻¹.

Problem 31: Unknown Particle in an Unknown Box

An unknown particle is confined in a one-dimensional box of length L with impenetrable barriers (i.e., the potential is infinite at the boundaries).

You have a magical device that can measure the amplitude of the particle's wavefunction ψ at a certain position. The position is defined by the dimensionless length $\bar{x} = \frac{x}{L}$, where x is the distance from one end of the box. It is known that the particle occupies a state corresponding to an odd quantum number (e.g. $n = 1, n = 3, n = 5, \dots$).

(a) Determine the length L of the box.

Leave your answer to 3 significant figures in units of nm. (2 points)

You are given 1 simulation query per problem attempt.

Another unknown particle of mass m is in a different 1 dimensional box. The particle is in one of the 5 lowest energy levels ($n \in \{1, 2, 3, 4, 5\}$), and the energy level it is at has energy $E = 4.244 \times 10^{-22}$ J.

(b) Determine the mass m of the particle.

Leave your answer to 3 significant figures in units of 10^{-26} kg. For example, if your answer is 2.56×10^{-26} kg, input 2.56. (3 points)

You are given 3 simulation queries per problem attempt.

Format

You are to input the dimensionless length $\bar{x}$ at which the amplitude of the wavefunction will be measured to **5 significant figures** where $0 \leq \bar{x} \leq 1$. The simulation will output the amplitude of the wavefunction ψ to **5 significant figures**.

Hypothetical Example	
Input: $\bar{x}$	Output: $\psi(\bar{x}) / \text{m}^{-1/2}$
0.65838	-18270
0.46537	71333

Hint: The following Wikipedia [link](#) may be helpful.

Problem 32: Squashing

(4 points)

Mario is studiously practising squash shots for his upcoming match against Luigi in a 10 m by 10 m square court ABCD with a very high ceiling. He wants to master his *ultimate shot*: the squash ball will bounce off wall DA, then wall AB, then wall BC before landing at corner D in that specific order.

Mario's ultimate shot can be carried out from many positions on the court. However, Mario is weak, so the maximum initial velocity he can launch the squash ball with is $v = 16 \text{ m s}^{-1}$. What area A of the court can he perform this *ultimate shot* from?

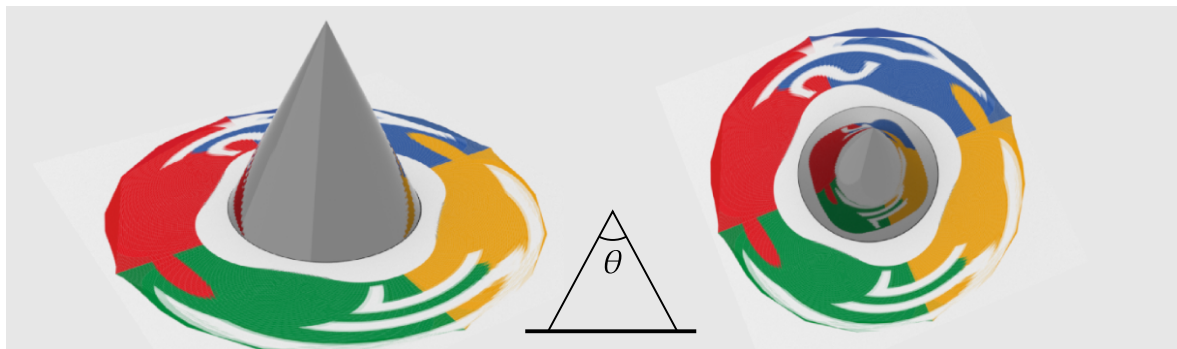
Assume that Mario launches the squash ball from ground level and that all collisions are perfectly elastic.

Leave your answer to 3 significant figures in units of m^2 .

Problem 33: Anamorphic SPhL

(4 points)

Anamorphosis is a perspective technique where the distortion in the object disappears when reflected off a curved mirror surface. The image below shows the distorted SPhL logo placed beneath a perfectly reflective cone.



When the cone is viewed directly from above, and from a sufficiently far distance away, the resultant image formed by the reflection off the cone looks perfectly undistorted (the black circle denotes the outer edge of the base of the cone). By making suitable measurements on the diagram below, find the apex angle of the cone θ .



Leave your answer to 2 significant figures in units of degree.

Problem 34: Flow State

(4 points)

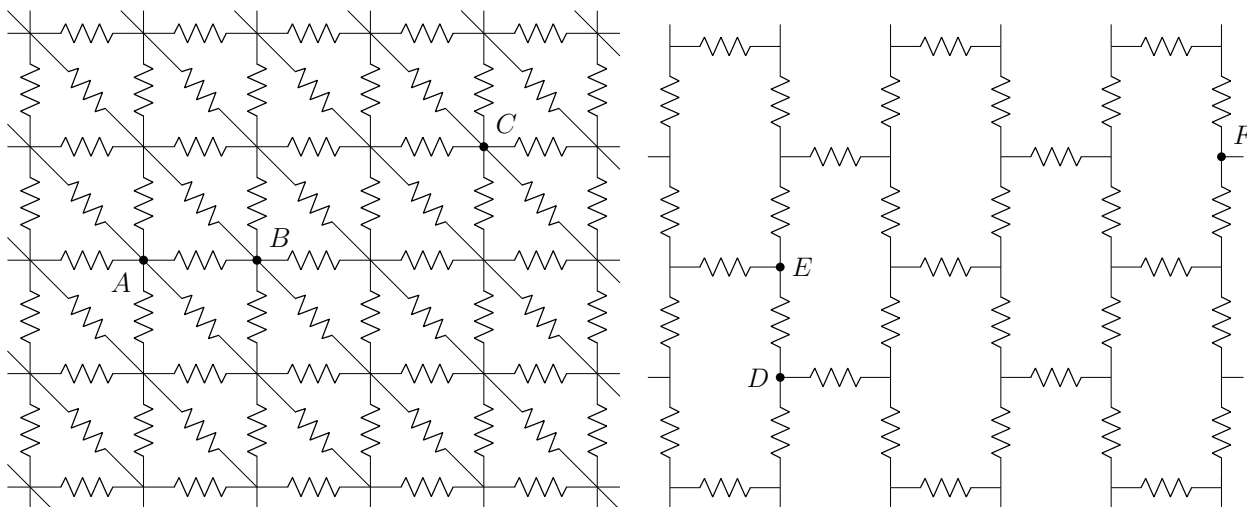
A cylindrical tube of infinite height with radius $r = 1.00$ m has a tiny leaking hole at height $z = 5.00$ m above bottom of area $A = 0.100$ m². At its bottom end, water is pumped in at a constant gauge pressure $P = 1800$ kPa.

Find h , the equilibrium water level in the pipe. Neglect the effects of turbulence, and assume that the open top of the cylindrical tube and the leaking hole are exposed to atmospheric pressure.

Leave your answer to 3 significant figures in units of m.

Problem 35: Resistor Grids

Two infinite resistor grids made of identical resistors are as shown below. Let R_{XY} denote the effective resistance between nodes X and Y that are on the same grid.



- (a) Determine $\frac{R_{AB}}{R_{DE}}$.

Leave your answer to 2 significant figures.

(2 points)

- (b) Determine $\frac{R_{AC}}{R_{DF}}$.

Leave your answer to 2 significant figures.

(4 points)

Problem 36: Simp 3

(4 points)

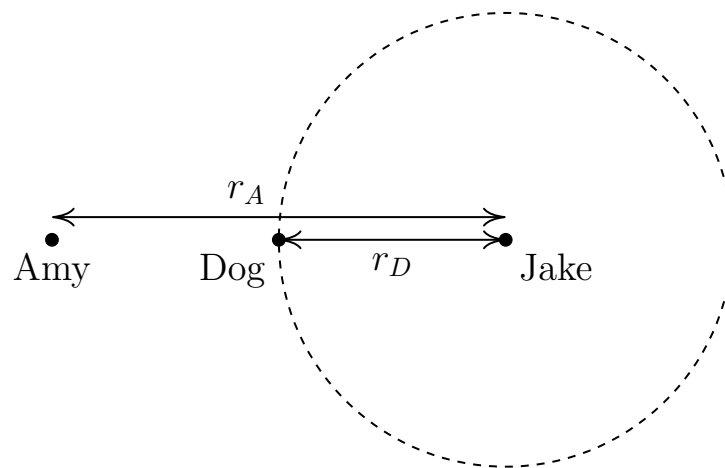
Unfortunately, Amy's clone was caught by the dog. Unfazed, Amy decides to play another round of the game with Jake and his dog, with the same rules as before:

- A “catch” occurs when two characters occupy the same position on the plane.
- If Amy catches Jake before his dog catches her, she wins.
- If Jake's dog catches Amy before she catches Jake, Jake and his dog win.

After what happened to Amy's clone, Jake has decided to keep his dog on a rigid leash. Meanwhile, Amy concludes that planning is the key to success, and commits to a fixed route she will follow regardless of the dog's movements. Unfortunately for her, Jake's dog manages to access her plan before the game begins. The game is therefore governed by the following additional rules:

- Jake remains stationary.
- Jake's dog must remain a fixed distance r_D from him, while Amy can move anywhere in the plane.
- Amy plans her path to catch Jake before the game starts and follows it regardless of the dog's movements.
- Jake's dog knows Amy's planned path before the game starts and can anticipate her movements.

Amy, Jake, and his dog begin the game in a line, with the dog between Amy and Jake. The starting distance between Amy and Jake is r_A . Amy and the dog's top speeds are v_A and v_D respectively, and they can change their velocities instantaneously.



They start the game with $r_A/r_D = 2$. Assuming all characters play optimally, Amy wins if $v_A/v_D > \chi$. Determine χ .

Leave your answer to 3 significant figures.

Problem 37: Hot Creep

(4 points)

While taking his customary midday rooftop siesta, Jed left his laptop in the sun and forgot to retrieve it. As the laptop warms during the day and cools at night, it repeatedly expands and contracts, causing it to slowly creep down the roof.

His roof is sloped at an angle $\theta = 20^\circ$ above the horizontal. You may treat the laptop as a square aluminium plate with side length $a = 30$ cm, linear thermal expansion coefficient $\alpha = 2.4 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$, and coefficient of friction with the roof $\mu = 0.9$. The difference in temperature between day and night is $\Delta T = 10^\circ\text{C}$.

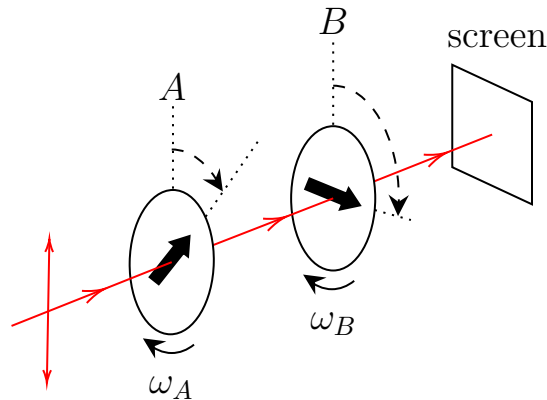
After one year of neglect, how far down the slope has the laptop moved?

Leave your answer to 3 significant figures in units of m.

Problem 38: Spin-2?

(4 points)

A beam of linearly polarised light travels towards 2 ideal linear polarisers A and B , arranged in a row. Initially, both polarisers are oriented upright, in the same direction as the light polarisation. Shanay decides to spin the two polarisers with constant angular velocities ω_A and ω_B respectively, starting at the same time. Both polarisers are spinning in the same direction.



Shanay observes [this](#) on his screen. Unfortunately, Shanay did not start recording immediately after spinning the polarisers, but he has managed to record a perfect loop¹ that is 24.00 s long. Let the periods of rotation of the polarisers be T_A and T_B . If $T_A > T_B$, determine $T_A - T_B$.

Leave your answer to 3 significant figures in units of s.

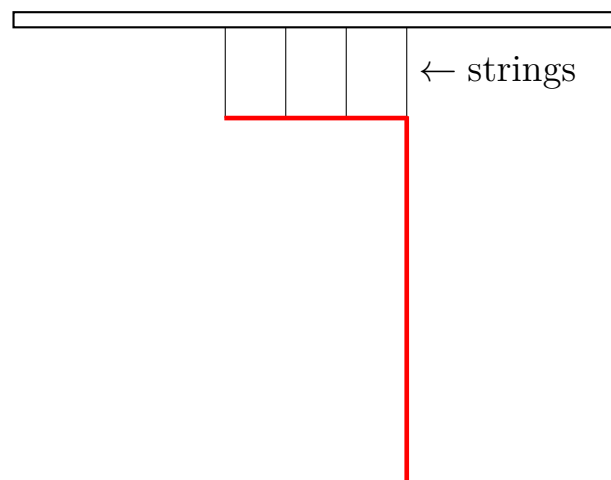
¹The frame that immediately follows the last frame is the first frame. Since this video is at 50 frames per second, we have $t = 23.98$ s on the last frame.

Problem 39: Hanging An L

(5 points)

A rigid structure in the shape of an “L” of total mass $M = 1.70$ kg consists of two rods of identical linear mass densities, one twice the length of the other, joined perpendicularly at their ends. Four identical strings hang vertically from the ceiling and are tied to the shorter rod at equidistant positions. Determine the tension T in the rightmost string.

Hint: You may assume each string acts as an extremely stiff Hookean spring when taut and exerts no force when slack.

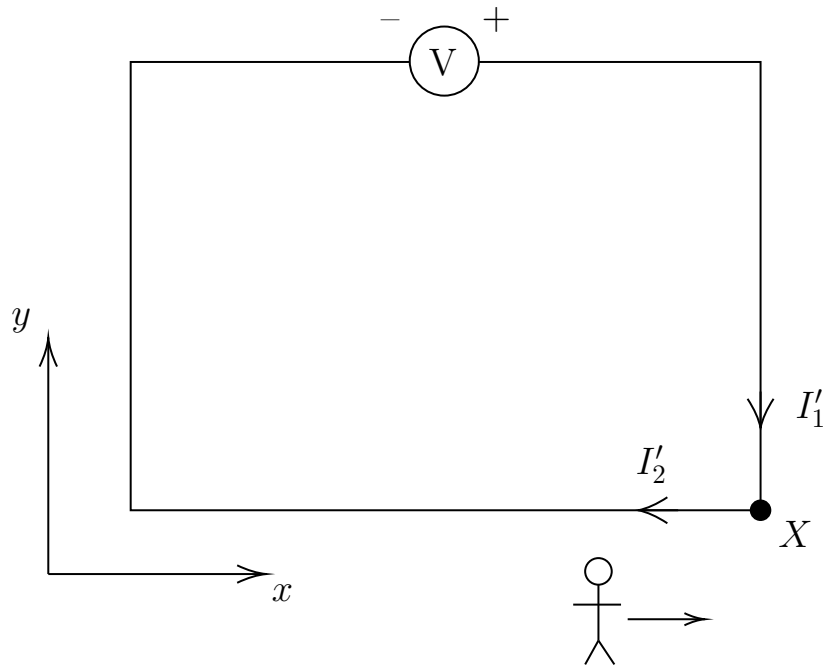


Leave your answer to 3 significant figures in units of N.

Problem 40: I of the Beholder

(4 points)

Sid has built a simple circuit, consisting of a voltage source $V = 5.00 \text{ V}$ connected to a resistive wire of resistance $R = 1.20 \Omega$. He checks that there is no current leakage and is satisfied with his design. However, Aaron, on his morning jog at $v = 0.7c$ past the circuit, notices something strange - he observes that the current I'_1 entering point X and I'_2 exiting point X are not the same! Compute this discrepancy, $I'_2 - I'_1$.

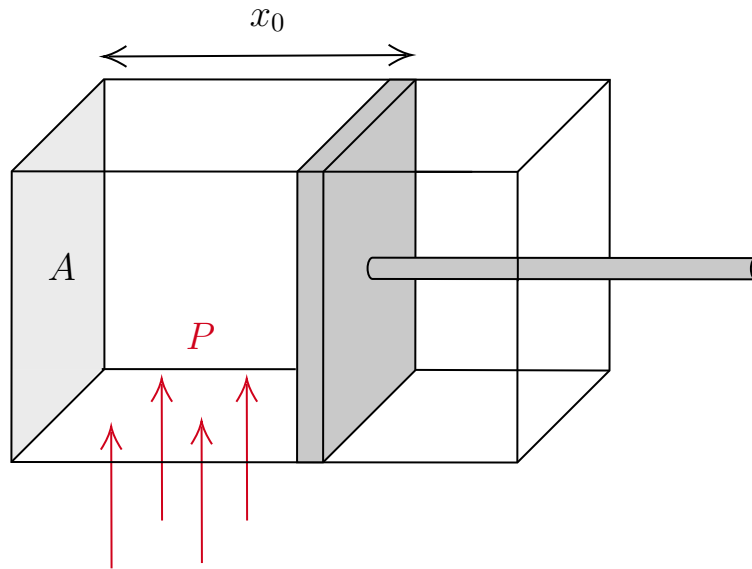


Leave your answer to 3 significant figures in units of A.

Hint: Charge density ρ and current density $\mathbf{j}$ (current per unit area) transform analogously to t and $\mathbf{x}$ respectively under a change in reference frame.

Problem 41: Full of Hot Air

A cuboid tank with a cross-sectional area A is placed horizontally on a hot plate and filled with an ideal monatomic gas. The tank is sealed by a frictionless piston of mass m located at an initial distance x_0 from the base of the tank. At this stage, the gas is at the same temperature and pressure as the surroundings. All faces of the tank are insulating except the face in contact with the hot plate. Let the external pressure be p_0 .



At $t = 0$, the hot plate is turned on and the gas is heated with power P . As a result, the gas expands and the piston moves to the right. It is found that the time t for the piston to move a small distance $\Delta x \ll x_0$ scales as $t \propto \Delta x^n$.

- (a) Determine the value of n in the limit $P^2 \ll \frac{(p_0 A)^3 x_0}{m}$.

Leave your answer to 2 significant figures.

(2 points)

- (b) Determine the value of n in the limit $P^2 \gg \frac{(p_0 A)^3 x_0}{m}$.

Leave your answer to 2 significant figures.

(3 points)

Problem 42: Love Triangle

(4 points)

Luc, a perennial egotist, has decided to enter a *ménage à trois* with 2 clones of himself.

They start off in space, at the vertices of an equilateral triangle with circumradius $r = 100$ m. The triangle is slowly rotating at angular velocity $\omega = 1.00 \times 10^{-14} \text{ s}^{-1}$. At $t = 0$ s, they impulsively move towards their rendezvous, *adding* a velocity $v = 12.0 \text{ m s}^{-1}$ towards the centre to their current velocity. All Lucs gravitationally interact with each other.

Unfortunately, as Luc is severely underweight, weighing only $m = 50.0$ kg, his gravitational interactions are not strong enough – so everyone ends up flying into the void. After a long time, what is the angle between each Luc's initial (shortly after $t = 0$ s) and final velocity?

Leave your answer to 3 significant figures in units of degrees.

Problem 43: Strategically Accelerating Frames

(4 points)

Aaron is currently training hard for his 2.4 km run, aiming to pass his IPPT and get a 2 month reduction to his national service. However, Sid has a better idea - by running to point P and back over the course of 2 years, he can simply make those 2 years of NS feel like only 1 year and 10 months!

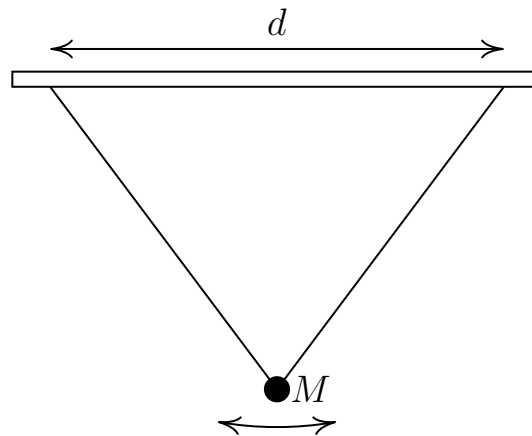
Sid has a top speed of $v = 0.6c$. Assume that he runs in a straight line at constant velocity on each leg of his journey and can only change his velocity once, at point P. As Sid is worried he might be breaking confinement, he wishes to stay as close to Singapore as possible. Determine x_P , the minimum distance of point P from the origin, in light-years.

Leave your answers to 3 significant figures in units of ly.

Problem 44: Rock and Roll

(4 points)

The ends of a massless string of length $\ell = 0.250$ m are fixed to a ceiling at a distance $d = 0.150$ m from each other. A bead of mass M is threaded through the string and positioned at equilibrium. The bead is then given a small displacement from equilibrium along the string and oscillates without friction. Determine the period T of oscillations.



Leave your answer to 3 significant figures in units of s.

Problem 45: A Light Round of Ping Pong

(5 points)

Monochromatic light of wavelength $\lambda = 600$ nm and total energy $E = 20$ J is trapped in a one-dimensional box of length $L = 1.00$ m. The two ends of the box are mirrors with mass $m = 0.10$ g and reflectance $R = 0.67$. What is the speed of each mirror after the energy of the light has halved?

Hint: The reflectance of a mirror is the ratio of the intensity of light it reflects to the intensity of light shone on it. Light not reflected is absorbed.

Leave your answer to 2 significant figures in the units of mm s^{-1} .

Problem 46: Lily Pond

(4 points)

Two identical planets with mass $m = 9.0 \times 10^{25}$ kg consisting entirely of water orbit each other. Each planet moves in an elliptical orbit with eccentricity $e = 0.85$ and semi-major axis $a = 2.5 \times 10^{11}$ m. The planets are covered in white lilies, making them perfectly reflective to incoming radiation. Because of this, they do not emit any blackbody radiation into space.

After a long time, tidal forces between the planets cause their orbits to become circular. Assuming that the temperature of each planet remains uniform, find the change in temperature ΔT after the planets assume circular orbits. The heat capacity of water is $c = 4184 \text{ J kg}^{-1} \text{ K}^{-1}$.

Leave your answer to 3 significant figures in units of K.

Problem 47: 67

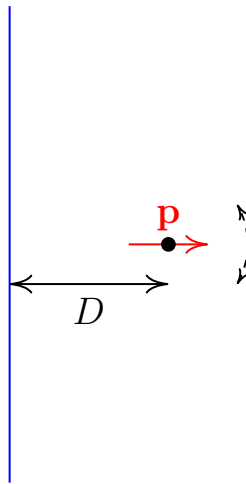
(5 points)

The 67 kid wishes to revive the 67 meme, but he has injured his arms. Determined, he instead places an electric dipole with dipole moment $p = 6.70 \times 10^{-6}$ C m and moment of inertia $I = 6.70 \times 10^{-3}$ kg m² pointing away from an infinite conducting plane at distance $D = 0.670$ m. The dipole is fixed in position but can freely rotate. The field by an electric dipole of dipole moment $\mathbf{p}$ at position vector $\mathbf{r}$ from the dipole is given by

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \frac{3(\mathbf{p} \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}}{|\mathbf{r}|^3}$$

He gives the dipole a small initial angular velocity to re-enact his signature 67 gesture. Determine the period T of small oscillations of the dipole.

conducting plane



Leave your answer to 3 significant figures in units of s.

Problem 48: Drag Me Down

In clouds, raindrops grow by collecting very small droplets as they fall. These droplets are evenly distributed in the cloud with mass density $\rho_d = 1.20 \text{ g m}^{-3}$, and are small enough that they remain effectively stationary in the cloud. The raindrop experiences quadratic air drag according to the equation

$$F_D = \frac{1}{2}C_D\rho_a v^2 A,$$

where $C_D = 0.460$, $\rho_a = 1.25 \text{ kg m}^{-3}$, v is the speed of the raindrop relative to the air and A is its cross-sectional area. The raindrop has a density of $\rho_w = 1000 \text{ kg m}^{-3}$.

A raindrop with radius $r_0 = 20.0 \text{ }\mu\text{m}$ starts in the cloud with no initial velocity, then allowed to fall. Assume that the raindrop is always spherical while falling and remains in the cloud.

- (a) Immediately after its release, the magnitude of the acceleration a of the raindrop can be approximated as a quadratic function

$$a = g - \alpha t^2,$$

where a is the magnitude of the raindrop's acceleration and g is the gravitational acceleration. Find the value of α .

Leave your answer to 3 significant figures in units of m s^{-4} . (3 points)

- (b) Some time after its release, the raindrop's acceleration becomes approximately constant at β . Determine β .

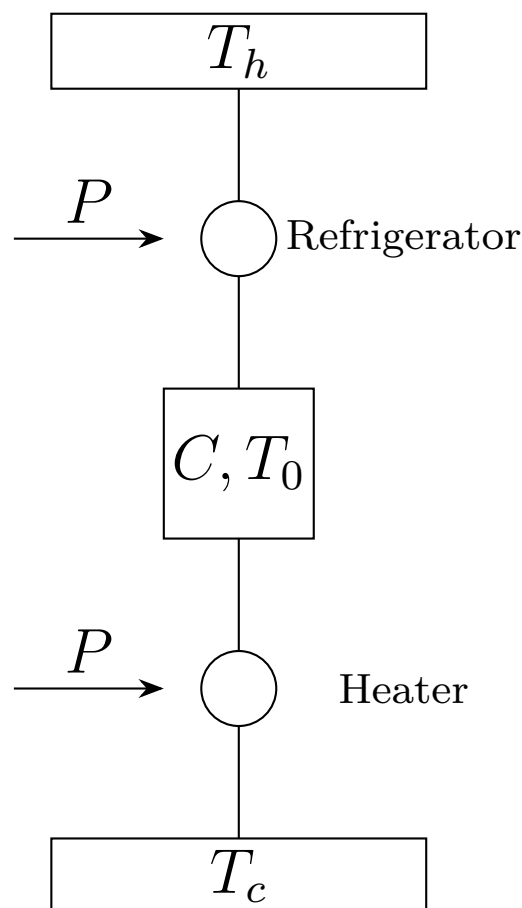
Leave your answer to 3 significant figures in units of m s^{-2} . (3 points)

Problem 49: Would You Like it Hot or Cold?

(5 points)

A block of heat capacity C and initial temperature $T_0 = 320$ K is connected to both a heater and a refrigerator as shown in the diagram below. The heater and refrigerator are respectively connected to a cold reservoir of temperature T_c and a hot reservoir of temperature T_h . Both have the same input power P and use reversible Carnot cycles to transfer heat from the cold reservoir to the block, or from the block to the hot reservoir.

When only the heater is turned on, it takes time t to double the block's temperature. When only the refrigerator is turned on, it takes the same time t to halve the block's temperature. If both are turned on simultaneously, what would the temperature of the block be after a long time?



Leave your answer to 3 significant figures in units of K.

Problem 50: Ergodicity Breaking

Consider two point particles A and B somewhere strictly inside a one-dimensional frictionless chamber of length $L = 1.000$ m, enclosed by fixed walls on both ends. Particle A starts with an initial velocity of $v_0 = 1.000$ m s⁻¹, while particle B starts off stationary. Collisions between the two particles and between a particle and a wall are completely elastic. Suppose that the subsequent motion of the particles is periodic, with period T .

- (a) If $m_A = 3m_B$, find the minimum possible T .

Leave your answers to 3 significant figures in units of s. (3 points)

If you think that the motion cannot be periodic, input your answer as -1 .

- (b) If $m_A = 4m_B$, find the minimum possible T .

Leave your answers to 3 significant figures in units of s. (5 points)

If you think the motion cannot be periodic, input your answer as -1 .

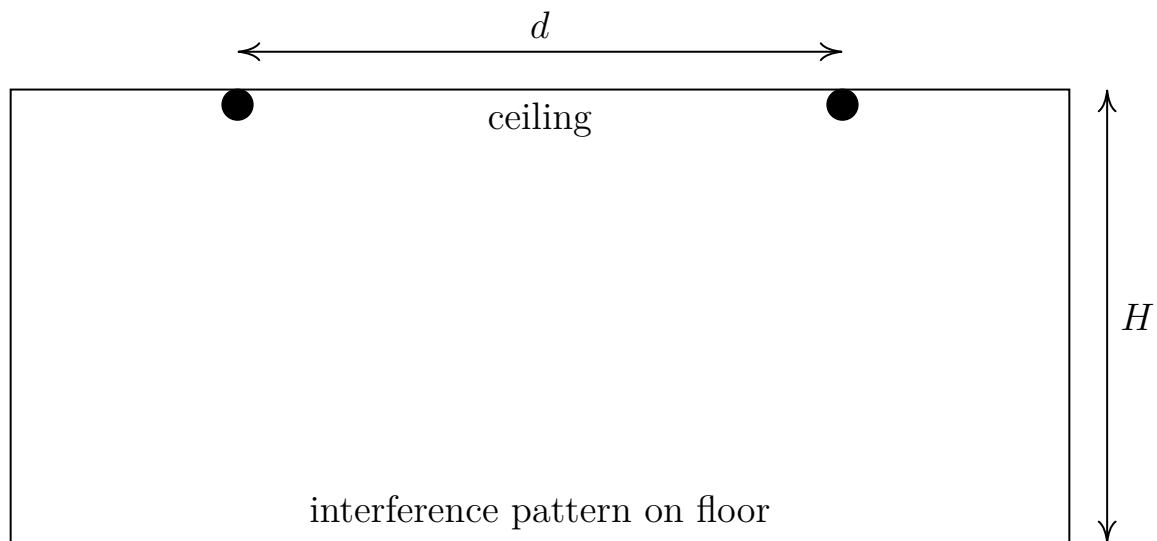
Hint: A similar system is analysed in the following [video](#).

Problem 51: Micromacromicrowave

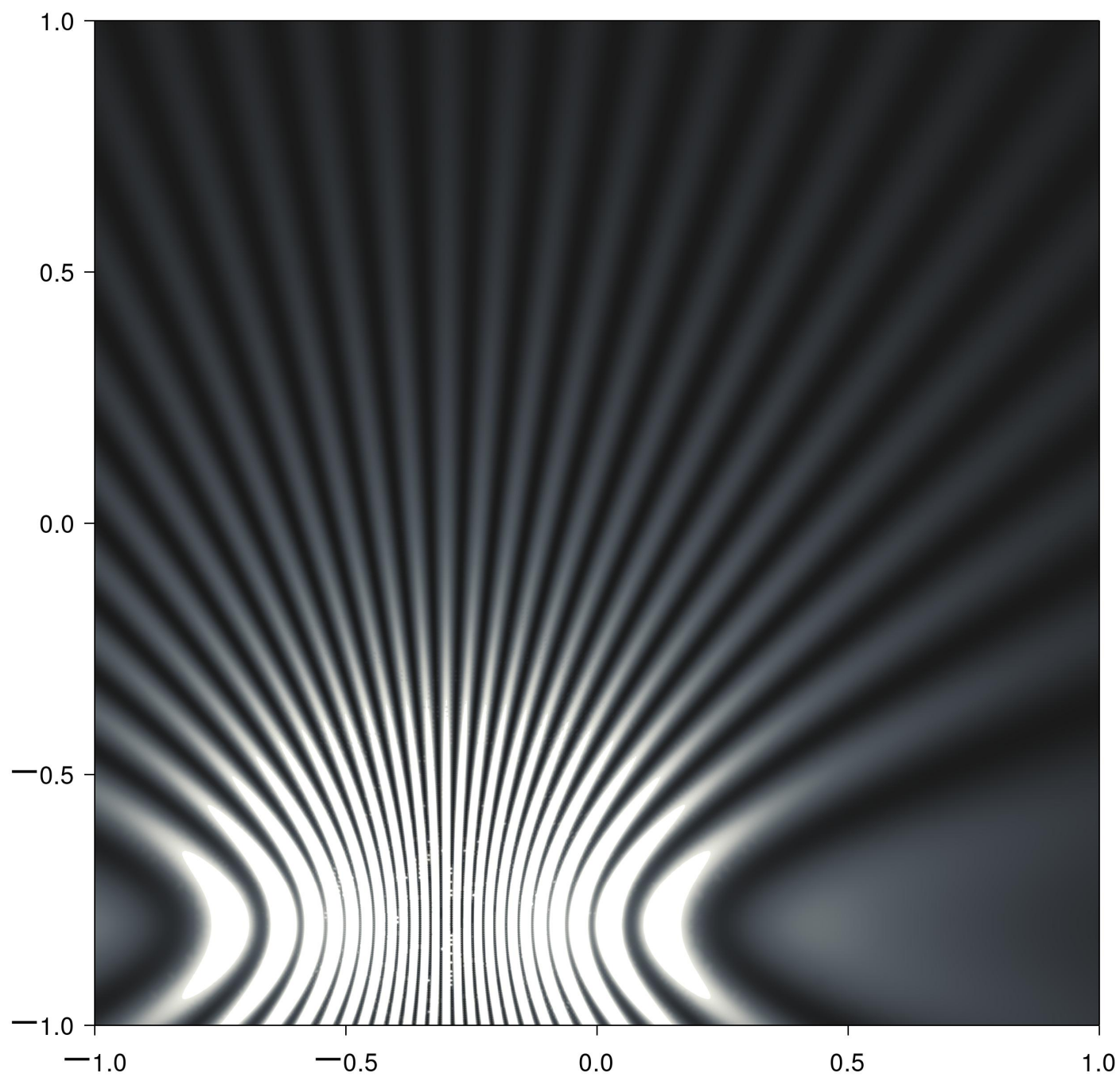
(5 points)

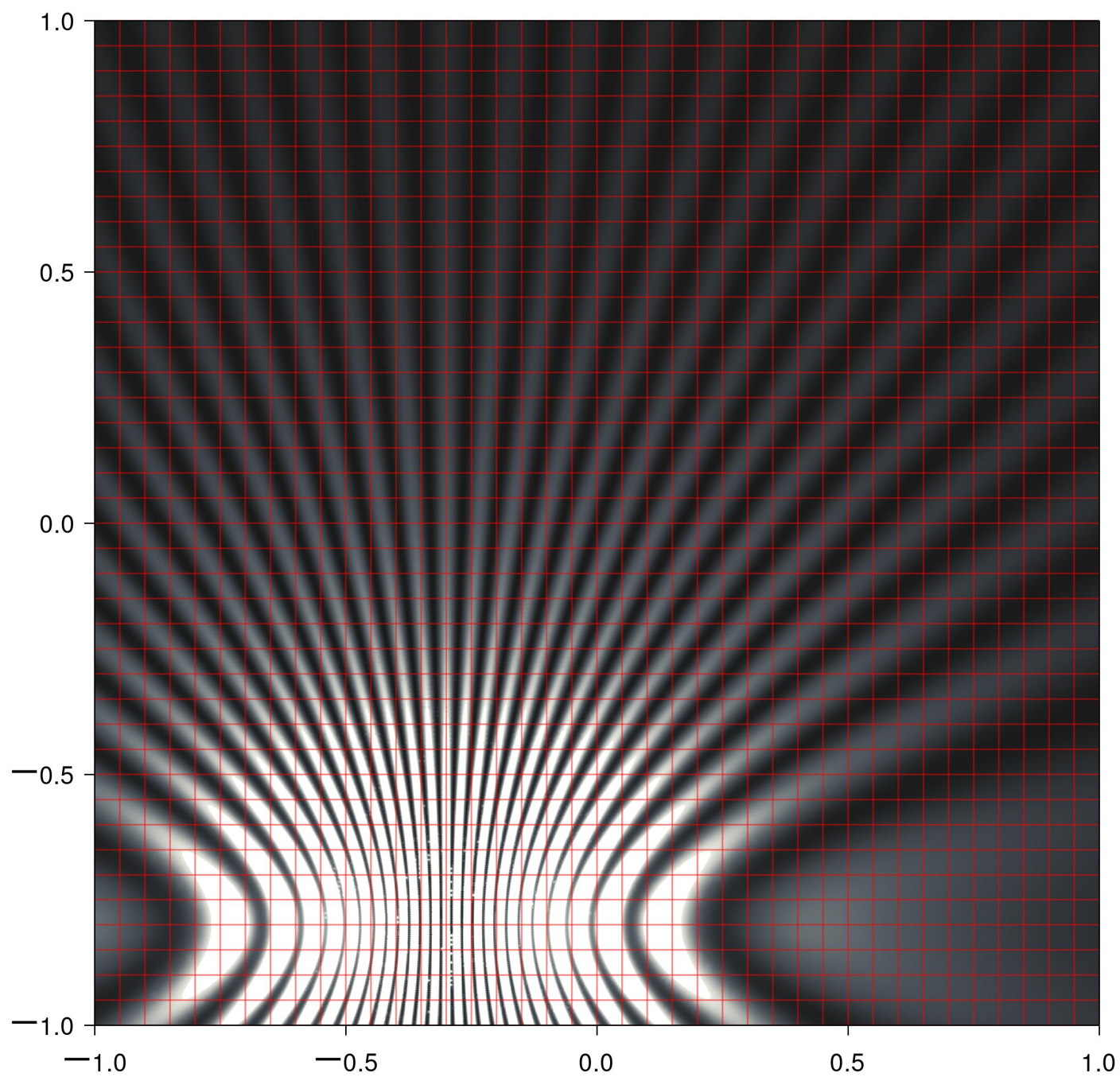
Aaron's microwave broke down suddenly, so he decides to build a new one from scratch. Unfortunately, he does not have access to the magnetrons used in conventional microwaves. Instead, Aaron fixes two small bulbs to the ceiling of the microwave, a distance d apart. Each bulb isotropically emits microwaves of wavelength λ . With his 20/20 vision, Aaron is able to observe the interference pattern the microwaves form on the floor, which can be found on the next two pages (both pictures are the same, but one of them has grid lines for annotations). Note that coordinates are given in metres.

If the ceiling is $H = 19$ cm above the floor, determine $5\lambda + \frac{d}{2}$.



Leave your answer to 2 significant figures in units of m.





Problem 52: Let Him Cook

(5 points)

James is tired of the food from the army cookhouse, and sneaked some ready-to-eat meals into camp. To heat the food, he has to use the ancient oven in the pantry. The heating element in the oven is maintained at a constant temperature $T_0 = 373$ K, and the surrounding air outside the oven at $T_2 = 298$ K. The food has heat capacity $C = 750$ J K⁻¹, and an initial temperature $T_1 = T_2$. Given that the ready-to-eat meal has to reach a minimum temperature of $T_f = 348$ K for it to be cooked, what is the minimum heat Q_0 that has to be supplied by the oven?

James has a lot of ideal heat engines with him.

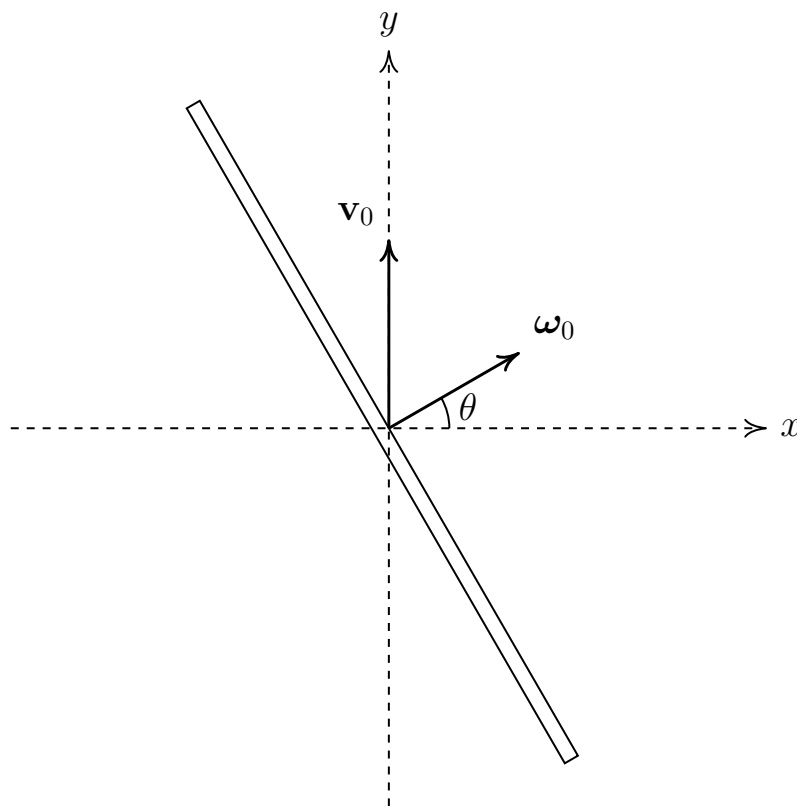
Leave your answer to 3 significant figures in units of kJ.

Problem 53: Kansei Dorifto?!

Ryab watched too much Initial D and wanted to try out some sick drift moves himself. Unfortunately, he can't afford a car with his NS pay, so he begrudgingly tries to drift a thin roll of tape like in [this video](#).

Ryab places a roll of tape (modelled as a thin ring) on a horizontal table and flicks it, giving it a translational velocity $\mathbf{v}_0 = v_0 \hat{\mathbf{y}}$ and a backspin angular velocity $\omega_0 = v_0/R$, where R is the radius of the roll of tape. The angular velocity vector $\boldsymbol{\omega}_0$ is at an angle θ anticlockwise from the x -axis, as shown in the **top-view** diagram below.

After some time, the roll transitions to pure rolling without slipping, with final velocity $\mathbf{v}_f$. The coefficient of kinetic friction between the tape roll and the table is $\mu = 0.86$.



- (a) Ryab wants the angle between $\mathbf{v}_0$ and $\mathbf{v}_f$ to be 86° . What is the required angle θ to achieve this?

Leave your answer to 3 significant figures in degrees.

(3 points)

- (b) If instead Ryab wants to maximise the displacement of the roll of tape from its starting position up to the instant it first attains pure rolling without slipping, what should the angle θ be?

Leave your answer to 3 significant figures in degrees.

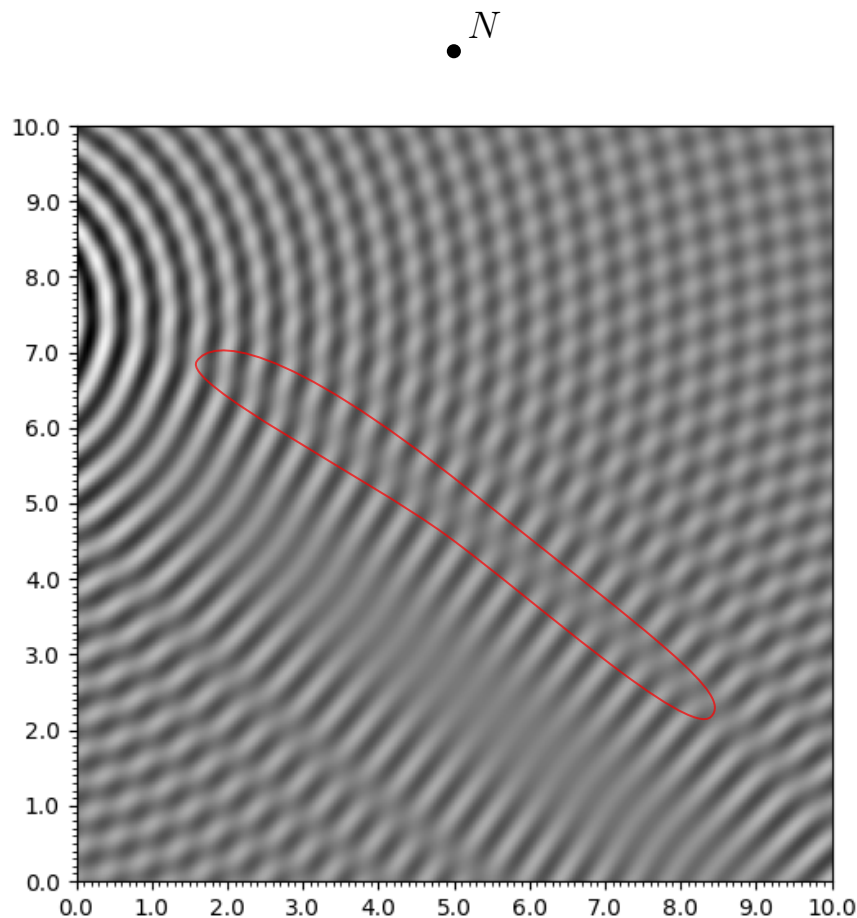
(4 points)

Problem 54: Mesmerizer

(6 points)

Miku and Teto are having a singing competition to see who can hold a note for the longest time. Both of them can be approximated as point sources in the 2-dimensional plane, each emitting a pure frequency f . The [video](#) shows the high and low pressure regions inside a $L \times L$ square, where $L = 10.0$ m, where black indicates regions with lower pressures and white indicates regions with higher pressures.

Note that both Miku and Teto are situated at points M and T outside of the square. Furthermore, **the video has been slowed down** by a factor of $\gamma = 1000$ (i.e. 1 s of the video corresponds to 1 ms of the actual physical setup).



The figure shows one example frame of the video. Observe the grey “wave” highlighted in the figure. Determine the speed of the “wave” v_w when it reaches point $N(5.0, 11.0)$ (shown in the figure above).

Leave your answer to 2 significant figures in units of m s^{-1} .

Hint: Clearly, it is not possible to find v_w by direct measurement at point N . Instead, try working out the positions of M and T .

Problem 55: Simp 4

(6 points)

While chasing Amy around Jake, the dog managed to break free of its leash. As such, Amy, Jake, and his dog decide to play one final game of catch, with the same rules as before:

- A “catch” occurs when two characters occupy the same position on the plane.
- If Amy catches Jake before his dog catches her, she wins.
- If Jake’s dog catches Amy before she catches Jake, Jake and his dog win.

Since Jake does not have a replacement leash for the dog, the following rules are used:

- Jake must remain stationary.
- Both Jake’s dog and Amy can move freely in the plane.
- Amy plans her path to catch Jake before the game starts and follows it exactly.
- Jake’s dog knows Amy’s planned path before the game starts and can anticipate her movements.

Amy, Jake, and his dog begin the game in a line, with the dog between Amy and Jake. The starting distance between Amy and Jake’s dog is r_A and that between Jake and his dog is r_J . Amy and the dog’s top speeds are v_A and v_D respectively, and they can change their velocities instantaneously.



They start the game with $r_A/r_J = \pi/2e$. Assuming all characters play optimally, Amy wins if $v_A/v_D > \chi$. Determine χ .

Leave your answer to 3 significant figures.

Half Hour Rush M1: Faro Shuffle

(3 points)

A faro shuffle, pictured below, consists of interweaving two halves of a deck together, such that the cards of each half alternate. Jindong performs a faro shuffle on his deck of $n = 52$ cards, sliding the cards slightly more than halfway into the weave, and places the deck on a frictionless table. Each card has mass $m = 1.70$ g, and static friction coefficient $\mu_s = 0.44$. Determine the minimum force F he must exert to separate the 2 halves by pulling on them.



Leave your answer to 3 significant figures, in units of N.

Half Hour Rush M2: Ribbon Spread

(3 points)

Jindong shows off his latest trick - ribbon spreading, without hands! He puts his patented frictionless deck of cards on a rough table and quickly pushes it sideways, giving it an initial velocity $v_0 = 10 \text{ cm s}^{-1}$ along the x -axis. Each card has a coefficient of kinetic friction with the table of $\mu_k = 0.562$ and negligible friction with other cards. You may treat the cards as thin rigid plates with width $w = 6.35 \text{ cm}$. Given the deck of cards consists of $N = 52$ cards, vertically stacked, determine the furthest distance x_{max} along the x -axis a playing card travels during the subsequent motion.

Leave your answer to 2 significant figures in units of cm.

Half Hour Rush M3: House of Cards

(5 points)

After mastering the faro shuffle and ribbon spread, Jindong decides to build a tower of cards to celebrate. The tower is composed of equilateral triangles, with the bottom layer on a rough surface. For added stability, Jindong glues the cards that make up each horizontal layer together, so that they can be treated as a single rigid body that evenly transfers the weight of the cards above to the cards below.



The static friction coefficient between the backs and edges of the cards is $\mu_s = 0.562$, while there is no friction between the faces and edges of the cards. You may assume the cards are rigid and have rounded edges. Determine the maximum number of levels, n , Jindong can build the tower to before it collapses.

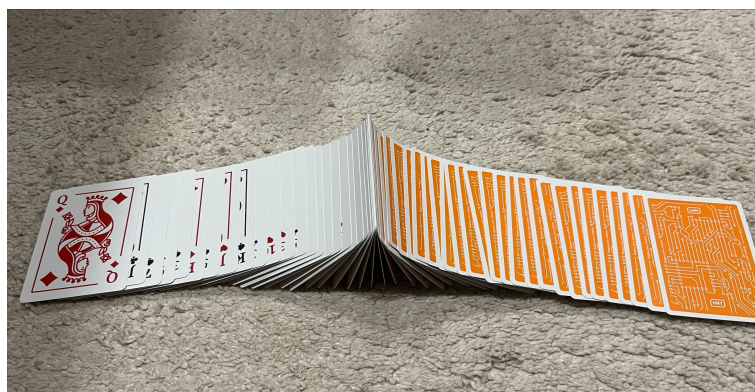
Leave your answer as an integer.

Answer -1 if the card tower can be built arbitrarily high.

Half Hour Rush M4: Soliton Solitaire

(5 points)

Jindong prepares his final flourish - by performing a ribbon spread, then turning the bottom card over, he can generate a “wave of cards” that travels down the spread.



Each card is very thin, has width $w = 6.35$ cm, and the gap between each card in the spread is $d = 0.10$ cm. You may assume that the table is sufficiently rough that cards do not slip, and that the total number of cards in the spread is very large. Jindong initiates the wave by turning over the first card at an angular speed of $\omega = 0.50 \text{ rad s}^{-1}$. Determine the speed of the wave when its peak passes the $n = 520^{\text{th}}$ card.

Leave your answer to 2 significant figures in units of m s^{-1} .

Half Hour Rush E1: Nyan Cat®

(3 points)

Nyan Cat® is a cartoon cat with a Pop-Tart torso. Nyan Cat® loves to fly through space, leaving a magnificent rainbow trail of width $w = 7.62$ cm. Nyan Cat® has a mass of $m = 48.0$ g and may be treated as a point particle.

Nyan Cat® decides to charge itself with an electric charge of $q = 1.00 \times 10^5$ C and fly through a region of space where there exists a uniform magnetic field $B = 5.50 \times 10^{-10}$ T. Nyan Cat® initially starts flying at a speed of $v = 5.00$ m s⁻¹ perpendicular to the magnetic field.

Determine the area A of Nyan Cat®'s rainbow trail once it loops back on itself, if it does. The centre of Nyan Cat®'s trail is formed by its trajectory and the trail lies in the same plane as the trajectory. Neglect all forces other than the magnetic force.

Leave your answer to 3 significant figures in units of m².

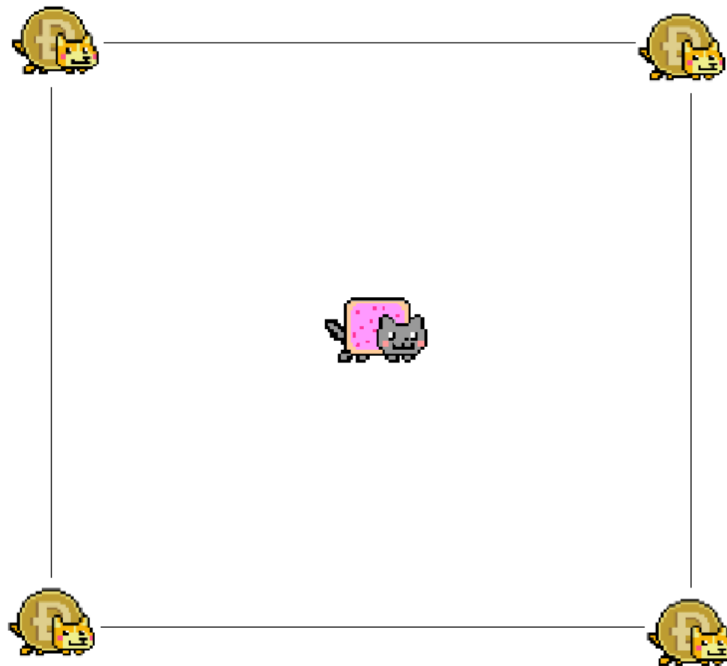
If you think the trail does not loop back on itself, enter -1.

Half Hour Rush E2: It Goes in the Square Hole

(4 points)

Nyan Cat[®] is trying to fly through a massless square frame of length $l = 3000$ m as shown in the diagram below (Nyan Cat[®]'s velocity is into the page). **The square frame is free to move in space.** However, as Nyan Cat[®] is not square enough, Square Hole Girl wants to stop it from flying through.

While Nyan Cat[®] is still far away, she attaches a Nyan Doge[®] with the same mass and charge as Nyan Cat[®] to each corner of the frame. If Nyan Cat[®] travels along the axis of symmetry perpendicular to the square, find the minimum velocity u it must have (when far from the frame) in order to pass through it. Nyan Cat[®]'s mass is 48.0 g and its charge is now $q = 1.00$ C.



Leave your answer to 3 significant figures in units of m s^{-1} .

Half Hour Rush E3: Megalovania

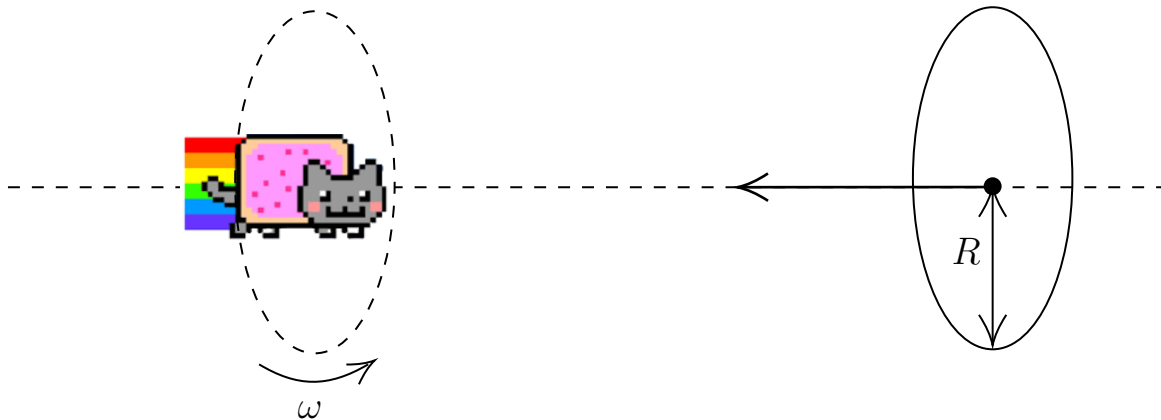
(4 points)

Angered by Nyan Cat[®]'s shenanigans, Sans[®] (from the indie game *Undertale*[®]) challenges Nyan Cat[®] to a boss fight. Unfortunately, Sans[®] ran out of bones, so he instead uses a circular, insulating and uniformly charged ring of radius $R = 0.240$ m, mass $M = 0.0960$ kg and electric charge $Q = 1.75 \times 10^3$ C as an obstacle. In retaliation, Nyan Cat[®] strips itself of its electric charge and, using its power of being fictional, charges itself with a *magnetic charge* $q_m = 1.25 \times 10^3$ A m, the hypothetical analogue of electric charge. As a result, Nyan Cat[®] produces a magnetic field around itself given by

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0 q_m}{4\pi r^2} \hat{\mathbf{r}}$$

where Nyan Cat[®] is centred at the origin.

Sans[®] attacks Nyan Cat[®] by propelling the initially non-rotating ring from very far away at a constant velocity $v = 12.0$ m s⁻¹ towards it. The ring's axis passes through Nyan Cat[®], who remains fixed in position. To Sans[®]' surprise, the ring begins to spin! Determine the angular velocity ω of the ring when its centre reaches Nyan Cat[®]. You may ignore the self-inductance of the ring.



Leave your answer to 3 significant figures in units of rad s⁻¹.

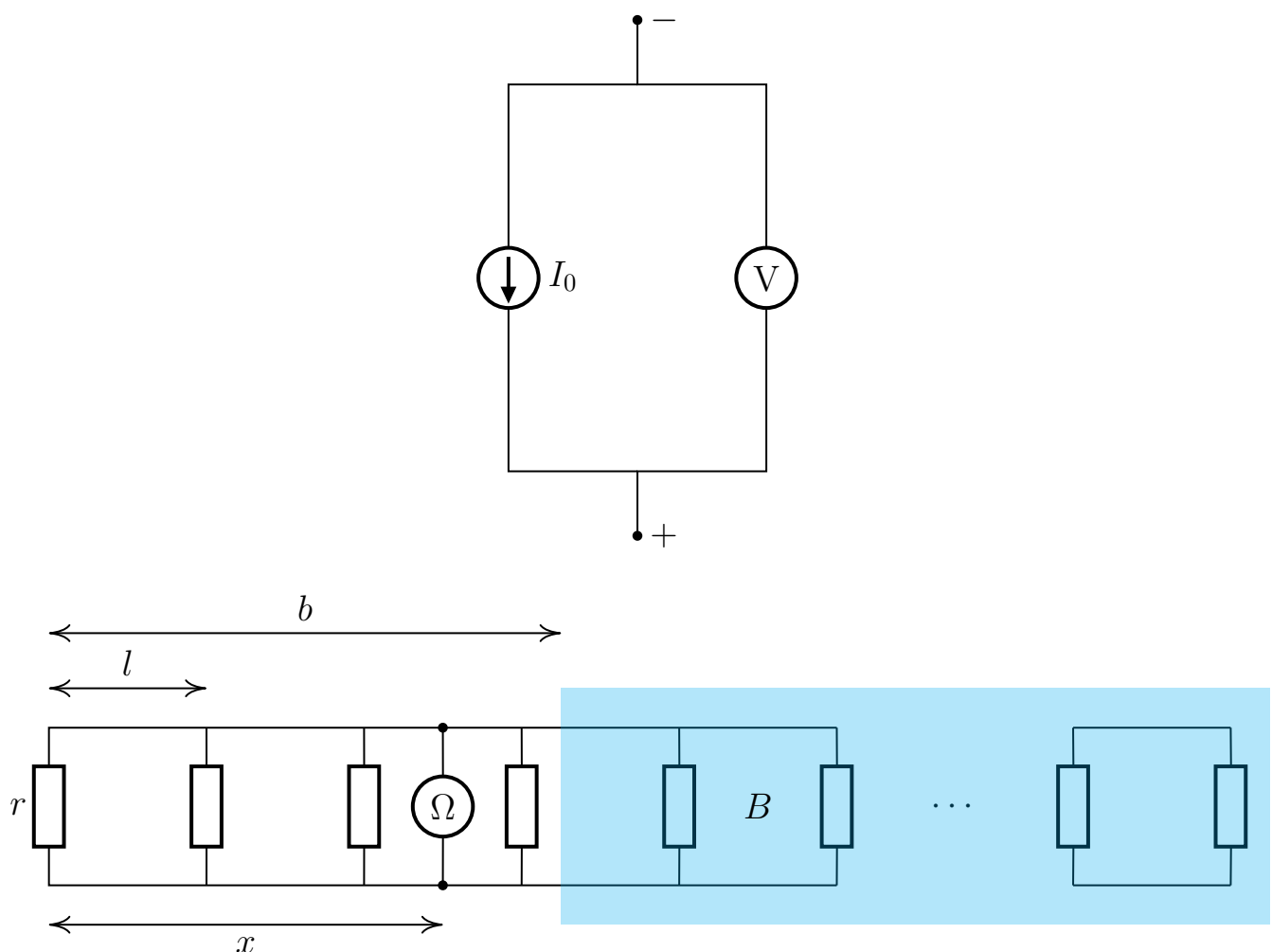
Half Hour Rush E4: I Am A Parallelogram

(5 points)

Being obsessed with parallelograms, Nyan Cat[®] decides to build a parallelogram circuit as shown below. Being traumatised by squares earlier, it builds N identical resistors r in parallel, with adjacent resistors spaced a distance l from each other. Let the origin be at the bottom left corner. Nyan Cat[®] uses an ohmmeter to measure the effective resistance of the circuit and gets a reading of $2.50 \text{ m}\Omega$. As shown below, the ohmmeter consists of an ideal current source I_0 in parallel with (and very close to) an ideal voltmeter. The negative end is connected to the top side of the circuit and the positive end is connected directly below to the bottom side. The ohmmeter reading is the voltmeter reading divided by I_0 .

Nyan Cat[®] then decides to turn on a uniform magnetic field in the blue region $x > b$ which increases linearly with time, $B = \alpha t$. Curiously, now the resistance reading on the ohmmeter R not only changes, but also depends on the position of the two terminals x as shown in the (incomplete) graph of R against $\frac{x}{l}$ on the next page. Find r .

Leave your answer to 3 significant figures in units of $\text{m}\Omega$.





Half Hour Rush X1: Spacing Out

(3 points)

After defeating Thor in [Avengers: Infinity War \(2018\)](#), Thanos uses the Space Stone to transport himself to distant locations in search of the remaining Infinity Stones. He teleports himself and all the air around him to a location extremely far away, leaving behind a perfect vacuum which rapidly collapses.

Thanos teleports a spherical region of air, of radius 2.00 m. The surrounding environment is at a pressure $P = 1.00 \text{ atm}$. Determine the total energy E released (as heat and sound) during the collapse of the vacuum.

Leave your answer to 2 significant figures in units of kJ.

Half Hour Rush X2: Hammer Time

(3 points)

Thor is determined to get revenge on Thanos. He decides to try reforging his broken hammer, Jonathan.

Thor brings a piece of Jonathan close to the Sun, which can be modelled as an infinite black plane at constant temperature $T_0 = 5700\text{ }^\circ\text{C}$. The fragment is then left to reach an equilibrium temperature of T_1 .

Assuming that the piece is an ideal blackbody in the shape of a cube with side length $L = 1\text{ m}$, determine T_1 . Note that there is no air in space.

Leave your answer to 3 significant figures in units of $^\circ\text{C}$.

Half Hour Rush X3: Particularly Strange

(4 points)

Sadly, Thor was unable to reforge his hammer, as the sun was not hot enough. Instead, Thor plans to travel to an alternate timeline to steal a different version of Jonathan. However, while handling a vial of Pym Particles, he accidentally breaks it, causing the particles to fall to the ground with near zero initial velocity. The Pym particles can be modelled as point masses with $m = 10^{-31}$ kg, released at a height $h = 10$ m above ground.

Thor initially expects all the particles to fall straight down. However, upon closer inspection, he realises that the landing positions of the Pym Particles form a distribution due to quantum uncertainty! Find Δr , the minimum standard deviation of the horizontal displacement from the expected landing point.

Assume that gravity causes each particle to accelerate downwards, air resistance is negligible, and ignore any higher-order quantum effects in the vertical axis.

Leave your answer to 2 significant figures in units of m.

Hint: The Heisenberg Uncertainty Principle states that:

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

Where h is Planck's constant and x and p refer to position and momentum in one axis.

Half Hour Rush X4: Forged from Flames

(5 points)

After failing several times, Thor finally decides to seek the help of Eitri, the dwarf, to reforge Jonathan. He travels to Nidavellir, where a giant neutron star is encased by a Dyson Sphere to harness its energy. The Dyson Sphere can be approximated as a thin spherical shell with negligible thickness.

However, Thor finds out that the outer surface of the Dyson sphere has become completely covered with a thick layer of ice, stopping it from working. Thor measures that the net radiative power transferred from the neutron star to the Dyson sphere is 10^{25} W.

The Dyson sphere and the neutron star are concentric spheres, both of which have emissivity $\epsilon = 0.5$. The neutron star surface is at constant temperature $T_1 = 10^6$ K, while the Dyson sphere is kept at a constant temperature $T_2 = 273.15$ K by the surrounding ice, which absorbs excess heat from the sphere via conduction. The region between the neutron star and Dyson sphere is a perfect vacuum.

Given that the inner radius of the Dyson sphere is $r_2 = 10^5$ m, find r_1 , the radius of the neutron star.

Leave your answer to 2 significant figures in km.

Hint: Kirchhoff's Law of Thermal Radiation applies here, so each object's absorptivity is equal to its emissivity.